STATISTICS IN TRANSITION-new series, Summer 2013
Vol. 14, No. 2, pp. 189–200

A CLASS OF PRODUCT-TYPE EXPONENTIAL ESTIMATORS OF THE POPULATION MEAN IN SIMPLE RANDOM SAMPLING SCHEME

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ABSTRACT

The present study proposes a class of product-type exponential estimators for estimating the population mean of the study variable, using known values of some population parameters of an auxiliary character, under the simple random sampling without replacement (SRSWOR) scheme. Furthermore, the study also proposes a modified exponential estimator based on both the ratio-type and the product-type exponential estimators. Properties of the proposed estimators, under the SRSWOR scheme, are obtained up to first order approximation. The modified exponential estimator under optimum conditions is shown to be more efficient than the simple sample mean and the ratio-type and product-type exponential estimators. The theoretical results are supported by an empirical illustration.

Key words: ratio-type and product-type exponential estimators, auxiliary character, simple random sampling, mean square error. 2010 AMS Classification: 62D05.

1. Introduction

The use of information on auxiliary character to improve estimates of population parameters of the study variable is a common practice in sample surveys, especially when there is a strong linear relationship between the study and auxiliary variables. Several authors have made contributions in this regard, including Sukhatme and Sukhatme (1970) and Cochran (1977). Singh and Tailor (2005) suggested the use of known correlation coefficient between auxiliary characters for the estimation of finite population mean. Khoshnevisan et al. (2007) suggested a family of estimators of the population mean using known values of population parameters in simple random sampling. Tailor and Sharma (2009) used known coefficient of variation and coefficient of kurtosis of an

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auxiliary character in estimating finite population mean of the study variable. Sharma and Tailor (2010) suggested a modified ratio-cum-dual to ratio estimator using known population mean of an auxiliary character. Onyeka (2012) used known values of population parameters of an auxiliary character to improve estimates of population mean in post-stratified random sampling. In using auxiliary information, different types of estimators have been considered, including the usual ratio-type, product-type and regression-type estimators. Recently, some authors have introduced dual and exponential estimators. Let y and x respectively denote the study and auxiliary characters in a finite population of N units; and let (y_i, x_i), i = 1, 2, … , n denote sample values of y and x in a random sample of n units drawn by simple random sampling without replacement (SRSWOR) method. In using known population mean $(\overline{X})$ of an auxiliary character x to improve estimates of the population mean $(\overline{Y})$ of the study variable y, Bahl and Tuteja (1991) introduced ratio and product type exponential estimators of the forms:

$$t_R = \overline{y}\exp\left[\frac{\overline{X} - \overline{x}}{\overline{X} + \overline{x}}\right] \tag{1.1}$$

and

$$t_P = \overline{y}\exp\left[\frac{\overline{x} - \overline{X}}{\overline{x} + \overline{X}}\right] \tag{1.2}$$

where $\overline{y}$ $(\overline{x})$ is the sample mean of the study (auxiliary) variable.

Singh and Vishwakama (2007) in their study used some modified exponential ratio-type and product-type estimators in estimating population mean in double sampling scheme. Singh et al. (2009a), following Kadilar and Cingi (2006) and Khoshnevisan et al. (2007), proposed a class of ratio-type exponential estimators of the population mean in simple random sampling, using known values of population parameters of an auxiliary character, of the form:

$$t_1 = \overline{y}\exp\left[\frac{(a\overline{X} + b) - (a\overline{x} + b)}{(a\overline{X} + b) + (a\overline{x} + b)}\right] = \overline{y}\exp\left[\frac{a(\overline{X} - \overline{x})}{a(\overline{X} + \overline{x}) + 2b}\right] \tag{1.3}$$

