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# CUMULATIVE SUM CONTROL CHARTS FOR TRUNCATED NORMAL DISTRIBUTION UNDER MEASUREMENT ERROR

**R. Sankle[1], J.R. Singh[2], I.K. Mangal[3]**

## ABSTRACT

In the present paper Cumulative Sum Control Chart (CSCC) for the truncated normal distribution under measurement error (r) is discussed. The sensitivity of the parameters of the V-Mask and the Average Run Length (ARL) is studied through numerical evaluation for different values of r.

**Key words:** Truncated Normal Distribution, Measurement Error, ARL and CSCC.

## 1. Introduction

The purpose of this paper is to show how truncated data affect the field performance of controlling manufacturing process. Failure to account for truncation can lead to biased inferences. Cumulative Sum (CUSUM) Control Charts are widely used instead of standard Shewhart charts when detection of small changes in a process parameter is important. Patel and Gajjar (1994) provide references on research performed on CUSUM charts. Errors of measurements are the differences between observed values recorded under "identical" conditions and some fixed true values. Hunter (1986) showed that finally all measurement should be traceable to nation standard the most important area for studying the effect of measurement error and misclassification in the sampling inspection plan and control charts.

Most of the standard statistical tests are derived on the assumption that the sample is combined from a "complete" population. But this assumption appears to be an unrealistic one. Truncations in the parents distributions at one or both the

[1] School of Studies in Statistics, Vikram University, Ujjain (M.P.), India. E-mail: rajshreesankle@gmail.com.
[2] School of Studies in Statistics, Vikram University, Ujjain (M.P.), India. E-mail: jrsinghstat@gmail.com.
[3] Madhav Science College, Ujjain (M.P.), India.

tails may have considerable effect on the CUCUM control charts. Woodroofe (1985) gave a comprehensive discussion of the exact estimation theory as well as consistency and asymptotic normality of the product limit estimator. Woodroofe also surveyed the history and application of this theory within astronomy but did not put it into the context of survival analysis. This was done by Wang et al. (1986) and by Keiding and Gill (1987) who went on to show how the exact and asymptotic properties of the estimators may be obtained as corollaries from statistical theory of counting processes and Markov processes, and Nelson (1990) stresses the distinction between truncation and censoring. Chang (1990) and Schneider (1986) contain some basic results concerning the properties of the truncated normal distribution.

Johnson and Leone (1962) considered mathematical procedure for construction of CUSUM Control Chart for Poisson variable using the relationship between Wald's Sequential Probability Ratio Test (SPRT) and CUSUM on the assumption that the probability of the second kind of error is small. They used this relationship to construct CUSUM charts for the mean and standard deviation of a normal distribution. Yeh et al. (2004) gave a unified CUSUM charts for monitoring process mean and variability. Cox (2009) studied control charts for monitoring observations from a truncated normal distribution. Nenes and Tagaras (2010) evaluated the properties of a CUSUM chart designed for monitoring the process mean in short production runs. Grigg and Spiegelhalter (2008) developed an empirical approximation to the null steady-state distribution of CUSUM statistics. Ryu et al. (2010) used ARL-based performance measure and proposed a method to optimally design a CUSUM chart based on expected weighted run length.

In this paper we have constructed CUSUM chart for mean under truncated normal distribution and measurement error. For different truncation points and different sizes of measurement error tables have been prepared for the average run length, lead distance and the angle of mask.

## 2. Determination of mask parameters under measurement error

Assuming that the true measurement x and the random error of measurement e are additive, we can write the observed measurement X as

$$X = x + e \tag{2.1}$$

where x and e are independent. The constants $\theta$ (unknown) and $\sigma_p$ (known) are the mean and the standard deviation of the true quality measurement x and e has the normal distribution, N (0, $\sigma_e^2$). The correlation coefficient $\rho$ between the true and the observed measurement can be written as

$$\rho = \frac{\sigma_p}{\sigma_X} \tag{2.2}$$

where

$\sigma_X = \frac{\sigma_p}{\rho} = \acute{\sigma}$ (say) is the standard deviation of X,

and the size of the measurement error

$$r = \frac{\sigma_p}{\sigma_e} \tag{2.3}$$

After some mathematical manipulation Singh (1984) showed that

$$\rho = \frac{r}{\sqrt{1+r^2}} \tag{2.4}$$

Let x be the true value of the variable which is distributed as

$$f(X, \theta, \sigma_p) = \frac{c(\theta, \sigma_p)}{\sigma_p\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{x-\theta}{\sigma_p}\right)^2\right] \; ; as\ a \le x \le b$$
$$= 0 \qquad ;\ otherwise \tag{2.5}$$

