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ESTIMATION OF FINITE POPULATION MEAN USING DECILES OF AN AUXILIARY VARIABLE

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ABSTRACT

The present paper deals with a class of modified ratio estimators for estimation of population mean of the study variable when the population deciles of the auxiliary variable are known. The biases and the mean squared errors of the proposed estimators are derived and compared with that of existing modified ratio estimators for certain known populations. Further, we have also derived the conditions for which the proposed estimators perform better than the existing modified ratio estimators. From the numerical study it is also observed that the proposed modified ratio estimators perform better than the existing modified ratio estimators.

Key words: mean squared error, natural populations, simple random sampling.

1. Introduction

Consider a finite population $\mathrm{U} = \{\mathrm{U}_1, \mathrm{U}_2, \dots, \mathrm{U}_\mathrm{N}\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $\mathrm{U}_\mathrm{i}, \mathrm{i} = 1,2,3, \dots, \mathrm{N}$ giving a vector $\mathrm{Y} = \{\mathrm{Y}_1, \mathrm{Y}_2, \dots, \mathrm{Y}_\mathrm{N}\}$. The problem is to estimate the population mean $\overline{\mathrm{Y}} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{Y}_\mathrm{i}$ on the basis of a random sample selected from the population U. The simple random sample mean is the simplest estimator of population mean. If an auxiliary variable X closely related to the study variable Y is available then one can improve the performance of the estimator of the study variable by using the known values of the population parameters of the auxiliary variable. That is, when the population parameters of the auxiliary variable X such as Population Mean, Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, etc., are known, a number of estimators such as ratio, product and linear regression estimators and their modifications are proposed in the literature. Among these estimators the ratio estimator and its modifications are widely used for the

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estimation of the mean of the study variable. Before discussing further about the modified ratio estimators and the proposed modified ratio estimators the notations to be used in this paper are described below:

- $N -$ Population size
- $n -$ Sample size
- $f = n/N$, Sampling fraction
- $Y -$ Study variable
- $X -$ Auxiliary variable
- $\bar{X}, \bar{Y} -$ Population means
- x, y - Sample totals
- $\bar{x}, \bar{y} -$ Sample means
- $S_x, S_y -$ Population standard deviations
- $C_x, C_y -$ Coefficient of variations
- $\rho -$ Coefficient of correlation
- $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$, Coefficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Coefficient of kurtosis of the auxiliary variable
- $B(.) -$ Bias of the estimator
- $MSE(.) -$ Mean squared error of the estimator
- $\hat{\bar{Y}}_i(\hat{\bar{Y}}_{pi}) -$ Existing (proposed) modified ratio estimator of $\bar{Y}$

The Ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\,\bar{X} \qquad (1.1)$$

where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$. The Ratio estimator given in (1.1) is used for improving the precision of estimate of the population mean compared to simple random sampling when there is a positive correlation between X and Y. Further improvements are achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, etc. The lists of modified ratio estimators, which are to be compared with that of the proposed estimators, are divided into two classes namely Class 1 and Class 2, and are given respectively in Table 1.1. and Table 1.2. As stated above, some of the existing modified ratio estimators together with their biases, mean squared errors and constants available in the literature are presented in the following tables:

Table 1.1. Existing modified ratio type estimators (Class 1) with their biases, mean squared errors and their constants

Estimator	**Bias - B(.)**	**Mean squared error MSE(.)**	**Constant θ_i**
$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$ Sisodia and Dwivedi [13]	$\frac{(1-f)}{n}\bar{Y}(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\bar{X}}{\bar{X}+C_x}$
$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$ Singh et.al [11]	$\frac{(1-f)}{n}\bar{Y}(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}$
$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$ Yan and Tian [15]	$\frac{(1-f)}{n}\bar{Y}(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}$
$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$ Singh and Tailor [10]	$\frac{(1-f)}{n}\bar{Y}(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$	$\theta_4 = \frac{\bar{X}}{\bar{X}+\rho}$
$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right]$ Upadhyaya and Singh [14]	$\frac{(1-f)}{n}\bar{Y}(\theta_5^2 C_x^2 - \theta_5 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho)$	$\theta_5 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}$
$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right]$ Upadhyaya and Singh [14]	$\frac{(1-f)}{n}\bar{Y}(\theta_6^2 C_x^2 - \theta_6 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_x C_y \rho)$	$\theta_6 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}$
$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{\bar{X}\beta_2+\beta_1}{\bar{x}\beta_2+\beta_1}\right]$ Yan and Tian [15]	$\frac{(1-f)}{n}\bar{Y}(\theta_7^2 C_x^2 - \theta_7 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_x C_y \rho)$	$\theta_7 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+\beta_1}$
$\hat{\bar{Y}}_8 = \bar{y}\left[\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}\right]$ Yan and Tian [15]	$\frac{(1-f)}{n}\bar{Y}(\theta_8^2 C_x^2 - \theta_8 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_x C_y \rho)$	$\theta_8 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$
$\hat{\bar{Y}}_9 = \bar{y}\left[\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right]$ Yan and Tian [15]	$\frac{(1-f)}{n}\bar{Y}(\theta_9^2 C_x^2 - \theta_9 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_x C_y \rho)$	$\theta_9 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$