where $a\ (\neq 0)$, b are either real numbers or functions of known parameters of the auxiliary variable x such as coefficient of variation (C_x), coefficient of skewness $(\beta_1(x))$, coefficient of kurtosis $(\beta_2(x))$, standard deviation (σ) and correlation coefficient (ρ). It is worth mentioning that the choice of the values of a and b, in practice, depends largely on the availability of known population parameters, which, admittedly, are not frequently known. The estimator in (1.3) is only useful when such population parameters are known, often from previous surveys. In sampling theory, preference, in terms of the known parameter to use, is usually given to those parameters that yield smaller variance or mean squared error when used to construct estimators. Again, notice that some of the population parameters like ρ and C_x are unitless. If such unitless quantities are used for the quantity b in (1.3), it implies that they are associated with the measurement unit of x and, in fact, they assume, temporarily, the same measurement unit of x. Ordinarily, this assumption would not cause any serious distortion in the value and measurement unit of the expression $a(\overline{X}+\overline{x})+2b$ in (1.3), if the value of b does not exceed unity. The estimator t_1 is biased for $\overline{Y}$ with mean square error, obtained up to first order approximation as:

$$MSE(t_1)=\left(\frac{1-f}{n}\right)\overline{Y}^2\left[C_y^2+\tfrac{1}{4}\theta C_x^2(\theta-4K)\right] \tag{1.4}$$

where

$$\theta=\frac{a\overline{X}}{a\overline{X}+b}\ ,\ K=\frac{\rho C_y}{C_x}\ ,\ f=\frac{n}{N} \tag{1.5}$$

and $C_y(C_x)$ and ρ are the coefficient of variation of y(x), and the correlation coefficient, respectively. Singh et al. (2009b) considered some ratio-type exponential estimators in stratified random sampling. Shabbir and Gupta (2010) suggested exponential estimator for finite population mean in two phase sampling scheme when auxiliary variables are attributes. Grover and Kaur (2011) improved on the work carried out by Abd-Elfattah et al. (2010) by proposing ratio-type exponential estimators of finite population mean in simple random sampling scheme using an auxiliary attribute. Singh et al. (2011) suggested exponential ratio and exponential product type estimators for estimating unknown population variance, using information on two auxiliary characters.

In the present study, we have extended the work carried out by Singh et al. (2009a), by suggesting a class of product-type exponential estimators for estimating the finite population mean of the study variable, using known values of population parameters of an auxiliary variable. The present study also proposes a modified exponential estimator based on both the ratio-type and product-type exponential estimators, and compares the efficiency of the modified exponential estimator with those of the simple sample mean, the ratio-type exponential estimator and the product-type exponential estimator. Properties of the proposed estimators, especially the biases and mean squared errors of the estimators, are obtained up to first order approximations under the SRSWOR scheme.

2. The proposed product-type exponential estimator

Following Singh et al. (2009a), we propose a class of product-type exponential estimators of the population mean $\overline{Y}$, in SRSWOR scheme as

$$t_2 = \overline{y}\exp\left[\frac{(a\overline{x}+b)-(a\overline{X}+b)}{(a\overline{x}+b)+(a\overline{X}+b)}\right] = \overline{y}\exp\left[\frac{a(\overline{x}-\overline{X})}{a(\overline{x}+\overline{X})+2b}\right] \quad (2.1)$$

Let,

$$e_0 = \frac{\overline{y}-\overline{Y}}{\overline{Y}} \text{ and } e_1 = \frac{\overline{x}-\overline{X}}{\overline{X}}. \quad (2.2)$$

so that

$$E(e_0) = E(e_1) = 0 \quad (2.3)$$

$$E(e_0^2) = \frac{V(\overline{y})}{\overline{Y}^2} = \left(\frac{1-f}{n}\right)C_y^2 \quad (2.4)$$

$$E(e_1^2) = \frac{V(\overline{x})}{\overline{X}^2} = \left(\frac{1-f}{n}\right)C_x^2 \quad (2.5)$$

$$E(e_0e_1) = \frac{Cov(\overline{y},\overline{x})}{\overline{Y}\overline{X}} = \left(\frac{1-f}{n}\right)KC_x^2 \quad (2.6)$$