where $\quad c(\theta, \sigma_p) = \frac{1}{\Phi\left(\frac{b-\theta}{\sigma_p}\right) - \Phi\left(\frac{a-\theta}{\sigma_p}\right)}$ (2.6)

and a and b are the points of truncation with

$$\Phi(t) = \int_{-\infty}^{t} (2\pi)^{-\frac{1}{2}} exp\left(-\frac{1}{2}y^2\right) dy$$

If $x_1, x_2, x_3, \dots, x_m$ are m independent random variables whose probability density function (p.d.f.) is given by (2.5). The likelihood ratio of the hypothesis $H_0: \theta = \theta_0$ against the alternative hypothesis $H_1: \theta = \theta_0 + \delta\sigma_p = \theta_1 (\delta > 0)$ is given by

$$\frac{f(x_1, x_2, \dots, x_m \mid \theta_1, \sigma_p)}{f(x_1, x_2, \dots, x_m \mid \theta_0, \sigma_p)} = R^m exp\left[-\frac{1}{2\sigma_p^2}\sum_{i=1}^{m}\{-2\delta\sigma_p(x_i - \theta_0) + \delta^2\sigma_p^2\}\right], \tag{2.7}$$

where

$$R = \frac{c(\theta_1, \sigma_p)}{c(\theta_0, \sigma_p)}$$

The continuation region of SPRT discriminating between the two hypotheses, $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ is given by

$$\frac{1}{\delta}\ln\left(\frac{\alpha_1}{1-\alpha_0}\right) + m\left[\frac{\delta}{2} - \left(\frac{\ln R}{\delta}\right)\right] < \frac{1}{\sigma_p}\sum_{i=1}^{m}(x_i - \theta_0) < \frac{1}{\delta}\ln\left(\frac{1-\alpha_1}{\alpha_0}\right) + m\left[\frac{\delta}{2} - \left(\frac{\ln R}{\delta}\right)\right] \tag{2.8}$$

where

$\alpha_i = Pr[Accept\ H_{1-i} \mid H_i]$ , $(i = 0,1)$.

For very small value of $\alpha_1$ the right hand inequality of (2.8) reduces to

$$\frac{1}{\sigma_p}\sum_{i=1}^{m}(x_i - \theta_0) < -\frac{1}{\delta}\ln\alpha_0 + m\left[\frac{\delta}{2} - \left(\frac{\ln R}{\delta}\right)\right] \tag{2.9}$$

But for the observed values

$\sigma_X^2 = \sigma_p^2 + \sigma_e^2$

$$\frac{1}{\acute{\sigma}}\sum_{i=0}^{m}(x_i - \theta_0) = \frac{\rho}{\sigma_p}\sum_{i=0}^{m}(x_i - \theta_0) < -\frac{1}{\delta}\ln\alpha_0 + m\left[\frac{\delta}{2} - \left(\frac{\ln R}{\delta}\right)\right]$$
$$= \frac{1}{\sigma_p}\sum_{i=0}^{m}(x_i - \theta_0) < -\frac{1}{\rho\delta}\ln\alpha_0 + \frac{m}{\rho}\left[\frac{\delta}{2} - \left(\frac{\ln R}{\delta}\right)\right]$$

For constructing a CUSUM Chart for the mean under observed measurements, we plot the sum

$S_m = \frac{1}{\acute{\sigma}}\sum_{i=1}^{m}(x_i - \theta_0)$ against the number of observations m.

## 3. The effect on the dimentions and the ARL of CUSUM chart for mean of a truncated normal distribution under measurment error

A narrow V-mask will detect change more quickly but it will give more frequent false alarms. On the other hand, we could reduce the frequency of false alarms by widening the angle of mask, but the average run length for real changes would be increased.