Table 1.2. Existing modified ratio type estimators (Class 2) with their biases, mean squared errors and their constants

Estimator	Bias-B(.)	Mean squared error MSE(.)	Constant R_i
$\hat{\bar{Y}}_{10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X}$ Kadilar and Cingi [2]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{10}^2$	$\frac{(1-f)}{n}\left(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{10} = \frac{\bar{Y}}{\bar{X}}$
$\hat{\bar{Y}}_{11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)} (\bar{X} + C_x)$ Kadilar and Cingi [2]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{11}^2$	$\frac{(1-f)}{n}\left(R_{11}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{11} = \frac{\bar{Y}}{\bar{X} + C_x}$
$\hat{\bar{Y}}_{12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)} (\bar{X} + \beta_2)$ Kadilar and Cingi [2]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{12}^2$	$\frac{(1-f)}{n}\left(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{12} = \frac{\bar{Y}}{\bar{X} + \beta_2}$
$\hat{\bar{Y}}_{13} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\,\beta_2 + C_x)} (\bar{X}\,\beta_2 + C_x)$ Kadilar and Cingi [2]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{13}^2$	$\frac{(1-f)}{n}\left(R_{13}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{13} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + C_x}$
$\hat{\bar{Y}}_{14} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\,C_x + \beta_2)} (\bar{X}\,C_x + \beta_2)$ Kadilar and Cingi [2]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{14}^2$	$\frac{(1-f)}{n}\left(R_{14}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{14} = \frac{\bar{Y}C_x}{\bar{X}C_x + \beta_2}$
$\hat{\bar{Y}}_{15} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)} (\bar{X} + \beta_1)$ Yan and Tian [15]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{15}^2$	$\frac{(1-f)}{n}\left(R_{15}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{15} = \frac{\bar{Y}}{\bar{X} + \beta_1}$
$\hat{\bar{Y}}_{16} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\,\beta_1 + \beta_2)} (\bar{X}\,\beta_1 + \beta_2)$ Yan and Tian [15]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{16}^2$	$\frac{(1-f)}{n}\left(R_{16}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{16} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \beta_2}$
$\hat{\bar{Y}}_{17} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)} (\bar{X} + \rho)$ Kadilar and Cingi [3]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{17}^2$	$\frac{(1-f)}{n}\left(R_{17}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{17} = \frac{\bar{Y}}{\bar{X} + \rho}$
$\hat{\bar{Y}}_{18} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \rho)} (\bar{X}C_x + \rho)$ Kadilar and Cingi [3]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{18}^2$	$\frac{(1-f)}{n}\left(R_{18}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{18} = \frac{\bar{Y}C_x}{\bar{X}C_x + \rho}$
$\hat{\bar{Y}}_{19} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)} (\bar{X}\rho + C_x)$ Kadilar and Cingi [3]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{19}^2$	$\frac{(1-f)}{n}\left(R_{19}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{19} = \frac{\bar{Y}\rho}{\bar{X}\rho + C_x}$
$\hat{\bar{Y}}_{20} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)} (\bar{X}\beta_2 + \rho)$ Kadilar and Cingi [3]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{20}^2$	$\frac{(1-f)}{n}\left(R_{20}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{20} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho}$
$\hat{\bar{Y}}_{21} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)} (\bar{X}\rho + \beta_2)$ Kadilar and Cingi [3]	$\frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{21}^2$	$\frac{(1-f)}{n}\left(R_{21}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{21} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2}$

It is to be noted that "the existing modified ratio estimators" mean the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean the entire list of modified ratio type estimators available in the literature. For a more detailed discussion on the ratio estimator and its modifications one may refer to Cochran [1], Kadilar and Cingi [2, 3], Koyuncu and Kadilar [4], Murthy [5], Prasad [6], Rao [7], Singh [9], Singh and Tailor [10,12], Singh et.al [11], Sisodia and Dwivedi [13], Upadhyaya and Singh [14], Yan and Tian [15] and the references cited therein.