Rewriting (2.1) in terms of e_0 and e_1, and expanding up to first order approximation in expected value gives:

$$(t_2 - \overline{Y}) = \overline{Y}\,[e_0 + \tfrac{1}{2}\theta e_1 + \tfrac{1}{2}\theta e_0 e_1 - \tfrac{1}{8}\theta^2 e_1^2\,] \quad (2.7)$$

and

$$(t_2 - \overline{Y})^2 = \overline{Y}^2\,[e_0^2 + \tfrac{1}{4}\theta^2 e_1^2 + \theta e_0 e_1\,] \quad (2.8)$$

Taking expectation of (2.7) and (2.8), and using (2.3) – (2.6) to make the necessary substitutions gives the bias and mean square error of the proposed product-type exponential estimator t_2, respectively as:

$$B(t_2) = \left(\frac{1-f}{n}\right)\left(\frac{\overline{Y}}{8}\right)\theta(4K-\theta)C_x^2 \quad (2.9)$$

and

$$MSE(t_2) = \left(\frac{1-f}{n}\right)\overline{Y}^2\left[C_y^2 + \tfrac{1}{4}\theta C_x^2(\theta + 4K)\right] \tag{2.10}$$

3. Modified exponential estimators

Following authors like Cochran (1977) and Singh et al. (2009a), we propose a modified class of exponential estimators (t_3) as a linear function of the exponential estimators t_1 and t_2. We give the modified exponential estimator as:

$$t_3 = \alpha_1 t_1 + \alpha_2 t_2 \tag{3.1}$$

where α_1 and α_2 are weighting fractions such that $\alpha_1 + \alpha_2 = 1$. In practice, α_1 and α_2 should be chosen so as to minimize the mean square error of the estimator t_3. However, the value of α_1 is expected to be greater than $\frac{1}{2}$, that is, greater than the value of α_2 when there is a strong positive linear relationship between the study and auxiliary variates, since most ratio-type estimators are known to perform well under such a condition. Conversely, the value of α_2 is expected to be greater than $\frac{1}{2}$, or greater than the value of α_1 when there is a strong negative correlation between the study and auxiliary variates, since most product-type estimators are known to perform well under such a condition. Following Cochran (1977), the best (optimal) choices of α_1 and α_2 are respectively obtained as:

$$\alpha_1 = \frac{V_{22} - V_{12}}{V_{11} + V_{22} - 2V_{12}} \tag{3.2}$$

and

$$\alpha_2 = \frac{V_{11} - V_{12}}{V_{11} + V_{22} - 2V_{12}} \tag{3.3}$$

with the resultant optimum mean square error of t_3 given as:

$$MSE_{opt}(t_3) = \frac{V_{11}V_{22} - V_{12}^2}{V_{11} + V_{22} - 2V_{12}} \tag{3.4}$$

where V_{ii} is the mean square error t_i $(i=1,2)$ and V_{12} is the covariance term of the estimators t_1 and t_2. Rewriting (1.3) in terms of e_0 and e_1, and expanding up to first order approximation in expected value gives:

$$(t_1-\overline{Y})=\overline{Y}\,[e_0-\tfrac{1}{2}\theta e_1-\tfrac{1}{2}\theta e_0e_1+\tfrac{3}{8}\theta^2e_1^2\,] \tag{3.5}$$

so that using (3.5) and (2.7), we obtain up to first order approximation in expected value:

$$(t_1-\overline{Y})(t_2-\overline{Y})=\overline{Y}^2\,[e_0^2-\tfrac{1}{4}\theta^2e_1^2\,] \tag{3.6}$$

Taking expectation of (3.6) gives the covariance term of the estimators t_1 and t_2 as

$$\mathrm{Cov}(t_1\,,t_2)=\left(\frac{1-f}{n}\right)\overline{Y}^2\left(C_y^2\text{ - }\tfrac{1}{4}\theta^2C_x^2\right) \tag{3.7}$$

Accordingly, it follows from (1.4), (2.10) and (3.7), that

$$V_{11}=\mathrm{MSE}(t_1)=\left(\frac{1-f}{n}\right)\overline{Y}^2\left[C_y^2+\tfrac{1}{4}\theta C_x^2(\theta-4K)\right] \tag{3.8}$$

$$V_{22}=\mathrm{MSE}(t_2)=\left(\frac{1-f}{n}\right)\overline{Y}^2\left[C_y^2+\tfrac{1}{4}\theta C_x^2(\theta+4K)\right] \tag{3.9}$$

$$V_{12}=\mathrm{Cov}(t_1\,,t_2)=\left(\frac{1-f}{n}\right)\overline{Y}^2\left(C_y^2\text{ - }\tfrac{1}{4}\theta^2C_x^2\right) \tag{3.10}$$