Under the measurement error the parameter of the mask, namely the angle of the mask $\phi$ and the lead distance d are given by

$$tan\phi = \frac{1}{\rho}\left[\frac{\delta}{2} - \left(\frac{lnR}{\delta}\right)\right] \tag{3.1}$$

where

$$R = \frac{c(\theta_1, \acute{\sigma})}{c(\theta_0, \acute{\sigma})} \tag{3.2}$$

and

$$d = \frac{(-\ln\alpha_0)}{\left(\frac{\delta^2}{2} - \ln R\right)} \tag{3.3}$$

For ARL we consider the situation where the true mean $\theta$ has shifted from $\theta_0$ to $\theta_1$. For very small value of $\alpha_1$ the ARL that is expected number of observation before the change from $\theta_0$ to $\theta_1$ is detected (see Mood and Graybill 1963) is approximately

$$(-\ln\alpha_0)E_{\theta_1}^{-1},$$

where

$$E_{\theta_1} = E\left[\ln\frac{f(X \mid \theta_1, \sigma_p)}{f(X \mid \theta_0, \sigma_p)} \mid \theta_1, \sigma_p\right]$$

For observed values

$$E_{\theta_1} = \int_a^b \left\{\left[\ln R - \frac{\delta^2}{2} + \frac{\delta}{\sigma_p}(x - \theta_0)\right]\frac{C(\theta_1, \sigma_p)}{\sigma_p\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x - \theta_1}{\sigma_p}\right)^2\right]\right\}dx,$$

$$= \ln R + \frac{\delta^2}{2} + \frac{\delta c(\theta_1, \sigma_p)}{\sqrt{2\pi}}\left[exp\left\{-\frac{1}{2}\left(\frac{a-\theta_1}{\acute{\sigma}}\right)^2\right\} - exp\left\{-\frac{1}{2}\left(\frac{b-\theta_1}{\acute{\sigma}}\right)^2\right\}\right] \tag{3.4}$$

## 4. Tabulation of results and conclusions

In Table-1, Table-2 and Table-3 the angle of the mask, lead distance and ARL for the mean chart are given, assuming $\theta_0 = 0, \sigma_p = 1$ and the size of the measurement errors $r$=2, 4, 6 and $\infty$. The truncation points have been taken as $\pm 2.5$, $\pm 2.25$, $\pm 2$, $(0,2.5)$, $(0,2)$. The values of $\delta$ considered are 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5 and 3.0 . The values of $\alpha_0$ are taken to be 0.05, 0.025, 0.01 and 0.005.

It is evident from the Table-1 that for fixed range of truncation with increasing magnitude of error term and $\delta$, an angle of mask increases. But for one sided truncation angle of mask deceases for increase in error term and increases with increase in $\delta$. From tables for lead distance and ARL it is seen that as the size of measurement error increases the lead distance and ARL increases for fixed range of truncation in case of symmetrical truncation. But for one sided truncation it is observed that there is less increase in lead distance as compared to symmetrical truncation and reverse is the case for ARL i.e. the decrease in ARL is seen with increase error term.

