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ALMOST UNBIASED RATIO AND PRODUCT TYPE EXPONENTIAL ESTIMATORS

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ABSTRACT

This paper considers the problem of estimating the population mean $\bar{Y}$ of the study variate y using information on auxiliary variate x. We have suggested a generalized version of Bahl and Tuteja (1991) estimator and its properties are studied. It is found that asymptotic optimum estimator (AOE) in the proposed generalized version of Bahl and Tuteja (1991) estimator is biased. In some applications, biasedness of an estimator is disadvantageous. So applying the procedure of Singh and Singh (1993) we derived an almost unbiased version of AOE. A numerical illustration is given in the support of the present study.

Key words: study variable, auxiliary variable, almost unbiased ratio-type and product-type exponential estimators, bias, mean squared error.

1. Introduction

It is well known that the use of auxiliary information provides efficient estimates of the population parameters. Ratio, regression and product methods of estimation are good illustrations in this context.

Let there be N units in population and information be available on auxiliary variable x. We draw a sample of size n using simple random sampling without replacement (SRSWOR) scheme and on the basis of which we can estimate the population mean of the character y understudy. Let $\left(\bar{y}, \bar{x}\right)$ be the sample means of $\left(y, x\right)$ respectively.

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Assume that we have information about the population mean $\bar{X}$ of the auxiliary character x and using $\bar{X}$ it is desired to estimate the population mean $\bar{Y}$ of the study character y.

When the correlation between the study variable y and the auxiliary variable x is positive, the classical ratio estimator for population mean $\bar{Y}$ is defined by

$$\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{1.1}$$

In such a situation, Bahl and Tuteja (1991) suggested a ratio-type exponential estimator t_1 for the population mean $\bar{Y}$ as

$$t_1 = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{1.2}$$

To the first degree of approximation, the biases and mean squared errors (MSEs) of $\bar{y}_R$ and t_1 are respectively given by

$$B\left(\bar{y}_R\right) = f_1\bar{Y}C_x^2\left(1-K\right) \tag{1.3}$$

$$B\left(t_1\right) = f_1\bar{Y}\frac{C_x^2}{8}\left(3-4K\right) \tag{1.4}$$

$$MSE\left(\bar{y}_R\right) = f_1\bar{Y}^2\left[C_y^2 + C_x^2\left(1-2K\right)\right] \tag{1.5}$$

$$MSE\left(t_1\right) = f_1\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}\left(1-4K\right)\right] \tag{1.6}$$

where $f_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$, $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $K = \rho_{yx}\frac{C_y}{C_x}$,

$$S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2, \quad S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2,$$

$$S_{xy} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)\left(y_i - \bar{Y}\right)$$

and ρ_{yx} is the correlation coefficient between y and x.

It is observed from (1.3) and (1.4) that the estimator t_1 due to Bahl and Tuteja (1991) is less biased than the classical ratio estimator $\bar{y}_R$ if

$$\left|B\left(t_1\right)\right| < \left|B\left(\bar{y}_R\right)\right|$$

$$\text{i.e. if } \left|\frac{1}{8}(3\text{-}4K)\right| < \left|(1\text{-}K)\right|$$

$$\text{i.e. if } \left(48K^2\text{-}104K+55\right) > 0 \tag{1.7}$$

From (1.5) and (1.6) it follows that the estimator t_1 due to Bahl and Tuteja (1991) is more efficient than the classical ratio estimator $\bar{y}_R$ if

$$MSE(t_1) < MSE(\bar{y}_R)$$

$$\text{if } \; K < \frac{3}{4} \quad \text{or} \quad \rho_{yx} < \frac{3}{4}\frac{C_x}{C_y} \tag{1.8}$$

Murthy (1964) suggested a product-type estimator

$$\bar{y}_P = \bar{y}\frac{\bar{x}}{\bar{X}} \tag{1.9}$$

for the population mean $\bar{Y}$ which is useful in the situation where the correlation between the study variable y and the auxiliary variable x is negative (high).