Using (3.8) – (3.10) to make the necessary substitutions in (3.2) – (3.4) gives the best (optimal) choices of α_1 and α_2, together with the associated minimum mean square error of the modified exponential estimator t_3, as:

$$\alpha_1=\frac{\theta+2K}{2\theta} \tag{3.11}$$

$$\alpha_2=\frac{\theta-2K}{2\theta} \tag{3.12}$$

$$MSE_{opt}(t_3) = \left(\frac{1-f}{n}\right)\overline{Y}^2(1-\rho^2)C_y^2 \tag{3.13}$$

In practice, the weighting fractions α_1 and α_2 should be chosen very close to the expressions given in (3.11) and (3.12), respectively. We note that (3.13) is the same as the variance of the customary regression estimator. This indicates that the efficiency of the proposed modified exponential estimators may not be improved beyond that of the usual regression-type estimator.

4. Efficiency comparison

The variance of the simple sample mean $\overline{y}$ in SRSWOR scheme is given by

$$V(\overline{y}) = \left(\frac{1-f}{n}\right)\overline{Y}^2 C_y^2 \tag{4.1}$$

Comparing (4.1) and (3.8), it follows that the ratio-type exponential estimator t_1, proposed by Singh et al. (2009a) would perform better than the sample mean in terms of having a smaller mean square error if

$$\frac{K}{\theta} = \left(\frac{\rho C_y}{C_x}\right)\left(\frac{a\overline{X}+b}{a\overline{X}}\right) > \frac{1}{4} \tag{4.2}$$

Notice that there is a slight mathematical error in the efficiency condition given by Singh et al. (2009a) with respect to the sign of the inequality of (5.2) of Singh et al. (2009a).

Comparing the product-type exponential estimator t_2 proposed in the present study with the sample mean $\overline{y}$, it follows from (4.1) and (3.9) that the proposed product-type exponential estimator t_2 would perform better than the sample mean in terms of having a smaller mean square error if

$$\frac{K}{\theta} = \left(\frac{\rho C_y}{C_x}\right)\left(\frac{a\overline{X}+b}{a\overline{X}}\right) < -\frac{1}{4} \tag{4.3}$$

To compare the efficiency of the proposed modified exponential estimator, t_3, with those of the estimators, $\overline{y}$, t_1 and t_2, we observe from (4.1), (3.8), (3.9) and (3.13), that:

Lemma I: The proposed modified exponential estimator t_3, under optimum conditions, (3.11) and (3.12), is more efficient than the sample mean $\bar{y}$ in terms of having a smaller mean square error, if

$$\rho^2 > 0 \text{ (which is always true)} \tag{4.4}$$

Lemma II: The proposed modified exponential estimator t_3, under optimum conditions, (3.11) and (3.12), is more efficient than the ratio-type exponential estimator t_1 proposed by Singh et al. (2009a) if

$$(\rho C_y - \tfrac{1}{2}\theta C_x)^2 > 0 \text{ (which is always true)} \tag{4.5}$$

Lemma III: The proposed modified exponential estimator t_3, under optimum conditions, (3.11) and (3.12), is more efficient than the product-type exponential estimator t_2 proposed in the present study if

$$(\rho C_y + \tfrac{1}{2}\theta C_x)^2 > 0 \text{ (which is always true)} \tag{4.6}$$

The implication of the above three lemmas is that once the weighting fractions α_1 and α_2 are chosen very close to their optimal values given in (3.11) and (3.12), respectively, then the proposed modified exponential estimator t_3 would be more efficient than the sample mean $\bar{y}$, the ratio-type exponential estimator t_1 proposed by Singh et al. (2009a), and the product-type exponential estimator t_2 proposed in the present study.