**Table 1.** Angle of the Mask for Mean for different Truncation Points (a, b)

| (a, b) | δ↓/r→ | 2 | 4 | 6 | ∞ |
|---|---|---|---|---|---|
| (-2.5,2.5) | 0.2 | 5.613 | 5.308 | 5.249 | 5.201 |
| | 0.4 | 11.091 | 10.493 | 10.377 | 10.283 |
| | 0.6 | 16.318 | 15.448 | 15.279 | 15.142 |
| | 0.8 | 21.207 | 20.091 | 19.874 | 19.697 |
| | 1.0 | 25.705 | 24.369 | 24.110 | 23.898 |
| | 1.5 | 35.166 | 33.381 | 33.031 | 32.747 |
| | 2.0 | 42.324 | 40.185 | 39.760 | 39.412 |
| | 2.5 | 47.701 | 45.258 | 44.762 | 44.355 |
| | 3.0 | 51.793 | 49.071 | 48.507 | 48.041 |
| (-2.25,2.25) | 0.2 | 5.250 | 4.966 | 4.914 | 4.872 |
| | 0.4 | 10.383 | 9.822 | 9.718 | 9.634 |
| | 0.6 | 15.297 | 14.468 | 14.314 | 14.190 |
| | 0.8 | 19.916 | 18.831 | 18.629 | 18.466 |
| | 1.0 | 24.189 | 22.864 | 22.614 | 22.414 |
| | 1.5 | 33.273 | 31.406 | 31.047 | 30.757 |
| | 2.0 | 40.258 | 37.921 | 37.460 | 37.085 |
| | 2.5 | 45.586 | 42.834 | 42.276 | 41.818 |
| | 3.0 | 49.696 | 46.571 | 45.921 | 45.382 |
| (-2,2) | 0.2 | 4.796 | 4.508 | 4.457 | 4.417 |
| | 0.4 | 9.497 | 8.923 | 8.821 | 8.741 |
| | 0.6 | 14.021 | 13.162 | 13.009 | 12.887 |
| | 0.8 | 18.302 | 17.162 | 16.956 | 16.793 |
| | 1.0 | 22.296 | 20.881 | 20.622 | 20.416 |
| | 1.5 | 30.922 | 28.850 | 28.457 | 28.140 |
| | 2.0 | 37.708 | 35.038 | 34.513 | 34.085 |
| | 2.5 | 42.994 | 39.787 | 39.135 | 38.599 |
| | 3.0 | 47.144 | 43.459 | 42.687 | 42.046 |
| (0,2.5) | 0.2 | 38.654 | 38.804 | 38.818 | 38.826 |
| | 0.4 | 40.338 | 40.106 | 40.045 | 39.991 |
| | 0.6 | 41.972 | 41.406 | 41.279 | 41.171 |
| | 0.8 | 43.552 | 42.695 | 42.510 | 42.356 |
| | 1.0 | 45.072 | 43.965 | 43.731 | 43.536 |
| | 1.5 | 48.598 | 47.003 | 46.673 | 46.400 |
| | 2.0 | 51.711 | 49.764 | 49.362 | 49.031 |
| | 2.5 | 54.415 | 52.184 | 51.722 | 51.340 |
| | 3.0 | 56.739 | 54.257 | 53.736 | 53.304 |
| (0,2) | 0.2 | 36.106 | 36.638 | 36.734 | 36.810 |
| | 0.4 | 37.660 | 37.751 | 37.762 | 37.769 |
| | 0.6 | 39.165 | 38.854 | 38.786 | 38.729 |
| | 0.8 | 40.618 | 39.940 | 39.801 | 39.685 |
| | 1.0 | 42.018 | 41.005 | 40.799 | 40.630 |
| | 1.5 | 45.277 | 43.546 | 43.194 | 42.906 |
| | 2.0 | 48.190 | 45.868 | 45.392 | 45.000 |