The modified ratio type estimators discussed above are biased but have minimum mean squared errors compared to the classical ratio estimator. The list of estimators given in Table 1.1. and Table 1.2. uses the known values of the parameters like $\bar{X}$, C_x, β_1, β_2, ρ and their linear combinations. However, it seems no attempt is made to use the known values of the population deciles of the auxiliary variable to improve the ratio estimator. Further, we know that the value of deciles is unaffected and robustness by the extreme values or the presence of outliers in the population values unlike the other parameters like the mean, coefficient of variation, coefficient of skewness and coefficient of kurtosis, etc. The points discussed above have motivated us to introduce modified ratio estimators using the known value of the population deciles of the auxiliary variable. It is observed that the proposed estimators perform better than the existing modified ratio type estimators listed in Table 1.1. and Table 1.2. The materials of this paper are arranged as follows: The proposed modified ratio estimators with known population deciles of an auxiliary variable are presented in section 2 whereas the conditions in which the proposed estimators perform better than the existing modified ratio estimators are derived in section 3. The performances of the proposed modified ratio estimators compared to the existing modified ratio estimators are assessed for certain natural populations in section 4 and the conclusion is presented in section 5.

2. Proposed modified ratio type estimators using deciles of the auxiliary variable

As we stated earlier one can always improve the performance of the estimator of the study variable by using the known population parameters of the auxiliary variable, which are positively correlated with that of study variable. In this section, we have suggested a class of modified ratio type estimators using the population deciles, denoted by D_j ; $j = 1, 2, 3, \ldots, 10$ of the auxiliary variable. The proposed modified ratio type estimators $\hat{\bar{Y}}_{pj}$, $j = 1,2, \ldots ,10$ for estimating the population mean $\bar{Y}$ together with the first degree of approximation, the biases, mean squared errors and the constants are given below:

Table 2.1. Proposed modified ratio type estimators (Class 3) with their biases, mean squared errors and their constants

Estimator	**Bias - B(.)**	**Mean squared error MSE(.)**	**Constant θ_i**
$\hat{\bar{Y}}_{p1} = \bar{y}\left[\frac{\bar{X}+D_1}{\bar{x}+D_1}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y \rho)$	$\theta_{p1} = \frac{\bar{X}}{\bar{X}+D_1}$
$\hat{\bar{Y}}_{p2} = \bar{y}\left[\frac{\bar{X}+D_2}{\bar{x}+D_2}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p2}^2 C_x^2 - \theta_{p2} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p2}^2 C_x^2 - 2\theta_{p2} C_x C_y \rho)$	$\theta_{p2} = \frac{\bar{X}}{\bar{X}+D_2}$
$\hat{\bar{Y}}_{p3} = \bar{y}\left[\frac{\bar{X}+D_3}{\bar{x}+D_3}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p3}^2 C_x^2 - \theta_{p3} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p3}^2 C_x^2 - 2\theta_{p3} C_x C_y \rho)$	$\theta_{p3} = \frac{\bar{X}}{\bar{X}+D_3}$
$\hat{\bar{Y}}_{p4} = \bar{y}\left[\frac{\bar{X}+D_4}{\bar{x}+D_4}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p4}^2 C_x^2 - \theta_{p4} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p4}^2 C_x^2 - 2\theta_{p4} C_x C_y \rho)$	$\theta_{p4} = \frac{\bar{X}}{\bar{X}+D_4}$
$\hat{\bar{Y}}_{p5} = \bar{y}\left[\frac{\bar{X}+D_5}{\bar{x}+D_5}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p5}^2 C_x^2 - \theta_5 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p5}^2 C_x^2 - 2\theta_{p5} C_x C_y \rho)$	$\theta_{p5} = \frac{\bar{X}}{\bar{X}+D_5}$
$\hat{\bar{Y}}_{p6} = \bar{y}\left[\frac{\bar{X}+D_6}{\bar{x}+D_6}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p6}^2 C_x^2 - \theta_{p6} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p6}^2 C_x^2 - 2\theta_{p6} C_x C_y \rho)$	$\theta_{p6} = \frac{\bar{X}}{\bar{X}+D_6}$
$\hat{\bar{Y}}_{p7} = \bar{y}\left[\frac{\bar{X}+D_7}{\bar{x}+D_7}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p7}^2 C_x^2 - \theta_{p7} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p7}^2 C_x^2 - 2\theta_{p7} C_x C_y \rho)$	$\theta_{p7} = \frac{\bar{X}}{\bar{X}+D_7}$
$\hat{\bar{Y}}_{p8} = \bar{y}\left[\frac{\bar{X}+D_8}{\bar{x}+D_8}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p8}^2 C_x^2 - \theta_{p8} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p8}^2 C_x^2 - 2\theta_{p8} C_x C_y \rho)$	$\theta_{p8} = \frac{\bar{X}}{\bar{X}+D_8}$
$\hat{\bar{Y}}_{p9} = \bar{y}\left[\frac{\bar{X}+D_9}{\bar{x}+D_9}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p9}^2 C_x^2 - \theta_{p9} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p9}^2 C_x^2 - 2\theta_{p9} C_x C_y \rho)$	$\theta_{p9} = \frac{\bar{X}}{\bar{X}+D_9}$
$\hat{\bar{Y}}_{p10} = \bar{y}\left[\frac{\bar{X}+D_{10}}{\bar{x}+D_{10}}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p10}^2 C_x^2 - \theta_{p10} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p10}^2 C_x^2 - 2\theta_{p10} C_x C_y \rho)$	$\theta_{p10} = \frac{\bar{X}}{\bar{X}+D_{10}}$

