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CONSISTENT ESTIMATION OF CROSS-CLASSIFIED DOMAINS

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ABSTRACT

Domain estimation has become an important area in survey sampling, however a lot of problems associate with it. One out of many is the lack of consistency between different surveys. The results of one survey do not coincide with the results of another survey done earlier or simultaneously, although the same variable is under study.

We study two methods, AC-calibration (A – auxiliary, C – common) and repeated weighting (RW), for achieving consistency. A short overview of these two methods is given, and we develop formulas for a specified case, for the cross-classified domains.

We assume that there are two sources of information on the study variables (either surveys or registers). The problem is that one source has information on domains formed by certain categorical variable, not considered or not identified in the other source. Instead, this second source has information on domains formed by another categorical variable. We are however interested in domains cross-classified with these categorical variables. A survey is done regarding these new domains, but the domain estimates will probably be inconsistent with marginal information, from earlier surveys. We require that the new domain estimates be consistent with the marginals. To achieve this we apply AC-calibration or repeated weighting.

The formulas of AC-calibration and RW for the cross-classified domains are tested in a simulation study. Simulations were done on a population composed of real data from the Estonian Household Survey.

Key words: Consistent estimation; calibration; AC-calibration; repeated weighting.

1. Introduction

In National Statistics Agencies it is standard practice to provide estimates for certain smaller groups or subpopulations in the population, called domains. The

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main research topics in domain estimation are how to increase accuracy and small area estimation (Estevao and Särndal, 2004).

Today's statistics consumers demand high quality statistics and several National Statistics Agencies have formulated their own definitions to the term "quality in (official) statistics" (Platek and Särndal, 2001). Elvers and Rosén (1999) give five principal dimensions: (1) Contents, (2) Accuracy, (3) Timeliness, (4) Coherence and Comparability, and (5) Availability and Clarity. Each has a number of sub-dimensions.

In modern practice it is quite usual that several surveys are carried out on the same population with some variables common in two or more surveys. Consistency between different surveys is one important indicator of quality, falling under Coherence and Comparability in Elvers and Rosén's quality dimensions. Here the term "consistent" refers not to "randomization consistent" but to "consistent with known aggregates".

In this study we assume that population totals, or totals of larger domains, for certain variables are estimated in one survey, that we call *the reference survey* (RFS). Estimates of the same variables are produced, but in greater detail (totals of domains) in another survey, that we call *the present survey* (PRS). The two surveys are carried out independently. Naturally consumers would assume that these two surveys produce numerically consistent estimates - e.g. domain total estimates in PRS sum up to the population total estimate in RFS. Sadly this does not always hold. Särndal and Traat (2011) treat the inconsistency problem and they propose a new method, the AC-calibration (A – auxiliary variables, C – common variables). Statistics Netherlands has also studied the problem and several articles have been published on repeated weighting (RW). The current study aims to use these two methods to achieve consistency in a more specific case.

We assume that there are two sources of information on the study variables (either surveys or registers). But the problem is that one source has information on domains formed by a certain categorical variable, not considered or not identified in the other source. Instead, this second source has information on domains formed by another categorical variable. We are however interested in domains formed by the cross-classification of the previous categorical variables. A survey is done regarding these new domains, but the domain estimates will probably be inconsistent with earlier information. So we take the earlier information, insert this as marginals to our 2-way table, and demand that the new found domain estimates are consistent with the marginals. To achieve this we apply AC-calibration or repeated weighting and test the new formulas in a simulation study.

First the necessary notations and terminology is given, then the concept of consistency is discussed, followed by an overview of the AC-calibration and the RW method. Formulas for our special case are derived in Section 4 and tested in a simulation study in Section 5.

2. Preliminaries

Let $U = (1, 2, \dots, N)$ denote a finite population of N units that is divided into D non-overlapping domains U_d, $d \in \mathfrak{D} = \{1, 2, \dots, D\}$. Let a random vector (design vector) $\mathbf{I} = (I_1, I_2, \dots, I_N)$ describe the sampling process on U and I_i is the sample inclusion indicator for unit $i \in U$. The probability sampling design generates for element i a known inclusion probability, $E(I_i) = \pi_i > 0$, and a corresponding sampling design weight $a_i = 1/\pi_i$. The column vector $\mathbf{y}_i: m \times 1$ of the study variables $\mathbf{y}$ is recorded for all $i \in s$ (complete response). Our objective is to estimate population totals $\mathbf{Y} = \sum_U \mathbf{y}_i$ and domain totals $\mathbf{Y}_d = \sum_{U_d} \mathbf{y}_i$.