| | 2.5 | 50.767 | 47.943 | 47.355 | 46.867 |
| | 3.0 | 53.035 | 49.768 | 49.073 | 48.494 |

**Table 2.** Lead Distance for Mean for different Truncation Points(a , b) and Measurement Error( r )

| r | $\delta\downarrow/\alpha_0\rightarrow$ | (a=-2.5,b=2.5) | | | | (a=-2.25,b=2.25) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.05 | 0.025 | 0.01 | 0.005 | 0.05 | 0.025 | 0.01 | 0.005 |
| 2 | 0.2 | 170.40 | 209.83 | 261.95 | 301.38 | 182.27 | 224.44 | 280.19 | 322.36 |
| | 0.4 | 42.71 | 52.60 | 65.66 | 75.55 | 45.70 | 56.27 | 70.25 | 80.83 |
| | 0.6 | 19.07 | 23.48 | 29.31 | 33.72 | 20.41 | 25.13 | 31.37 | 36.10 |
| | 1.0 | 6.96 | 8.57 | 10.70 | 12.31 | 7.46 | 9.18 | 11.46 | 13.19 |
| | 2.0 | 1.84 | 2.26 | 2.83 | 3.25 | 1.98 | 2.44 | 3.04 | 3.50 |
| | 3.0 | 0.88 | 1.08 | 1.35 | 1.55 | 0.95 | 1.17 | 1.46 | 1.67 |
| 4 | 0.2 | 166.18 | 204.63 | 255.46 | 293.91 | 177.68 | 218.79 | 273.14 | 314.25 |
| | 0.4 | 41.68 | 51.32 | 64.07 | 73.71 | 44.59 | 54.91 | 68.55 | 78.87 |
| | 0.6 | 18.62 | 22.93 | 28.63 | 32.94 | 19.95 | 24.56 | 30.66 | 35.28 |
| | 1.0 | 6.82 | 8.39 | 10.48 | 12.06 | 7.32 | 9.02 | 11.26 | 12.95 |
| | 2.0 | 1.83 | 2.25 | 2.81 | 3.23 | 1.98 | 2.44 | 3.05 | 3.51 |
| | 3.0 | 0.89 | 1.10 | 1.37 | 1.58 | 0.97 | 1.20 | 1.50 | 1.72 |
| 6 | 0.2 | 165.29 | 203.54 | 254.09 | 292.34 | 176.62 | 217.49 | 271.51 | 312.38 |
| | 0.4 | 41.46 | 51.05 | 63.74 | 73.33 | 44.34 | 54.60 | 68.16 | 78.41 |
| | 0.6 | 18.53 | 22.82 | 28.48 | 32.77 | 19.84 | 24.43 | 30.50 | 35.09 |
| | 1.0 | 6.79 | 8.36 | 10.43 | 12.00 | 7.29 | 8.98 | 11.21 | 12.89 |
| | 2.0 | 1.83 | 2.25 | 2.81 | 3.23 | 1.98 | 2.44 | 3.05 | 3.51 |
| | 3.0 | 0.90 | 1.10 | 1.38 | 1.58 | 0.98 | 1.21 | 1.51 | 1.73 |
| ∞ | 0.2 | 164.56 | 202.64 | 252.97 | 291.05 | 175.73 | 216.39 | 270.14 | 310.80 |
| | 0.4 | 41.28 | 50.83 | 63.46 | 73.01 | 44.12 | 54.33 | 67.82 | 78.03 |
| | 0.6 | 18.45 | 22.72 | 28.36 | 32.63 | 19.75 | 24.31 | 30.35 | 34.92 |
| | 1.0 | 6.76 | 8.33 | 10.39 | 11.96 | 7.26 | 8.94 | 11.17 | 12.85 |
| | 2.0 | 1.82 | 2.24 | 2.80 | 3.22 | 1.98 | 2.44 | 3.05 | 3.50 |
| | 3.0 | 0.90 | 1.11 | 1.38 | 1.59 | 0.99 | 1.21 | 1.51 | 1.74 |