3. Efficiency of the proposed estimators

For want of space, for the sake of convenience to the readers and for the ease of comparisons, the modified ratio type estimators given in Table 1.1, Table 1.2 and the proposed modified ratio estimators given in Table 2.1 are represented into three classes as given below:

Class 1: The biases, the mean squared errors and the constants of the modified ratio type estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_9$ listed in the Table 1.1 are represented in a single class (say Class 1), which will be very much useful for comparing with that of proposed modified ratio estimators, and are given below:

$$B\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_i^2 C_x^2 - \theta_i C_x C_y\,\rho)$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i C_x C_y\,\rho\right)\ i = 1,2,3,\ldots,9 \qquad (3.1)$$

where $\theta_1 = \frac{\bar{X}}{\bar{X}+C_x}, \theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}, \theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}, \theta_4 = \frac{\bar{X}}{\bar{X}+\rho}, \theta_5 = \frac{\bar{X}\,C_x}{\bar{X}\,C_x+\beta_2}, \theta_6 = \frac{\bar{X}\,\beta_2}{\bar{X}\,\beta_2+C_x},$

$\theta_7 = \frac{\bar{X}\,\beta_2}{\bar{X}\,\beta_2+\beta_1}, \theta_8 = \frac{\bar{X}\,\beta_1}{\bar{X}\,\beta_1+\beta_2}$ and $\theta_9 = \frac{\bar{X}\,C_x}{\bar{X}\,C_x+\beta_1}$

Class 2: The biases, the mean squared errors and the constants of the 12 modified ratio estimators $\hat{\bar{Y}}_{10}$ to $\hat{\bar{Y}}_{21}$ listed in the Table 1.2. are represented in a single class (say Class 2), which will be very much useful for comparing with that of proposed modified ratio estimators, and are given below:

$$B\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\frac{S_x^2}{\bar{Y}}\,R_i^2$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right)\ i = 10,11,12,\ldots,21 \qquad (3.2)$$

where

$R_{10} = \frac{\bar{Y}}{\bar{X}}, R_{11} = \frac{\bar{Y}}{\bar{X}+C_x}, R_{12} = \frac{\bar{Y}}{\bar{X}+\beta_2}, R_{13} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}, R_{14} = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}, R_{15} = \frac{\bar{Y}}{\bar{X}+\beta_1}, R_{16} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2},$

$R_{17} = \frac{\bar{Y}}{\bar{X}+\rho}, R_{18} = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho}, R_{19} = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x}, R_{20} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$ and $R_{21} = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