The basic design unbiased estimator of $\mathbf{Y}$ is $\widehat{\mathbf{Y}} = \sum_s a_i \mathbf{y}_i$, the Horwitz-Thopmson (HT) estimator. For domains it can be written $\widehat{\mathbf{Y}}_d = \sum_{s_d} a_i \mathbf{y}_i = \sum_s a_i \delta_i^d \mathbf{y}_i$, $d \in \mathfrak{D}$, where

$$\delta_i^d := \begin{cases} 1, & i \in U_d, \\ 0, & \text{otherwise.} \end{cases}$$

It is, however, inefficient when strong auxiliary information is available for use at the estimation stage.

Auxiliary information vector $\mathbf{x}_i: p \times 1$ can be found for every element $i \in s$ (or every $i \in U$ if it is compiled from comprehensive registers). It is assumed that the auxiliary variable totals $\mathbf{X} = \sum_U \mathbf{x}_i$ are known.

One way of using the auxiliary information is by calibrating. There are several approaches to calibration, but the two main methods are the distance minimization, used by Deville and Särndal (1992), and the instrument vector method considered in Estevao and Särndal (2000) and Kott (2006). A good overview of both methods is given by Särndal (2007), and he shows that the distance minimization method is a special case of the instrument vector method (also shown by Estevao and Särndal (2000) and Kott (2006)). We will use the instrument vector approach, since it is a more general method.

We find the new weights w_i, such that they satisfy the calibration equations

$$\sum_U \mathbf{x}_i = \sum_s w_i \mathbf{x}_i = \mathbf{X}. \tag{2.1}$$

The new weights are in the form

$$w_i = a_i(1 + \boldsymbol{\lambda}' \mathbf{z}_i) \tag{2.2}$$

where $\boldsymbol{\lambda}' = (\mathbf{X} - \widehat{\mathbf{X}})' \mathbf{M}^{-1}$, $\mathbf{M} = \sum_s a_i \mathbf{z}_i \mathbf{x}_i'$, $\widehat{\mathbf{X}} = \sum_s a_i \mathbf{x}_i$ and vector $\mathbf{z}_i: p \times 1$ is called an instrument vector and is chosen freely.

3. Consistent estimation

Let us assume that the population totals $\mathbf{Y}$ are known, either from registers or RFS where the population was not divided into desired domains. It is natural to demand that the domain totals in the PRS sum up to the corresponding known totals.

For this, however, we have to make the following assumptions:

- data from registers and surveys is taken on the same time moment;
- all sources consider the same population;
- all common variables have the same definition.

If any of these assumptions is not met, then we are trying to achieve numerical consistency with variables that do not have to be consistent. Fulfilling these assumptions can become an obstacle. The first assumption is satisfied if the RFS data is taken from a register which is constantly updated in real-time, RFS and PRS are carried out simultaneously or the time between them is deemed acceptable. The satisfaction of the last two assumptions can be ensured in the planning phase of the PRS - using the same population and defining variables as they are in RFS.

The two methods discussed below can be used to achieve consistency with common variables.

AC-calibration

In this section we give a short overview of the AC-calibration method developed by Särndal and Traat (2011). The authors named it after the two sources of information used to calibrate new weights - auxiliary (A) and common (C) variables (further the terms A-variables, A-information and C-variables, C-information are used).

AC-calibration is basically standard calibration where the auxiliary variable vector is extended with C-variables. For each element $i \in s$, we can find vectors $\mathbf{x}_i: p \times 1$ and $\mathbf{y}_i: m \times 1$. We define new $(p + m)$ dimensional vectors:

$$\begin{pmatrix}\mathbf{x}_i\\ \mathbf{y}_i\end{pmatrix}, i \in s, \qquad \begin{pmatrix}\mathbf{X}\\ \mathbf{Y}_0\end{pmatrix}, \qquad \begin{pmatrix}\widehat{\mathbf{X}}\\ \widehat{\mathbf{Y}}\end{pmatrix}, \tag{3.3}$$