**Table 2.** Lead Distance for Mean for different Truncation Points(a , b) and Measurement Error( r ) (cont.)

| r | $\delta\downarrow/\alpha_0\rightarrow$ | (a=-2,b=2) | | | | (a=0,b=2.5) | | | | (a=0,b=2) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.05 | 0.025 | 0.01 | 0.005 | 0.05 | 0.03 | 0.01 | 0.01 | 0.05 | 0.03 | 0.01 | 0.01 |
| 2 | 0.2 | 199.61 | 245.80 | 306.85 | 353.04 | 20.94 | 25.78 | 32.19 | 37.03 | 22.96 | 28.27 | 35.30 | 40.61 |
| | 0.4 | 50.05 | 61.63 | 76.94 | 88.52 | 9.86 | 12.14 | 15.16 | 17.44 | 10.85 | 13.36 | 16.68 | 19.19 |
| | 0.6 | 22.35 | 27.53 | 34.36 | 39.53 | 6.21 | 7.64 | 9.54 | 10.98 | 6.85 | 8.44 | 10.53 | 12.12 |
| | 1.0 | 8.17 | 10.06 | 12.56 | 14.45 | 3.34 | 4.11 | 5.14 | 5.91 | 3.72 | 4.58 | 5.71 | 6.57 |
| | 2.0 | 2.17 | 2.67 | 3.33 | 3.83 | 1.32 | 1.63 | 2.03 | 2.34 | 1.50 | 1.84 | 2.30 | 2.65 |
| | 3.0 | 1.04 | 1.28 | 1.59 | 1.83 | 0.73 | 0.90 | 1.13 | 1.30 | 0.84 | 1.03 | 1.29 | 1.49 |
| 4 | 0.2 | 195.84 | 241.16 | 301.06 | 346.37 | 19.20 | 23.64 | 29.52 | 33.96 | 20.76 | 25.56 | 31.91 | 36.72 |
| | 0.4 | 49.17 | 60.55 | 75.59 | 86.97 | 9.17 | 11.29 | 14.09 | 16.21 | 9.97 | 12.28 | 15.33 | 17.63 |
| | 0.6 | 22.01 | 27.10 | 33.83 | 38.92 | 5.84 | 7.19 | 8.97 | 10.32 | 6.39 | 7.87 | 9.82 | 11.30 |
| | 1.0 | 8.09 | 9.97 | 12.44 | 14.32 | 3.20 | 3.94 | 4.92 | 5.66 | 3.55 | 4.37 | 5.46 | 6.28 |
| | 2.0 | 2.20 | 2.71 | 3.38 | 3.89 | 1.31 | 1.61 | 2.01 | 2.31 | 1.50 | 1.84 | 2.30 | 2.65 |
| | 3.0 | 1.09 | 1.34 | 1.67 | 1.92 | 0.74 | 0.91 | 1.14 | 1.31 | 0.87 | 1.07 | 1.34 | 1.54 |
| 6 | 0.2 | 194.81 | 239.89 | 299.48 | 344.55 | 18.87 | 23.24 | 29.01 | 33.38 | 20.35 | 25.06 | 31.28 | 35.99 |
| | 0.4 | 48.93 | 60.25 | 75.21 | 86.53 | 9.03 | 11.12 | 13.89 | 15.98 | 9.80 | 12.07 | 15.07 | 17.34 |
| | 0.6 | 21.91 | 26.98 | 33.68 | 38.75 | 5.77 | 7.10 | 8.86 | 10.20 | 6.30 | 7.76 | 9.68 | 11.14 |
| | 1.0 | 8.07 | 9.94 | 12.41 | 14.27 | 3.17 | 3.91 | 4.88 | 5.61 | 3.52 | 4.33 | 5.41 | 6.22 |
| | 2.0 | 2.21 | 2.72 | 3.39 | 3.91 | 1.30 | 1.60 | 2.00 | 2.30 | 1.50 | 1.84 | 2.30 | 2.65 |
| | 3.0 | 1.10 | 1.35 | 1.69 | 1.94 | 0.74 | 0.91 | 1.14 | 1.31 | 0.88 | 1.08 | 1.35 | 1.55 |
| $\infty$ | 0.2 | 193.90 | 238.77 | 298.08 | 342.94 | 18.61 | 22.92 | 28.61 | 32.92 | 20.01 | 24.65 | 30.77 | 35.40 |
| | 0.4 | 48.71 | 59.98 | 74.88 | 86.15 | 8.93 | 10.99 | 13.72 | 15.79 | 9.67 | 11.90 | 14.86 | 17.10 |
| | 0.6 | 21.82 | 26.87 | 33.55 | 38.60 | 5.71 | 7.03 | 8.78 | 10.10 | 6.23 | 7.67 | 9.57 | 11.01 |
| | 1.0 | 8.05 | 9.91 | 12.37 | 14.23 | 3.15 | 3.88 | 4.85 | 5.58 | 3.49 | 4.30 | 5.37 | 6.18 |
| | 2.0 | 2.21 | 2.73 | 3.40 | 3.91 | 1.30 | 1.60 | 2.00 | 2.30 | 1.50 | 1.84 | 2.30 | 2.65 |
| | 3.0 | 1.11 | 1.36 | 1.70 | 1.96 | 0.74 | 0.92 | 1.14 | 1.32 | 0.88 | 1.09 | 1.36 | 1.56 |