Class 3: The biases, the mean squared errors and the constants of the 10 proposed modified ratio estimators $\hat{\bar{Y}}_{p1}\ to\ \hat{\bar{Y}}_{p10}$ listed in the Table 2.1. are represented in a single class (say Class 3), which will be very much useful for comparing with that of existing modified ratio estimators given in Class1 and Class 2, and are given below:

$$B\left(\hat{\bar{Y}}_{pj}\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_{pj}^2 C_x^2 - \theta_{pj} C_x C_y\,\rho)$$

$$MSE\left(\hat{\bar{Y}}_{pj}\right) = \frac{(1-f)}{n}\,\bar{Y}^2\left(C_y^2 + \theta_{pj}^2 C_x^2 - 2\theta_{pj} C_x C_y\,\rho\right),\quad j = 1,2,3,\ldots,10 \qquad (3.3)$$

where $\theta_{p1} = \frac{\overline{X}}{\overline{X}+D_1}, \theta_{p2} = \frac{\overline{X}}{\overline{X}+D_2}, \theta_{p3} = \frac{\overline{X}}{\overline{X}+D_3}, \theta_{p4} = \frac{\overline{X}}{\overline{X}+D_4}, \theta_{p5} = \frac{\overline{X}}{\overline{X}+D_5}, \theta_{p6} = \frac{\overline{X}}{\overline{X}+D_6}$,

$$\theta_{p7} = \frac{\overline{X}}{\overline{X}+D_7}, \theta_{p8} = \frac{\overline{X}}{\overline{X}+D_8}, \theta_{p9} = \frac{\overline{X}}{\overline{X}+D_9} \text{ and } \theta_{p10} = \frac{\overline{X}}{\overline{X}+D_{10}}$$

From the expressions given in (3.1) and (3.3) we have derived the conditions for which the proposed estimator $\hat{\overline{Y}}_{pj}$ is more efficient than the existing modified ratio type estimators given in Class 1, $\hat{\overline{Y}}_i\,; i = 1, 2, 3, \ldots, 9$, and which are given below.

$$MSE\left(\hat{\overline{Y}}_{pj}\right) < MSE\left(\hat{\overline{Y}}_i\right) \text{ if } \rho < \frac{(\theta_{pj}+\theta_i)}{2}\frac{C_x}{C_y}\;; i = 1, 2, 3, \ldots, 9, j = 1, 2, 3, \ldots, 10 \tag{3.4}$$

From the expressions given in (3.2) and (3.3) we have derived the conditions for which the proposed estimator $\hat{\overline{Y}}_{pj}$ is more efficient than the existing modified ratio estimators given in Class 2, $\hat{\overline{Y}}_i\,; i = 10, 11, 12, \ldots, 21$, and which are given below:

$$MSE\left(\hat{\overline{Y}}_{pj}\right) < MSE\left(\hat{\overline{Y}}_i\right) \text{ if } \frac{\theta_{pj}C_x - R_i^* S_x}{C_y} < \rho < \frac{R_i^* S_x + \theta_{pj}C_x}{C_y} \text{ or } \frac{R_i^* S_x + \theta_{pj}C_x}{C_y} < \rho < \frac{\theta_{pj}C_x - R_i^* S_x}{C_y} \tag{3.5}$$

$i = 10, 11, 12, \ldots, 21,\; j = 1, 2, 3, \ldots, 10$

where $R_i^* = \frac{R_i}{\overline{Y}}$.

4. Empirical study

The performances of the proposed modified ratio estimators listed in Table 2.1. are assessed with that of existing modified ratio estimators listed in Table 1.1. and Table 1.2. for certain natural populations. In this connection, we have considered three natural populations for the assessment of the performances of the proposed modified ratio estimators with that of existing modified ratio estimators. They are: Population 1 is the closing price of the industry ACC in the National Stock Exchange from 2, January 2012 to 27, February 2012 [16]; Population 2 and Population 3 are taken from Singh and Chaudhary [8] given in page 177. The population parameters and the constants computed from the above populations are given below:

Parameters	Population 1	Population 2	Population 3
N	40	34	34
n	20	20	20
$\overline{Y}$	5141.5363	856.4117	856.4117