where $\mathbf{X} = \sum_U \mathbf{x}_i$, $\mathbf{Y}_0$ is the vector of known C-information totals from RFS and $\widehat{\mathbf{X}} = \sum_s a_i \mathbf{x}_i$ and $\widehat{\mathbf{Y}} = \sum_s a_i \mathbf{y}_i$ are HT estimates found from PRS. The calibration weights are then

$$w_{ACi} = a_i(1 + \boldsymbol{\lambda}'_{AC}\mathbf{z}_{ACi}) \tag{3.4}$$

where $\mathbf{z}_{ACi}$ is the instrument vector and

$$\boldsymbol{\lambda}_{AC}^{'} = \begin{pmatrix} \mathbf{X} - \widehat{\mathbf{X}} \\ \mathbf{Y}_0 - \widehat{\mathbf{Y}} \end{pmatrix}^{'} \mathbf{M}^{-1}, \qquad \mathbf{M} = \sum_s a_i \mathbf{z}_{ACi} \begin{pmatrix} \mathbf{x}_i \\ \mathbf{y}_i \end{pmatrix}^{'}.$$

The new weights are consistent with A-information as well as C-information, meaning

$$\sum_s w_{ACi} \begin{pmatrix} \mathbf{x}_i \\ \mathbf{y}_i \end{pmatrix} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y}_0 \end{pmatrix}.$$

The inverse of the matrix $\mathbf{M}$ exists if $\mathbf{x}$ and $\mathbf{y}$ are linearly independent.

Repeated weighting

The repeated weighting method has been developed in Statistics Netherlands. The following is a brief overview of the method, more can be read from Kroese and Renssen (1999), Houbiers (2004), Knottnerus and Van Duin (2006).

The idea of calibration is that we change the initial weights a_i so that they satisfy the calibration equations (2.1). Repeated weighting is basically calibrating the already calibrated weights w_i once again, but in the second step the C-information $\mathbf{Y}_0$ is used in the calibration equations.

With RW we get weights

$$w_{RWi} = w_i(1 + \boldsymbol{\lambda}_{RW}^{'} \mathbf{z}_{RWi}) \tag{3.5}$$

where $\mathbf{z}_{RWi}$ is the instrument vector and

$$\boldsymbol{\lambda}_{RW}^{'} = \left(\mathbf{Y}_0 - \widehat{\mathbf{Y}}^{CAL}\right)^{'} \mathbf{M}^{-1}, \qquad \mathbf{M} = \sum_s w_i \mathbf{z}_{RWi} \mathbf{y}_i^{'}, \qquad \widehat{\mathbf{Y}}^{CAL} = \sum_s w_i\, \mathbf{y}_i.$$

The new weights are consistent only with C-information, meaning

$$\sum_s w_{RWi} \mathbf{y}_i = \mathbf{Y}_0.$$

4. Cross-classfied domain estimation

In the previous section we assumed that total of the C-variables were known from RFS and demanded consistency between them and domain estimates in the PRS. But what if we have access to more detailed information - domain level totals? Then by using them we can achieve more accurate results. Let us assume that there are two or more sources of common variable(s), but all had different categorical variables for domain identification. Let these variables be measured in the new, present survey. We are interested in the domains obtained by crossing these categorical variables. In order to use the known C-information we define cross-classified domains, insert the known information as marginals and demand that the row and column sums are consistent with them.

5. Cross-classified domains by two variables

From this point forward we assume that our population is divided into domains by two categorical variables B and D. Let variable B have r levels and variable D c levels, resulting in a $r \times c$ 2-way table. Naturally one could want domains by crossing three (or more) variables, then assuming that two of them come from the same source, we can redefine a $r \times c \times d$ table into a $(rc) \times d$ table. If all three (or more) categorical domain indication variables come from different sources, then 2-way tables can be applied as many times as needed. If a 3-way table still has to be used then a large sample size is also needed, since the number of cells in the table increases by $r \times c$ with each level of the third categorical variable.