**Table 3.** ARL for Mean for different Truncation Points (a ,b) and Measurement Error( r )

| r | $\delta\downarrow/\alpha_0\rightarrow$ | (a=-2.5,b=2.5) | | | | (a=-2.25,b=2.25) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.05 | 0.025 | 0.01 | 0.005 | 0.05 | 0.025 | 0.01 | 0.005 |
| 2 | 0.2 | 176.48 | 217.31 | 271.29 | 312.12 | 192.53 | 237.07 | 295.96 | 340.50 |
| | 0.4 | 44.54 | 54.85 | 68.47 | 78.78 | 48.65 | 59.91 | 74.78 | 86.04 |
| | 0.6 | 20.11 | 24.76 | 30.91 | 35.56 | 22.00 | 27.09 | 33.82 | 38.92 |
| | 1.0 | 7.60 | 9.35 | 11.68 | 13.44 | 8.36 | 10.29 | 12.84 | 14.78 |
| | 2.0 | 2.31 | 2.84 | 3.55 | 4.08 | 2.57 | 3.17 | 3.96 | 4.55 |
| | 3.0 | 1.31 | 1.62 | 2.02 | 2.32 | 1.47 | 1.82 | 2.27 | 2.61 |
| 4 | 0.2 | 167.67 | 206.47 | 257.76 | 296.55 | 180.22 | 221.91 | 277.03 | 318.73 |
| | 0.4 | 42.35 | 52.15 | 65.10 | 74.90 | 45.61 | 56.16 | 70.11 | 80.67 |
| | 0.6 | 19.14 | 23.57 | 29.42 | 33.85 | 20.68 | 25.47 | 31.79 | 36.58 |
| | 1.0 | 7.26 | 8.94 | 11.16 | 12.84 | 7.92 | 9.75 | 12.17 | 14.00 |
| | 2.0 | 2.26 | 2.78 | 3.47 | 3.99 | 2.52 | 3.10 | 3.87 | 4.46 |
| | 3.0 | 1.33 | 1.64 | 2.04 | 2.35 | 1.50 | 1.85 | 2.31 | 2.66 |
| 6 | 0.2 | 166.13 | 204.57 | 255.39 | 293.83 | 177.98 | 219.17 | 273.60 | 314.79 |
| | 0.4 | 41.96 | 51.67 | 64.50 | 74.21 | 45.06 | 55.48 | 69.26 | 79.69 |
| | 0.6 | 18.97 | 23.35 | 29.16 | 33.54 | 20.44 | 25.17 | 31.42 | 36.15 |
| | 1.0 | 7.20 | 8.86 | 11.07 | 12.73 | 7.84 | 9.65 | 12.05 | 13.86 |
| | 2.0 | 2.25 | 2.77 | 3.45 | 3.97 | 2.51 | 3.09 | 3.86 | 4.44 |
| | 3.0 | 1.33 | 1.64 | 2.05 | 2.36 | 1.51 | 1.86 | 2.32 | 2.67 |
| ∞ | 0.2 | 164.93 | 203.09 | 253.54 | 291.70 | 176.22 | 216.99 | 270.89 | 311.66 |
| | 0.4 | 41.66 | 51.30 | 64.04 | 73.67 | 44.62 | 54.94 | 68.59 | 78.91 |
| | 0.6 | 18.83 | 23.19 | 28.95 | 33.30 | 20.25 | 24.93 | 31.13 | 35.81 |
| | 1.0 | 7.15 | 8.80 | 10.99 | 12.64 | 7.77 | 9.57 | 11.95 | 13.74 |
| | 2.0 | 2.24 | 2.76 | 3.44 | 3.96 | 2.50 | 3.08 | 3.85 | 4.43 |
| | 3.0 | 1.34 | 1.65 | 2.06 | 2.36 | 1.52 | 1.87 | 2.33 | 2.68 |