In negative correlation situation, Bahl and Tuteja (1991) suggested a product-type exponential estimator t_2 for the population mean $\bar{Y}$ is defined as

$$t_2 = \bar{y}\,\exp\left(\frac{\bar{x}\text{-}\bar{X}}{\bar{x}+\bar{X}}\right) \tag{1.10}$$

To the first degree of approximation, the biases and mean squared errors (MSEs) of $\bar{y}_P$ and t_2 are respectively given by

$$B(\bar{y}_P) = f_1\bar{Y}C_x^2K \tag{1.11}$$

$$B(t_2) = f_1\bar{Y}\frac{C_x^2}{8}(4K\text{-}1) \tag{1.12}$$

$$MSE(\bar{y}_P) = f_1\bar{Y}^2\left[C_y^2 + C_x^2(1+2K)\right] \tag{1.13}$$

$$MSE(t_2) = f_1\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1+4K)\right] \tag{1.14}$$

It is observed from (1.11) and (1.12) that the product-type exponential estimator t_2 is less biased than the usual product estimator $\bar{y}_P$ if

$$\left|B(t_2)\right| < \left|B(\bar{y}_P)\right|$$

$$\text{i.e. if } \left|\frac{1}{8}(4K-1)\right| < |(K)|$$

$$\text{i.e. if } \left(48K^2+8K-1\right) > 0 \tag{1.15}$$

From (1.13) and (1.14), it follows that the Bahl and Tuteja (1991) product-type exponential estimator t_2 is more efficient than the product estimator $\bar{y}_P$ if

$$MSE(t_2) < MSE(\bar{y}_P)$$

$$\text{if } K > -\frac{3}{4} \quad \text{or} \quad \rho_{yx} > -\frac{3}{4}\frac{C_x}{C_y} \tag{1.16}$$

2. Generalized version of t_1 and t_2

We define a generalized version of t_1 and t_2 as

$$t_c = \bar{y} \exp\left[c\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right] \tag{2.1}$$

where c is 'non-zero' constant. For c=1, t_c reduces to t_1 while for c=-1 it reduces to t_2.

To obtain the bias and mean squared error (MSE) of the estimator t_c up to the first degree of approximation, we write

$$\bar{y}=\bar{Y}(1+e_0) \quad \text{and} \quad \bar{x}=\bar{X}(1+e_1)$$

such that $E(e_0)=E(e_1)=0,$

$$E(e_0^2)=f_1 C_Y{}^2, \quad E(e_1^2)=f_1 C_X^2 \quad \text{and} \quad E(e_0 e_1)=f_1 \rho_{yx} C_Y C_X.$$

Expressing t_c in terms of e's, we have

$$t_c=\bar{Y}(1+e_0)\exp\left[c\left(\frac{-e_1}{2+e_1}\right)\right]$$

Now, expanding the right hand side of the above, multiplying out and neglecting terms of e's having power greater than two, we have

$$t_c=\bar{Y}\left[1+e_0-\frac{c}{2}e_1-\frac{c}{2}e_0e_1+\frac{c}{4}e_1^2+\frac{c^2}{8}e_1^2\right]$$

or

$$\left(t_c-\bar{Y}\right)=\bar{Y}\left[e_0-\frac{c}{2}e_1-\frac{c}{2}e_0e_1+\frac{c}{4}e_1^2+\frac{c^2}{8}e_1^2\right] \tag{2.2}$$

Taking expectations of both sides of (2.2), we get the bias of the estimator t_c up to the first order of approximation as

$$B\left(t_c\right)=f_1\bar{Y}\frac{c}{2}C_x^2\left(2+c-4K\right) \tag{2.3}$$

Squaring both sides of (2.2) and neglecting terms of e's having power greater than two, we have

$$\left(t_c-\bar{Y}\right)^2=\bar{Y}^2\left(e_0-\frac{c}{2}e_1\right)^2 \tag{2.4}$$

Taking expectation of both sides of (2.4), we get mean squared error (MSE) of the estimator t_c up to the first order of approximation as

$$MSE\left(t_c\right)=f_1\bar{Y}^2\left[C_y^2+cC_x^2\left(\frac{c}{4}-K\right)\right] \tag{2.5}$$

Differentiating (2.5) w.r.t. c and equating it to zero, we get the optimum value of c as

$$c=2K \tag{2.6}$$

Thus, the substitution of optimum value c=2K in (2.1) yields the asymptotic optimum estimator (AOE) in the class of estimator t_c as

$$t_K=\bar{y}\exp\left[2K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right] \tag{2.7}$$

The bias and mean squared error (MSE) of the estimator t_K are respectively given by

$$B\left(t_K\right)=f_1\bar{Y}C_x^2\left\{K\left(1-K\right)/2\right\} \tag{2.8}$$

$$MSE\left(t_K\right)=f_1\bar{Y}^2C_Y^2\left(1-\rho^2\right) \tag{2.9}$$

It is observed from (2.9) that the MSE of the AOE t_K is equal to the approximate variance of the regression estimator $\bar{y}_{lr}=\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right)$, which is biased, where $\hat{\beta}$ is sample estimate of the population regression coefficient β. Expression (2.8) clearly indicates that the AOE is a biased estimator. So our objective is to obtain an almost unbiased estimator for the population mean $\bar{Y}$. In the following section, we meet with our objective using Singh and Singh (1993) approach.