We denote the cross-classified domains as U_{kl} ($k = \{1, 2, \dots, r\}; l = \{1, 2, \dots, c\}$) so that $\bigcup_{k=1}^{r} \bigcup_{l=1}^{c} U_{kl} = U$. Row marginals are denoted $U_{k\cdot} := \bigcup_{l=1}^{c} U_{kl}$ and column marginals $U_{\cdot l} := \bigcup_{k=1}^{r} U_{kl}$. In order to estimate the domains and marginals we have to define a row indicator:

$$\delta_i^k = \begin{cases} 1, & i \in U_{k\cdot}, \\ 0, & \text{otherwise} \end{cases} \tag{5.6}$$

and a column indicator

$$\gamma_i^l = \begin{cases} 1, & i \in U_{\cdot l}, \\ 0, & \text{otherwise.} \end{cases} \tag{5.7}$$

By combining indicators into vectors $\boldsymbol{\delta}_i = (\delta_i^1, \delta_i^2, \dots, \delta_i^r)'$ and $\boldsymbol{\gamma}_i = (\gamma_i^1, \gamma_i^2, \dots, \gamma_i^c)'$ we can obtain an indicator for the 2-way table:

$$\mathbb{I}_i = \boldsymbol{\delta}_i \boldsymbol{\gamma}_i^{'} = \begin{pmatrix} \delta_i^1 \gamma_i^1 & \cdots & \delta_i^1 \gamma_i^c \\ \vdots & \ddots & \vdots \\ \delta_i^r \gamma_i^1 & \cdots & \delta_i^r \gamma_i^c \end{pmatrix}.$$

The vectors $\boldsymbol{\delta}_i$ and $\boldsymbol{\gamma}_i$, and the matrix $\mathbb{I}_i$ contain only one „1“, all other elements are zeros. Using these indicators one can easily find the HT estimates for all $\mathbf{y}_i$. But instead of a normal multiplication, we have to use the Kronecker product($\otimes$)

$$\hat{\mathbf{Y}} = \sum_s a_i \mathbb{I}_i \otimes \mathbf{y}_i = \sum_s a_i (\boldsymbol{\delta}_i \boldsymbol{\gamma}_i') \otimes \mathbf{y}_i. \tag{5.8}$$

As a result we get a $(rm) \times c$ matrix, where we have (rc) m-dimensional vectors, each being a vector of m total estimates of one domain.

To get estimates for marginals we can use the earlier indicators (5.6) and (5.7):

$$\hat{\mathbf{Y}}_{k+} = \sum_U a_i \delta_i^k \mathbf{y}_i, \;\; k = \{1, 2, \dots, r\}, \tag{5.9}$$

$$\hat{\mathbf{Y}}_{+l} = \sum_U a_i \gamma_i^l \mathbf{y}_i, \;\; l = \{1, 2, \dots, c\}. \tag{5.10}$$

Already Deming and Stephan (1940) adjusted multi-dimensional tables with known marginals to achieve consistency, but they achieved it iteratively. With the formulas in the next section we can achieve consistency within one calculation step. They did however develop formulas for a 3-way table, not discussed here. Deville, Särndal and Sautory (1993) dealt with calibration on marginal sums, but they used marginals as A-information and with post-stratification in mind i.e. the known marginals were the sizes of margin domains. Knottnerus (2003, Chapter 12.9) touches on the issue at hand in the chapter on the General Restriction (GR) estimator, but he uses the covariance matrix of initial estimators to achieve consistency. Knottnerus and van Duin (2006) derive RW method formulas for a contingency table where marginal sums are known, but they use a GREG estimator and consider a one-dimensional case with domain sizes N_{kl}.

AC-calibration in caseS of cross-classified domains

In order to get AC-calibration estimators for our special case we first have to operate with the marginals by putting them into a single vector:

$$\mathbf{Y}_{0marg} := (\mathbf{Y}'_{1+}, \mathbf{Y}'_{2+}, \dots, \mathbf{Y}'_{r+}, \mathbf{Y}'_{+1}, \mathbf{Y}'_{+2}, \dots, \mathbf{Y}'_{+c})'.$$

The same has to be done with initial estimates:

$$\widehat{\mathbf{Y}}_{marg} := \left(\widehat{\mathbf{Y}}'_{1+}, \widehat{\mathbf{Y}}'_{2+}, \dots, \widehat{\mathbf{Y}}'_{r+}, \widehat{\mathbf{Y}}'_{+1}, \widehat{\mathbf{Y}}'_{+2}, \dots, \widehat{\mathbf{Y}}'_{+c}\right)'$$

Additionally we will need a combination of the indicator vectors $\boldsymbol{\delta}_i$ and $\boldsymbol{\gamma}_i$:

$$\boldsymbol{\Gamma}_i := (\boldsymbol{\delta}'_i \boldsymbol{\gamma}'_i)' = (\delta_i^1, \delta_i^2, \dots, \delta_i^r, \gamma_i^1, \gamma_i^2, \dots, \gamma_i^c)'.$$