**Table 3.** ARL for Mean for different Truncation Points (a ,b) and Measurement Error( r ) (cont.)

| r | δ↓/$\alpha_0$→ | (a=-2,b=2) | | | | (a=0,b=2.5) | | | | (a=0,b=2) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.05 | 0.025 | 0.01 | 0.005 | 0.05 | 0.03 | 0.01 | 0.01 | 0.05 | 0.03 | 0.01 | 0.005 |
| 2 | 0.2 | 217.11 | 267.35 | 333.75 | 383.99 | 136.66 | 168.28 | 210.08 | 241.70 | 156.88 | 193.18 | 241.17 | 277.47 |
| | 0.4 | 54.89 | 67.59 | 84.37 | 97.07 | 52.71 | 64.91 | 81.03 | 93.23 | 62.73 | 77.24 | 96.42 | 110.94 |
| | 0.6 | 24.84 | 30.59 | 38.19 | 43.94 | 28.32 | 34.87 | 43.53 | 50.08 | 34.68 | 42.70 | 53.31 | 61.33 |
| | 1.0 | 9.45 | 11.64 | 14.53 | 16.72 | 12.06 | 14.84 | 18.53 | 21.32 | 15.43 | 19.00 | 23.72 | 27.29 |
| | 2.0 | 2.93 | 3.60 | 4.50 | 5.18 | 3.52 | 4.33 | 5.40 | 6.22 | 4.79 | 5.90 | 7.36 | 8.47 |
| | 3.0 | 1.68 | 2.07 | 2.58 | 2.97 | 1.80 | 2.21 | 2.77 | 3.18 | 2.49 | 3.06 | 3.82 | 4.40 |
| 4 | 0.2 | 200.20 | 246.52 | 307.76 | 354.08 | 271.73 | 334.60 | 417.71 | 480.59 | 324.08 | 399.07 | 498.20 | 573.18 |
| | 0.4 | 50.74 | 62.48 | 77.99 | 89.73 | 82.80 | 101.96 | 127.29 | 146.44 | 103.11 | 126.97 | 158.51 | 182.37 |
| | 0.6 | 23.06 | 28.39 | 35.44 | 40.78 | 38.89 | 47.89 | 59.78 | 68.78 | 49.89 | 61.43 | 76.69 | 88.23 |
| | 1.0 | 8.88 | 10.93 | 13.65 | 15.70 | 14.28 | 17.58 | 21.95 | 25.25 | 19.14 | 23.57 | 29.42 | 33.85 |
| | 2.0 | 2.87 | 3.53 | 4.41 | 5.07 | 3.67 | 4.52 | 5.64 | 6.49 | 5.23 | 6.44 | 8.04 | 9.25 |
| | 3.0 | 1.72 | 2.12 | 2.65 | 3.05 | 1.87 | 2.30 | 2.87 | 3.31 | 2.69 | 3.31 | 4.14 | 4.76 |
| 6 | 0.2 | 197.04 | 242.63 | 302.90 | 348.49 | 347.05 | 427.34 | 533.49 | 613.79 | 425.28 | 523.68 | 653.76 | 752.16 |
| | 0.4 | 49.96 | 61.52 | 76.80 | 88.36 | 94.59 | 116.48 | 145.41 | 167.29 | 120.23 | 148.05 | 184.82 | 212.64 |
| | 0.6 | 22.72 | 27.98 | 34.93 | 40.18 | 42.31 | 52.10 | 65.04 | 74.83 | 55.17 | 67.93 | 84.81 | 97.57 |
| | 1.0 | 8.77 | 10.80 | 13.48 | 15.51 | 14.85 | 18.29 | 22.83 | 26.26 | 20.16 | 24.83 | 30.99 | 35.66 |
| | 2.0 | 2.86 | 3.52 | 4.39 | 5.05 | 3.70 | 4.55 | 5.68 | 6.54 | 5.32 | 6.56 | 8.18 | 9.42 |
| | 3.0 | 1.73 | 2.13 | 2.66 | 3.06 | 1.88 | 2.32 | 2.90 | 3.33 | 2.74 | 3.37 | 4.21 | 4.84 |
| ∞ | 0.2 | 194.51 | 239.52 | 299.01 | 344.02 | 452.67 | 557.41 | 695.87 | 800.61 | 578.01 | 711.75 | 888.54 | 1022.28 |
| | 0.4 | 49.34 | 60.76 | 75.85 | 87.26 | 107.45 | 132.31 | 165.17 | 190.03 | 139.81 | 172.16 | 214.92 | 247.27 |
| | 0.6 | 22.45 | 27.65 | 34.52 | 39.71 | 45.67 | 56.24 | 70.21 | 80.78 | 60.56 | 74.57 | 93.09 | 107.11 |
| | 1.0 | 8.68 | 10.69 | 13.35 | 15.35 | 15.36 | 18.92 | 23.62 | 27.17 | 21.10 | 25.98 | 32.43 | 37.31 |
| | 2.0 | 2.85 | 3.51 | 4.38 | 5.04 | 3.72 | 4.58 | 5.72 | 6.58 | 5.40 | 6.65 | 8.30 | 9.55 |
| | 3.0 | 1.74 | 2.14 | 2.68 | 3.08 | 1.90 | 2.33 | 2.91 | 3.35 | 2.77 | 3.41 | 4.26 | 4.90 |

Also it is seen that the angle of mask increases as the range of the truncation is increased, and is bigger in case of one sided truncation. The ARL and lead distance deceases with increase in range of truncation. With increase in $\delta$ the angle of mask increases, while the ARL and the lead distance decreases. The ARL and lead distance increases as $\alpha_0$ is deceased and as $\delta$ is deceased.

For one sided truncation we see that lead distance is less and angle of mask is greater compared to symmetrical truncation. This shows that in case of symmetrical truncation, as angle of mask is smaller, it will detect change more quickly, but it will give more frequent false alarms, as seen by comparing table for ARL for error free case for both symmetrical truncation and one sided truncation.

This paper considered the normal distribution case. For other symmetrical distribution this model will not be suitable. We have to develop another model for other symmetrical distribution.

From the above discussion it is clear that truncated normal distribution under measurement error appears frequently in quality control problems. In order to meet certain specification the supplier uses some technique to ensure that the quality characteristic must satisfy error free observations, error prone data create a serious problem for determining the value of the mask parameter and the ARL. This paper may be *practically implicated* within engineering, finance, medicine, environmental statistics, and many other fields.

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