Parameters	Population 1	Population 2	Population 3
$\bar{X}$	1221.6463	208.8823	199.4412
ρ	0.9244	0.4491	0.4453
S_y	256.1464	733.1407	733.1407
C_y	0.0557	0.8561	0.8561
S_x	102.5494	150.5059	150.2150
C_x	0.0839	0.7205	0.7531
β_2	-1.5154	0.0978	1.0445
β_1	0.3761	0.9782	1.1823
D_1	1111.8150	70.3000	60.6000
D_2	1119.4800	76.8000	83.0000
D_3	1139.2000	108.2000	102.7000
D_4	1159.8400	129.4000	111.2000
D_5	1184.2250	150.0000	142.5000
D_6	1252.5500	227.2000	210.2000
D_7	1307.1000	250.4000	264.5000
D_8	1345.7200	335.6000	304.4000
D_9	1366.7850	436.1000	373.2000
D_{10}	1389.3000	564.0000	634.0000

The constants, the biases and the mean squared errors of the existing and proposed modified ratio estimators for the above populations are respectively given in the Tables 4.1. to 4.3.

Table 4.1. The constants of the existing and proposed modified ratio type estimators

Estimator	Constants θ_i or R_i		
	Population 1	Population 2	Population 3
$\hat{\bar{Y}}_1$ Sisodia and Dwivedi [13]	0.9999	0.9966	0.9962
$\hat{\bar{Y}}_2$ Singh et.al [11]	1.0012	0.9995	0.9948
$\hat{\bar{Y}}_3$ Yan and Tian [15]	0.9997	0.9953	0.9941
$\hat{\bar{Y}}_4$ Singh and Tailor [10]	0.9992	0.9979	0.9978
$\hat{\bar{Y}}_5$ Upadhyaya and Singh [14]	1.0150	0.9994	0.9931
$\hat{\bar{Y}}_6$ Upadhyaya and Singh [14]	1.0000	0.9658	0.9964
$\hat{\bar{Y}}_7$ Yan and Tian [15]	1.0002	0.9542	0.9944
$\hat{\bar{Y}}_8$ Yan and Tian [15]	1.0033	0.9995	0.9956
$\hat{\bar{Y}}_9$ Yan and Tian [15]	0.9963	0.9935	0.9922
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi [2]	4.2087	4.1000	4.2941
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi [2]	4.2084	4.0859	4.2779
$\hat{\bar{Y}}_{12}$ Kadilar and Cingi [2]	4.2139	4.0981	4.2717
$\hat{\bar{Y}}_{13}$ Kadilar and Cingi [2]	4.2089	3.9598	4.2786
$\hat{\bar{Y}}_{14}$ Kadilar and Cingi [2]	4.2718	4.0973	4.2644

Table 4.1. The constants of the existing and proposed modified ratio type estimators (cont.)

Estimator	Constants θ_i or R_i		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_{15}$ Yan and Tian [15]	4.2074	4.0809	4.2688
$\widehat{\bar{Y}}_{16}$ Yan and Tian [15]	4.2226	4.0980	4.2751
$\widehat{\bar{Y}}_{17}$ Kadilar and Cingi [3]	4.2055	4.0912	4.2845
$\widehat{\bar{Y}}_{18}$ Kadilar and Cingi [3]	4.1711	4.0878	4.2814
$\widehat{\bar{Y}}_{19}$ Kadilar and Cingi [3]	4.2084	4.0687	4.2579
$\widehat{\bar{Y}}_{20}$ Kadilar and Cingi [3]	4.2108	4.0115	4.2849
$\widehat{\bar{Y}}_{21}$ Kadilar and Cingi [3]	4.2143	4.0957	4.2441
$\widehat{\bar{Y}}_{p1}$(Proposed estimator)	0.5235	0.7482	0.7670
$\widehat{\bar{Y}}_{p2}$(Proposed estimator)	0.5218	0.7312	0.7061
$\widehat{\bar{Y}}_{p3}$(Proposed estimator)	0.5175	0.6588	0.6601
$\widehat{\bar{Y}}_{p4}$(Proposed estimator)	0.5130	0.6175	0.6420
$\widehat{\bar{Y}}_{p5}$(Proposed estimator)	0.5078	0.5820	0.5833
$\widehat{\bar{Y}}_{p6}$(Proposed estimator)	0.4938	0.4790	0.4869
$\widehat{\bar{Y}}_{p7}$(Proposed estimator)	0.4831	0.4548	0.4299
$\widehat{\bar{Y}}_{p8}$(Proposed estimator)	0.4758	0.3836	0.3958
$\widehat{\bar{Y}}_{p9}$(Proposed estimator)	0.4720	0.3239	0.3483
$\widehat{\bar{Y}}_{p10}$(Proposed estimator)	0.4679	0.2703	0.2393