From (3.3) we get

$$\begin{pmatrix} \mathbf{x}_i \\ \boldsymbol{\Gamma}_\mathrm{i} \otimes \mathbf{y}_i \end{pmatrix}, i \in s, \qquad \begin{pmatrix} \mathbf{X} \\ \mathbf{Y}_{0marg} \end{pmatrix}, \qquad \begin{pmatrix} \widehat{\mathbf{X}} \\ \widehat{\mathbf{Y}}_{marg} \end{pmatrix}.$$

Similarly to (3.4) the new weights are then

$$w_{ACi} = a_i(1 + \boldsymbol{\lambda}'_{AC} \mathbf{z}_{ACi}) \tag{5.11}$$

where $\mathbf{z}_{ACi}$ is the instrument vector and

$$\boldsymbol{\lambda}'_{AC} = \begin{pmatrix} \mathbf{X} - \widehat{\mathbf{X}} \\ \mathbf{Y}_{0marg} - \widehat{\boldsymbol{Y}}_{marg} \end{pmatrix}' \mathbf{M}^{+}, \qquad \mathbf{M} = \sum_s a_i \mathbf{z}_{ACi} \begin{pmatrix} \mathbf{x}_i \\ \boldsymbol{\Gamma}_\mathrm{i} \otimes \mathbf{y}_i \end{pmatrix}'.$$

Since $\mathbf{M}$ is singular because of the definition of $\boldsymbol{\Gamma}_i$, we have to use Moore-Penrose generalized inverse of the matrix $\mathbf{M}$ (denoted by $\mathbf{M}^{+}$). Deville, Särndal and Sautory (1993) suggest to leave one element out in the vector $\boldsymbol{\Gamma}_i$ when finding the matrix $\mathbf{M}$, e.g. γ_i^c (vectors $\mathbf{Y}_{0marg}$ and $\widehat{\mathbf{Y}}_{marg}$ also have to be shortened correspondingly). Then the linear dependency in matrix $\mathbf{M}$ is gone and $\mathbf{M}^{-1}$ can be found, the consistency remains. Since $\mathbf{M}^{+}$ is approximated in most of the cases the author suggests to use $\mathbf{M}^{-1}$. During simulations the use of $\mathbf{M}^{+}$ produced inconsistent results, meaning that the domain totals did not sum up to the marginal totals, although, the differences were very near to zero. When applying $\mathbf{M}^{-1}$, this disturbing fact disappeared.

Finally from (5.8) and (5.11) we get

$$\hat{\mathbf{Y}}^{AC} = \sum_s w_{ACi}\mathbb{I}_i \otimes \mathbf{y}_i.$$

When finding new weights w_{ACi} we only use those study variables $\mathbf{y}_i$ with which we wish to achieve consistency, but these weights can be used to calculate our 2-way table domain total estimates for all other variables of interest.

RW in caseS of cross-classified domains

With RW we first found weights that were calibrated on A-information. Since no assumptions have been made on auxiliary variables, then A-calibrated estimates weights w_i can be found using (2.2). Replacing weights a_i with w_i in (5.9) and (5.10) we get the A-calibrated estimates of the marginal sums $\hat{\mathbf{Y}}_{k+}^{CAL}$ and $\hat{\mathbf{Y}}_{+l}^{CAL}$, $k = \{1, 2, \dots, r\}; l = \{1, 2, \dots, c\}$. We define a new vector

$$\hat{\mathbf{Y}}_{marg}^{CAL} := \left(\hat{\mathbf{Y}}_{1+}^{CAL\prime}, \hat{\mathbf{Y}}_{2+}^{CAL\prime}, \dots, \hat{\mathbf{Y}}_{r+}^{CAL\prime}, \hat{\mathbf{Y}}_{+1}^{CAL\prime}, \hat{\mathbf{Y}}_{+2}^{CAL\prime}, \dots, \hat{\mathbf{Y}}_{+c}^{CAL\prime}\right)'.$$