3. Almost unbiased exponential estimator

We consider the estimators

$$t_1=\bar{y}\exp\left[2K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right] \tag{3.1}$$

$$t_2=\bar{y}\exp\left[4K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right] \tag{3.2}$$

$$t_3=\bar{y}\exp\left[6K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right] \tag{3.3}$$

such that $t_1, t_2, t_3 \in H$, where H denotes the set of all possible estimators for estimating the population mean $\bar{Y}$.

To the first degree of approximation, the biases and mean squared errors (MSEs) of the estimators t_1, t_2 and t_3 are respectively given by

$$B(t_1)=f_1\bar{Y}(K/2)C_x^2(1-K). \tag{3.4}$$

$$B(t_2)=f_1\bar{Y}KC_x^2 \tag{3.5}$$

$$B(t_3)=f_1\bar{Y}(3/2)KC_x^2(1-K) \tag{3.6}$$

$$MSE(t_1)=f_1\bar{Y}^2C_y^2(1-\rho^2) \tag{3.7}$$

$$MSE(t_2)=f_1\bar{Y}^2C_y^2 \tag{3.8}$$

$$MSE(t_3)=f_1\bar{Y}^2\left[C_y^2+3K^2C_x^2\right] \tag{3.9}$$

Now, considering the estimators (3.1), (3.2) and (3.3), we suggest a class of exponential estimators for $\bar{Y}$ as

$$t_{Kh}=\sum_{j=1}^{3}h_jt_j \in H \tag{3.10}$$

$$\text{with} \quad \sum_{j=1}^{3}h_j=1,\ h_j \in R, \tag{3.11}$$

where h_j (j=1, 2, 3) denotes the statistical constants and R denotes the set of real numbers.

Expressing t_{Kh} in terms of e's, we have

$$t_{Kh}=\left[h_1\bar{Y}(1+e_0)\exp\left\{-Ke_1\left(1+\frac{e_1}{2}\right)^{-1}\right\}+h_2\bar{Y}(1+e_0)\exp\left\{-2Ke_1\left(1+\frac{e_1}{2}\right)^{-1}\right\}\right.$$
$$\left.+h_3\bar{Y}(1+e_0)\exp\left\{-3Ke_1\left(1+\frac{e_1}{2}\right)^{-1}\right\}\right]$$
$$=\bar{Y}\left[h_1(1+e_0)\left\{1-Ke_1\left(1+\frac{e_1}{2}\right)^{-1}+\frac{K^2e_1^2}{2}\left(1+\frac{e_1}{2}\right)^{-2}-...\right\}\right.$$
$$+h_2(1+e_0)\left\{1-2Ke_1\left(1+\frac{e_1}{2}\right)^{-1}+2K^2e_1^2\left(1+\frac{e_1}{2}\right)^{-2}-...\right\}$$
$$\left.+h_3(1+e_0)\left\{1-3Ke_1\left(1+\frac{e_1}{2}\right)^{-1}+\frac{9}{2}K^2e_1^2\left(1+\frac{e_1}{2}\right)^{-2}-...\right\}\right]$$

$$=\bar{Y}\left[h_1(1+e_0)\left\{1-Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+\frac{K^2e_1^2}{2}(1-e_1+...)-...\right\}\right.$$
$$+h_2(1+e_0)\left\{1-2Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+2K^2e_1^2(1-e_1+...)-...\right\}$$
$$\left.+h_3(1+e_0)\left\{1-3Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+\frac{9}{2}K^2e_1^2(1-e_1+...)-...\right\}\right]$$

$$=\bar{Y}\left[h_1\left\{1+e_0-Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+\frac{K^2e_1^2}{2}(1-e_1+...)-Ke_0e_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+...\right\}\right.$$

$$+h_2\left\{1+e_0-2Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+2K^2e_1^2(1-e_1+...)-2Ke_0e_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+...\right\}$$
$$\left.+h_3\left\{1+e_0-3Ke_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+\frac{9}{2}K^2e_1^2(1-e_1+...)-3Ke_0e_1\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-...\right)+...\right\}\right]$$

Neglecting the terms of e’s having power greater than two, we have

$$t_{Kh}=\bar{Y}\left[1+e_0-hK\left(e_1+e_0e_1\right)+\frac{Ke_1^2}{2}\left\{h+\left(h_1+4h_2+9h_3\right)K\right\}\right] \tag{3.12}$$

or

$$\left(t_{Kh}-\bar{Y}\right)=\bar{Y}\left[e_0-hK\left(e_1+e_0e_1\right)+\frac{Ke_1^2}{2}\left\{h+\left(h_1+4h_2+9h_3\right)K\right\}\right] \tag{3.13}$$

where $\left(h_1+2h_2+3h_3\right)=h$ (a constant). (3.14)