Table 4.2. The biases of the existing and proposed modified ratio type estimators

Estimator	Bias $B(.)$		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_{1}$Sisodia and Dwivedi [13]	0.3505	4.2233	4.8836
$\widehat{\bar{Y}}_{2}$ Singh et.al [11]	0.3522	4.2631	4.8621
$\widehat{\bar{Y}}_{3}$ Yan and Tian [15]	0.3502	4.2070	4.8519
$\widehat{\bar{Y}}_{4}$ Singh and Tailor [10]	0.3497	4.2406	4.9064
$\widehat{\bar{Y}}_{5}$ Upadhyaya and Singh [14]	0.3697	4.2607	4.8369
$\widehat{\bar{Y}}_{6}$ Upadhyaya and Singh [14]	0.3507	3.8212	4.8860
$\widehat{\bar{Y}}_{7}$ Yan and Tian [15]	0.3509	3.6732	4.8556
$\widehat{\bar{Y}}_{8}$ Yan and Tian [15]	0.3548	4.2630	4.8739
$\widehat{\bar{Y}}_{9}$Yan and Tian [15]	0.3460	4.1831	4.8236
$\widehat{\bar{Y}}_{10}$ Kadilar and Cingi [2]	0.9058	9.1539	10.0023
$\widehat{\bar{Y}}_{11}$ Kadilar and Cingi [2]	0.9056	9.0911	9.9272
$\widehat{\bar{Y}}_{12}$ Kadilar and Cingi [2]	0.9080	9.1454	9.8983

Table 4.2. The biases of the existing and proposed modified ratio type estimators (cont.)

Estimator	Bias B(.)		
	Population 1	**Population 2**	**Population 3**
$\hat{\bar{Y}}_{13}$ Kadilar and Cingi [2]	0.9058	8.5387	9.9303
$\hat{\bar{Y}}_{14}$ Kadilar and Cingi [2]	0.9331	9.1420	9.8646
$\hat{\bar{Y}}_{15}$ Yan and Tian [15]	0.9052	9.0688	9.8847
$\hat{\bar{Y}}_{16}$ Yan and Tian [15]	0.9118	9.1452	9.9143
$\hat{\bar{Y}}_{17}$ Kadilar and Cingi [3]	0.9044	9.1147	9.9578
$\hat{\bar{Y}}_{18}$ Kadilar and Cingi [3]	0.8896	9.0995	9.9432
$\hat{\bar{Y}}_{19}$ Kadilar and Cingi [3]	0.9056	9.0149	9.8348
$\hat{\bar{Y}}_{20}$ Kadilar and Cingi [3]	0.9066	8.7630	9.9597
$\hat{\bar{Y}}_{21}$ Kadilar and Cingi [3]	0.9081	9.1349	9.7711
$\hat{\bar{Y}}_{p1}$(Proposed estimator)	0.0424	1.4697	2.0008
$\hat{\bar{Y}}_{p2}$(Proposed estimator)	0.0430	1.3223	1.4125
$\hat{\bar{Y}}_{p3}$(Proposed estimator)	0.0447	0.7548	1.0164
$\hat{\bar{Y}}_{p4}$(Proposed estimator)	0.0464	0.4741	0.8726
$\hat{\bar{Y}}_{p5}$(Proposed estimator)	0.0483	0.2581	0.4499
$\hat{\bar{Y}}_{p6}$(Proposed estimator)	0.0533	0.2394	0.0939
$\hat{\bar{Y}}_{p7}$(Proposed estimator)	0.0568	0.3281	0.3279
$\hat{\bar{Y}}_{p8}$(Proposed estimator)	0.0591	0.5266	0.4367
$\hat{\bar{Y}}_{p9}$(Proposed estimator)	0.0602	0.6218	0.5499
$\hat{\bar{Y}}_{p10}$(Proposed estimator)	0.0614	0.6515	0.6387

Table 4.3. The mean squared errors of the existing and proposed modified ratio type estimators