The definitions of $\mathbf{Y}_{0marg}$ and $\mathbf{\Gamma}_i$ remain the same. The auxiliary information vector takes the form $(\mathbf{\Gamma}_i \otimes \mathbf{y}_i), i \in s$ and by recalibrating, so that $\sum_s w_{RWi}\mathbf{\Gamma}_i \otimes \mathbf{y}_i = \mathbf{Y}_{0marg}$, from (3.5) we get new weights

$$w_{RWi} = w_i(1 + \boldsymbol{\lambda}_{RW}'\mathbf{z}_{RWi}) \tag{5.12}$$

where

$$\boldsymbol{\lambda}_{RW}' = \left(\mathbf{Y}_0 - \hat{\mathbf{Y}}_{marg}^{CAL}\right)'\mathbf{M}^+, \qquad \mathbf{M} = \sum_s w_i \mathbf{z}_{RWi}(\mathbf{\Gamma}_i \otimes \mathbf{y}_i)'.$$

The argumentation around $\mathbf{M}^+$ is the same as in the last subsection.

Finally we recieve RW estimates for all domain totals

$$\hat{\mathbf{Y}}^{RW} = \sum_s w_{RWi}\mathbb{I}_i \otimes \mathbf{y}_i.$$

6. Simulations

In order to test the formulas developed in the previous section and to compare the two methods discussed above, a simulation study was carried out in the SAS IML environment. A population was composed on real data from the Estonian Household Survey, with 17 540 households.

Simulation set-up

The population was divided into 12 cross-classified domains according to gender and economic activity of the head of the household (the variables were combined to form one dimension), and education level of the head of the household (second dimension). The resulting domains and their sizes are presented in Table 1, immediately one can notice that sizes differ quite considerably. The largest being 23,3% of the population, and three domains contain only about 2% of households. Since we will use simple random sampling to produce samples, we have an opportunity to study the behavior of our estimators in cases of small areas.

Table 1. Domain sizes and percentage of the population total

Head of the household		Education level							
Activity	Gender	Primary		Secondary		Higher		TOTAL	
Employed	Male	712	4,1%	4 094	23,3%	1 840	10,5%	6 646	37,9%
	Female	364	2,1%	2 393	13,6%	2 015	11,5%	4 772	27,2%
Unemployed	Male	923	2,0%	1 172	6,7%	370	2,1%	2 465	14,1%
	Female	1 559	8,9%	1 478	8,4%	620	3,5%	3 657	20,8%
TOTAL		3 558	20,3%	9 137	52,1%	4 845	27,6%	17 540	100%

(1) Household net income, (2) household expenditure and (3) domain membership were taken as study variables. Auxiliary variable vector consisted of (1) income from benefits, (2) number of household members and (3) number of children in the household under the age of 16.

In the simulation study we will explore two important cases:

a) The marginals $\mathbf{Y}_{0marg}$ are known;
b) The marginals $\mathbf{Y}_{0marg}$ are estimated in a previous survey (RFS).

To emphasize the flexibility of the calibration technique and previously developed formulas, we will assume that only marginals for net income and domain size are known (estimated for the second case) and later use the obtained weights to calculate domain estimates for household expenditure.

As said, the instrument vectors in formulas (2.2), (5.11) and (5.12) can be chosen freely (as long as the dimension coincides with the auxiliary vector), so they were fixed to $\mathbf{z}_i = \mathbf{x}_i$, $\mathbf{z}_{ACi} = (\mathbf{x}_i', (\mathbf{\Gamma}_i \otimes \mathbf{y}_i)')'$ and $\mathbf{z}_{RWi} = (\mathbf{\Gamma}_i \otimes \mathbf{y}_i)'$, which is a standard choice (Särndal, 2007). The auxiliary variable vector $\mathbf{x}_i$, study variable vector y_i and the matrix $\mathbf{\Gamma}_i$ are constructed using the variables mentioned above, with the exeption that household expenditure is not used in the calibration step.

Performance of the AC and RW estimators was assessed by relative root mean square error (RRMSE) and relative bias (RB).

$$RRMSE(\hat{Y}_d) = \frac{\sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\hat{Y}_d^{(m)} - Y_d\right)^2}}{Y_d}, \qquad RB(\hat{Y}_d) = \frac{\frac{1}{M}\sum_{m=1}^{M}\hat{Y}_d^{(m)} - Y_d}{Y_d}$$

where M is the number of sample repetitions, $\hat{Y}_d^{(m)}$ is the domain total estimate of the study variable in the m-th sample and Y_d is the actual domain total. To set a baseline for comparison and assessing performance HT estimates were also found, accompanied by RRMSE and RB.