Taking expectation of both sides of (3.13), we get the bias of t_{Kh} to the first degree of approximation as

$$B\left(t_{Kh}\right)=f_1\bar{Y}\left(\frac{KC_x^2}{2}\right)\left[\left(1-2K\right)h+K\left(h_1+4h_2+9h_3\right)\right] \tag{3.15}$$

Squaring both sides of (3.13) and neglecting terms of e’s having power greater than two, we have

$$\left(t_{Kh}-\bar{Y}\right)^2=\bar{Y}^2\left[e_0^2+h^2K^2e_1^2-2hKe_0e_1\right] \tag{3.16}$$

Taking expectation of both sides of (3.16), we get the MSE of t_{Kh} to the first degree of approximation as

$$MSE\left(t_{Kh}\right)=f_1\bar{Y}^2\left[C_y^2+h^2K^2C_x^2-2hk\rho C_yC_x\right] \tag{3.17}$$

Minimizing (3.17) with respect to h, we get the optimum value of h as

$$\left(h_1+2h_2+3h_3\right)=h=1 \tag{3.18}$$

Substitution of (3.18) in (3.17) yields minimum MSE of t_{Kh} as

$$\text{min. } MSE\left(t_{Kh}\right)=f_1\bar{Y}^2C_Y^2\left(1-\rho^2\right) \tag{3.19}$$

In order to get unique solution of h_j’s $\left(j=1, 2, 3\right)$, we shall impose the linear restriction as we have only two equations in three unknowns.

$$\sum_{j=1}^{3}h_jB\left(t_j\right)=0, \tag{3.20}$$

where $B\left(t_j\right)$ represents the bias of the j^{th} estimator.

So, we have three equations (3.11), (3.18) and (3.20) with three unknowns. These can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ B(t_1) & B(t_2) & B(t_3) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad (3.21)$$

Using (3.21), we get the unique values of h_j's (j=1, 2, 3) as

$$\left.\begin{aligned} h_1 &= \left(\frac{3}{4}+\frac{1}{4K}\right) \\ h_2 &= \left(\frac{1}{2}-\frac{1}{2K}\right) \\ h_3 &= \left(-\frac{1}{4}+\frac{1}{4K}\right) \end{aligned}\right\} \qquad (3.22)$$

Using these h_j's (j=1, 2, 3), we can remove the bias of the estimator t_c up to the terms of order $o(n^{-1})$.

Thus, an almost unbiased exponential estimator for population mean $\bar{Y}$ is defined as

$$t_K^{(u)} = \bar{y}\left[\frac{(3K+1)}{4K}\exp\left\{2K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\}+\frac{(K-1)}{2K}\exp\left\{4K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\}+\frac{(1-K)}{4K}\exp\left\{6K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\}\right] \qquad (3.23)$$

It can be shown to the first degree of approximation that the mean squared error of $t_K^{(u)}$ is

$$MSE\left(t_K^{(u)}\right)=f_1\bar{Y}^2C_Y^2\left(1-\rho^2\right) \qquad (3.24)$$

4. Efficiency comparison

It is well known under SRSWOR that the variance of usual unbiased estimator $\bar{y}$ is

$$Var\left(\bar{y}\right)=MSE\left(\bar{y}\right)=f_1\bar{Y}^2C_y^2 \qquad (4.1)$$

From (1.5), (1.6), (1.13), (1.14), (3.24) and (4.1), we have

(i) $\mathrm{MSE}\left(\bar{y}\right)-\mathrm{MSE}\left(t_K^{(u)}\right) > 0$ if

$$\rho > 0 \tag{4.2}$$

(ii) $\mathrm{MSE}\left(\bar{y}_R\right)-\mathrm{MSE}\left(t_K^{(u)}\right) > 0$ if

$$\left(K^2\text{-}2K+1\right) > 0$$

$$K>1 \text{ or } \rho > \frac{C_x}{C_y} \tag{4.3}$$

(iii) $\mathrm{MSE}\left(t_1\right)-\mathrm{MSE}\left(t_K^{(u)}\right) > 0$ if

$$\left(4K^2\text{-}4K+1\right) > 0$$

$$K > \frac{1}{2} \quad \text{or} \quad \rho > \frac{1}{2}\frac{C_x}{C_y} \tag{4.4}$$