Estimator	Mean Squared Error MSE(.)		
	Population 1	**Population 2**	**Population 3**
$\hat{\bar{Y}}_{1}$ Sisodia and Dwivedi [13]	995.2787	10514.2250	10929.0458
$\hat{\bar{Y}}_{2}$ Singh et.al [11]	1000.0116	10535.8620	10916.9080
$\hat{\bar{Y}}_{3}$ Yan and Tian [15]	994.4171	10505.3563	10911.1914
$\hat{\bar{Y}}_{4}$ Singh and Tailor [10]	992.8028	10523.6171	10941.9491
$\hat{\bar{Y}}_{5}$ Upadhyaya and Singh [14]	1050.6525	10534.5417	10902.7384
$\hat{\bar{Y}}_{6}$ Upadhyaya and Singh [14]	995.6899	10298.4432	10930.3879
$\hat{\bar{Y}}_{7}$ Yan and Tian [15]	996.2592	10220.4736	10913.2804
$\hat{\bar{Y}}_{8}$ Yan and Tian [15]	1007.5083	10535.7860	10923.6103
$\hat{\bar{Y}}_{9}$ Yan and Tian [15]	982.4136	10492.3779	10895.2039

Table 4.3. The mean squared errors of the existing and proposed modified ratio type estimators (cont.)

Estimator	Mean Squared Error MSE(.)		
	Population 1	Population 2	Population 3
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi [2]	4954.6195	16673.4489	17437.6451
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi [2]	4953.9796	16619.6435	17373.3111
$\hat{\bar{Y}}_{12}$ Kadilar and Cingi [2]	4966.1946	16666.1389	17348.6192
$\hat{\bar{Y}}_{13}$ Kadilar and Cingi [2]	4955.0419	16146.6142	17376.0389
$\hat{\bar{Y}}_{14}$ Kadilar and Cingi [2]	5095.3661	16663.3064	17319.7468
$\hat{\bar{Y}}_{15}$ Yan and Tian [15]	4951.7534	16600.5393	17336.9770
$\hat{\bar{Y}}_{16}$ Yan and Tian [15]	4985.4911	16665.9758	17362.2582
$\hat{\bar{Y}}_{17}$ Kadilar and Cingi [3]	4947.5796	16639.8457	17399.5196
$\hat{\bar{Y}}_{18}$ Kadilar and Cingi [3]	4871.7809	16626.8702	17387.0811
$\hat{\bar{Y}}_{19}$ Kadilar and Cingi [3]	4953.9273	16554.4002	17294.1864
$\hat{\bar{Y}}_{20}$ Kadilar and Cingi [3]	4959.2739	16338.6465	17401.1397
$\hat{\bar{Y}}_{21}$ Kadilar and Cingi [3]	4967.1427	16657.1867	17239.6579
$\hat{\bar{Y}}_{p1}$(Proposed estimator)	334.8577	9194.9620	9454.2668
$\hat{\bar{Y}}_{p2}$(Proposed estimator)	336.2980	9139.9570	9214.1709
$\hat{\bar{Y}}_{p3}$(Proposed estimator)	340.0837	8956.7638	9074.5845
$\hat{\bar{Y}}_{p4}$(Proposed estimator)	344.1636	8889.1069	9029.7423
$\hat{\bar{Y}}_{p5}$(Proposed estimator)	349.1280	8852.3417	8922.5155
$\hat{\bar{Y}}_{p6}$(Proposed estimator)	363.7720	8857.3224	8874.7609
$\hat{\bar{Y}}_{p7}$(Proposed estimator)	376.1193	8882.6263	8921.3976
$\hat{\bar{Y}}_{p8}$(Proposed estimator)	385.1501	9010.2560	8975.8044
$\hat{\bar{Y}}_{p9}$(Proposed estimator)	390.1632	9178.8233	9085.0541
$\hat{\bar{Y}}_{p10}$(Proposed estimator)	395.5824	9377.5847	9481.5539

From the values of Table 4.2 it is observed that the biases of the proposed modified ratio estimators are lower than the biases of all the 21 existing modified ratio estimators. Similarly, from the values of Table 4.3, it is observed that the mean squared errors of the proposed modified ratio estimators are lower than the mean squared errors of all the 21 existing modified ratio estimators.

5. Conclusion

In this paper we have proposed a class of modified ratio type estimators using known values of population deciles of the auxiliary variable. The biases and mean squared errors of the proposed estimators are obtained and compared with that of existing modified ratio estimators. Further, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators. We have also assessed the performances of the proposed estimators for some known populations. It is observed that the biases and mean squared errors of the proposed estimators are lower than the biases and mean squared errors of the existing modified ratio estimators for certain known populations. Hence, we strongly recommend that the proposed modified estimators may be preferred over the existing modified ratio estimators for the use of practical applications.

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