Simulations and results

In case (a) 1000 independent samples were drawn with simple random sampling (SRS), each sample consisted of 1000 households (5,7% of the population) from which domain estimates with HT, AC and RW methods were found. For case (b) marginals were first estimated with HT estimator from a sample of 2000 households (SRS sample) and then the domain estimates were calculated using weights received from calibrating on the estimated marginals (and auxiliary variables), this was repeated 1000 times. Finally the performance indicators were calculated over the simulations for both cases. Resulting RRMSE and RB can be seen in Table and Table .

Consistency was achieved using AC-calibration and RW in both cases, when marginal sums were known, and when they were first estimated. From Table it can be seen that both the RW and the AC methods have lower RRMSE than the HT estimator. Even when the marginal sums were estimated, AC and RW outperform the HT estimator in all domains. The AC estimator has relatively the same RRMSE as RW in all domains and in all cases. When we compare the performance of the estimators in cases (a) and (b), then it is clear that using actual information gives better results.

In Table we see that using AC and RW typically increases bias compared to the HT estimator, but the bias is still close to 0. This is because calibration is approximately unbiased. HT showed worse results only where the variance within the domain was small. In the case of known marginals the bias is typically slightly lower than with estimated marginals, in some cases it is vice versa, but the differences are small.

Table 2. RRMSE of net income, domain size and expenditure over simulations

		Net income					Domain size					Expenditure				
		$\mathbf{Y}_{0marg}$			$\hat{\mathbf{Y}}_{0marg}$		$\mathbf{Y}_{0marg}$			$\hat{\mathbf{Y}}_{0marg}$		$\mathbf{Y}_{0marg}$			$\hat{\mathbf{Y}}_{0marg}$	
D	n_d	HT	AC	RW	AC	RW	HT	AC	RW	AC	RW	HT	AC	RW	AC	RW
1	712	,20	,12	,12	,17	,17	,15	,12	,12	,14	,14	,21	,15	,15	,19	,19
2	4 094	,08	,03	,03	,06	,06	,06	,03	,03	,04	,04	,08	,06	,06	,07	,07
3	1 840	,14	,06	,06	,11	,11	,09	,06	,06	,08	,08	,13	,09	,09	,12	,12
4	364	,28	,26	,26	,27	,27	,21	,19	,19	,20	,20	,32	,31	,30	,31	,31
5	2 393	,11	,08	,08	,10	,10	,08	,05	,05	,07	,07	,11	,09	,09	,10	,10
6	2 015	,13	,07	,07	,10	,10	,08	,05	,05	,07	,07	,13	,09	,09	,11	,11
7	923	,17	,14	,14	,16	,16	,14	,10	,10	,12	,12	,19	,16	,16	,18	,18
8	1 172	,17	,11	,11	,14	,14	,12	,08	,08	,10	,10	,21	,20	,20	,22	,21
9	370	,32	,26	,26	,29	,29	,22	,19	,19	,20	,20	,30	,28	,28	,30	,30
10	1 559	,13	,10	,10	,12	,12	,10	,06	,06	,08	,08	,14	,12	,12	,13	,13
11	1 478	,14	,10	,10	,12	,12	,10	,07	,07	,09	,09	,15	,12	,12	,13	,13
12	620	,24	,19	,19	,21	,21	,16	,13	,13	,15	,15	,27	,25	,25	,26	,26
U	17 540	,04	0	0	,02	,02	0	0	0	,01	,01	,04	,03	,03	,03	,03

Table 3. RB of net income, domain size and expenditure over simulations ($\times 10^{-2}$)