(iv) $\mathrm{MSE}\left(\bar{y}_P\right)\text{-}\mathrm{MSE}\left(t_K^{(u)}\right)>0$ if

$$\left(K^2+2K+1\right) > 0$$

$$K > \text{-}1 \quad \text{or} \quad \rho > \text{-}\frac{C_x}{C_y} \tag{4.5}$$

(v) $\mathrm{MSE}\left(t_2\right)-\mathrm{MSE}\left(t_K^{(u)}\right) > 0$

$$\left(4K^2+4K+1\right) > 0$$

$$K > \text{-}\frac{1}{2} \quad \text{or} \quad \rho > \text{-}\frac{1}{2}\frac{C_x}{C_y} \tag{4.6}$$

5. Empirical study

To see the performance of the estimators $\bar{y}_R$, $\bar{y}_P$, t_1, t_2 and $t_K^{(u)}$ over $\bar{y}$, we consider two population data sets. Using the formula

$$\mathrm{PRE}\left(.,\,\bar{y}\right)=\frac{\mathrm{MSE}\left(\bar{y}\right)}{\mathrm{MSE}\left(.\right)}\times 100, \qquad \left(.\right)=\bar{y},\ \bar{y}_R,\ \bar{y}_P,\ t_1,\ t_2 \text{ and } t_K^{(u)}$$

We have computed the percent relative efficiencies (PREs) of the estimators $\bar{y}_R$, $\bar{y}_P$, t_1, t_2 and $t_K^{(u)}$ over $\bar{y}$ and compiled in Table 1.

The values of scalars h_j's (j=1, 2, 3) of the almost unbiased exponential estimator are calculated for different population data sets and compiled in Table 2.

The description of the populations is given below:

Positive correlated variables:

Population- I: [Source: Murthy (1967, pp. 228)]

It consists of 80 factories in a region, the characters x and y being fixed capital and output respectively. The variates are defined as follows:

Y: output

X: the number of fixed capital

$C_y = 0.3519$, $C_x = 0.7459$, $\rho = 0.9413$, $K = 0.4440$

Negative correlated variables:

Population- II: [Source: Steel and Torrie (1960, pp. 282)]

Y: log of leaf burn in secs,

X: chlorine percentage

$C_y = 0.4803$, $C_x = 0.7493$, $\rho = -0.4996$, $K = -0.32$

Table 1. Percent relative efficiencies (PREs) of the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, t_1, t_2 and $t_K^{(u)}$ with respect to $\bar{y}$

S. No.	Estimator	$PRE(., \bar{y})$	
		Population I	Population II
1.	$\bar{y}$	100.00	100.00
2.	$\bar{y}_R$	66.5233	20.0277
3.	$\bar{y}_P$	10.5433	53.2809
4.	t_1	783.5443	41.8739
5.	t_2	24.2792	120.5436
6.	$t_K^{(u)}$	878.0141	133.2177

Table 2. Values of h_j's (j=1, 2, 3) for almost unbiased exponential estimator

Scalars	Population I	Population II
h_1	1.3130	-0.0312
h_2	-0.6261	2.0625
h_3	0.3130	-1.0312

From Table 1, it is observed that the suggested almost unbiased exponential estimator $t_K^{(u)}$ is more efficient than the usual unbiased estimator $\bar{y}$, classical ratio estimator $\bar{y}_R$, classical product estimator $\bar{y}_P$ and Bahl and Tuteja (1991) ratio-type estimator t_1 and product-type estimator t_2 respectively.

From Table 2, we can say that by using these values of scalars h_j's (j=1, 2, 3), one can reduce the bias of the estimator $t_K^{(u)}$ up to the first degree of approximation.

6. Conclusions

It is observed from (1.3) and (1.11) that the classical ratio estimator $\bar{y}_R$ and the product estimator $\bar{y}_P$ are biased. In some applications, biasedness of an estimator is disadvantageous. So keeping this in view, first we have suggested a generalized version of Bahl and Tuteja (1991) ratio-type and product-type estimators. It is observed that the suggested generalized estimator is also biased. So using the technique as adopted by Singh and Singh (1993), we have suggested an almost unbiased estimator for the population mean $\bar{Y}$ with its variance formula. From Table 1 and Table 2, we have observed that the suggested almost unbiased exponential estimator $t_K^{(u)}$ is more efficient than $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, t_1, t_2 and $t_K^{(u)}$. We shall see that the suggested almost unbiased estimator depends only on the well known parameter $K=\rho_{yx}\left(C_Y/C_X\right)$, the value of which can be obtained quite accurately from some earlier survey or a pilot study.

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