		Net income					Domain size					Expenditure				
			$\mathbf{Y}_{0marg}$		$\hat{\mathbf{Y}}_{0marg}$			$\mathbf{Y}_{0marg}$		$\hat{\mathbf{Y}}_{0marg}$			$\mathbf{Y}_{0marg}$		$\hat{\mathbf{Y}}_{0marg}$	
D	n_d	HT	AC	RW	AC	RW	HT	AC	RW	AC	RW	HT	AC	RW	AC	RW
1	712	-,17	-1,56	-1,63	-1,78	-1,86	-,32	-,31	-,32	-,27	-,29	-,27	-1,14	-1,14	-1,25	-1,28
2	4 094	,06	,30	,30	,31	,32	-,01	,09	,10	,10	,11	-,18	,12	,15	,12	,15
3	1 840	,02	-,10	-,10	,15	,15	-,13	-,08	-,09	,24	,23	-,21	,10	,16	,38	,46
4	364	-,19	-,10	,05	-,43	-,26	-,61	-,74	-,66	-,82	-,72	-,37	-,17	-,03	-,54	-,31
5	2 393	-,25	-,72	-,70	-,85	-,84	,06	-,13	-,13	-,14	-,14	-,21	-,32	-,29	-,44	-,41
6	2 015	-,08	,64	,61	,85	,82	,12	,29	,27	,55	,54	-,02	,82	,78	1,02	,99
7	923	-,71	1,81	1,86	1,65	1,70	-,42	,26	,27	,16	,17	-,84	1,59	1,64	1,38	1,42
8	1 172	-,50	-,62	-,74	-,88	-,98	-,13	-,18	-,21	-,44	-,46	,39	1,89	1,77	1,56	1,45
9	370	,50	-1,74	-1,51	-1,36	-1,19	,50	-,08	,02	,08	,13	,12	,88	,99	,94	1,02
10	1 559	-,44	,56	,54	,44	,42	-,24	,16	,14	,21	,20	-,52	,45	,46	,39	,41
11	1 478	1,37	,62	,64	,45	,46	,59	,10	,11	,05	,05	1,56	1,04	1,04	,87	,84
12	620	-,07	-2,41	-2,42	-2,65	-2,64	,34	-,64	-,61	-,42	-,40	,59	-,46	-,41	-,28	-,19
U	17 540	-,03	0	0	,02	,02	0	0	0	,05	,05	-,07	,28	,30	,30	,33

In terms of comparing AC *versus* RW they both give almost equal estimates. Neither estimator is superior to the other over all domains. The same conclusion was reached by Särndal and Traat (2011).

In the simulations AC and RW outperformed the HT estimator, but all of the mentioned estimators performed not so well in smaller domains, hence they are not appropriate for small area estimation.

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REFERENCES

Deming, W.E., Stephan, F.F., 1940. On a Least Squares Adjustment of a Sampled Frequency Table When the Expected marginal Totals are Known. *The Annals of Mathematical Statistics*, 11, pp.427-444.

Deville, J-C., Särndal, C.-E., 1992. Calibration Esitmators in Survey Sampling. *Journal of the American Statistical Association*, 87, pp.376-382.

Deville, J.-C., Särndal, C.-E., Sautory, O., 1993. Generalized Procedures in Survey Sampling. *Journal of the American Statistical Association*, 88, pp.1013-1020.

Elvers, E., Rosén, B., 1999. Quality Concept for Official Statistics. In: S. Kotz, C.B. Read, and D.L. Banks eds. *Encyclopedia of Statistical Sciences*, *Update Volume 3.* New York: Wiley, pp.621-629.

Estevao, V.M., Särndal, C.-E., 2000. A Functional Approach to Calibration. *Journal of Official Statistics*, 16, pp.379-399.

Estevao, V.M., Särndal, C.-E., 2004. Borrowing Strength Is Not the Best Technique Within a Wide Range of Design-Consistent Domain Estimators. *Journal of Official Statistics*, 20, pp.645-669.

Houbiers, M., 2004. Towards a Social Statistical Database and Unified Estimates at Statistics Netherlands. *Journal of Official Statistics*, 20, pp.55-75.

Knottnerus, P., 2003. *Sample Survey Theory. Some Pythagorean Perspectives*. New York: Springer

Knottnerus, P., van Duin, C., 2006. Variances in Repeated Weighting with an Application to the Dutch Labour Force Survey. *Journal of Official Statistics*, 22, pp.565-584.

Kott, P.S., 2006. Using calibration weighting to adjust for nonresponse and coverage errors. *Survey Methodology*, 32, pp.133-142.

Kroese, A.H., Renssen, R.H., 1999. Weighting and Imputation at Statistics Netherlands. *Proceedings of IASS Satellite Conference on Small Area Estimation*, Riga:Latvia, pp.109-120.

Platek, R., Särndal, C.-E., 2001. Can a Statistician Deliver?. *Journal of Official Statistics*, 17, pp.1-20.

Särndal, C.-E., 2007. The Calibration Approach in Survey Theory and Practice. *Survey Methodology*, 33, pp.99-119.

Särndal, C.-E., Traat, I., 2011. Domain Estimators Calibrated on Information from Another Survey. *Acta et Commentationes Universitatis Tartuensis de Mathematica*, vol. 15 (to appear).