5. Numerical Illustration

For the purpose of empirical illustration of our theoretical results, consider the following five (5) members $t_j(i)$, $(j = 1, 2;\ i = 1, 2, 3, 4, 5)$, each of the estimators t_1 and t_2.

Estimator (i)	$t_1(i)$	$t_2(i)$	a	b
1	$t_1(1) = \bar{y}\exp\left[\dfrac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$	$t_2(1) = \bar{y}\exp\left[\dfrac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right]$	1	1
2	$t_1(2) = \bar{y}\exp\left[\dfrac{\bar{X} - \bar{x}}{\bar{X} + \bar{x} + 2C_x}\right]$	$t_2(2) = \bar{y}\exp\left[\dfrac{\bar{x} - \bar{X}}{\bar{x} + \bar{X} + 2C_x}\right]$	1	C_x

Estimator (i)	$t_1(i)$	$t_2(i)$	a	b
3	$t_1(3)=\bar{y}\exp\left[\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2\rho}\right]$	$t_2(3)=\bar{y}\exp\left[\dfrac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}+2\rho}\right]$	1	ρ
4	$t_1(4)=\bar{y}\exp\left[\dfrac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\rho}\right]$	$t_2(4)=\bar{y}\exp\left[\dfrac{C_x(\bar{x}-\bar{X})}{C_x(\bar{x}+\bar{X})+2\rho}\right]$	C_x	ρ
5	$t_1(5)=\bar{y}\exp\left[\dfrac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2C_x}\right]$	$t_2(5)=\bar{y}\exp\left[\dfrac{\rho(\bar{x}-\bar{X})}{\rho(\bar{x}+\bar{X})+2C_x}\right]$	ρ	C_x

The mean square errors of the estimators $t_j(i)$, $(j=1,2;\ i=1,2,3,4,5)$, are easily obtained using (3.8), (3.9) and (1.5). To illustrate the efficiency of the estimators, consider the following two sets of data given in Tailor et al. (2012).

<u>Data I:</u> Johnston, page 171

y = Percentage of hives affected by disease

x = Date of flowering of a particular summer species (number of days from January 1)

$$N=10,\ n=4,\ \bar{Y}=52,\ \bar{X}=200,\ C_y=0.1562,\ C_x=0.04583,\ \rho=-0.94$$

Using Data I, the computed percentage relative efficiencies of the estimators over the simple sample mean estimator $\bar{y}$ are displayed in Table 1.

Table 1. Percentage Relative Efficiency (PRE) over the Sample Mean $\bar{y}$

Estimators	$\bar{y}$	i = 1	i = 2	i = 3	i = 4	i = 5	t_3
$t_1(i)$	100	77.18	77.09	76.99	74.96	77.08	859.11
$t_2(i)$	100	133.89	134.09	134.30	139.00	134.11	859.11

<u>Data II:</u> Johnston, page 171

y = Percentage of hives affected by disease

x = Mean January Temperature (^{0}C)

$$N=10,\ n=4,\ \bar{Y}=52,\ \bar{X}=42,\ C_y=0.1562,\ C_x=0.13038,\ \rho=0.80$$

Using Data II, the computed percentage relative efficiencies of the estimators over the simple sample mean estimator $\bar{y}$ are displayed in Table 2.

Table 2. Percentage Relative Efficiency (PRE) over the Sample Mean $\bar{y}$

Estimators	$\bar{y}$	i					t_3
		1	2	3	4	5	
$t_1(i)$	100	194.57	197.08	195.14	181.83	196.98	277.78
$t_2(i)$	100	54.99	54.38	54.85	58.30	54.41	277.78

Tables 1 and 2 reveal that the ratio-type exponential estimator t_1 proposed by Singh et al. (2009a), and the product-type exponential estimator t_2 proposed in the present study, are not always or uniformly more efficient than the simple sample mean $\bar{y}$, except when the efficiency conditions (4.2) and (4.3) are respectively satisfied. Again, the numerical results in Tables 1 and 2 confirmed, as expected, that the proposed product-type exponential estimator t_2 is preferred over the ratio-type exponential estimator t_1 proposed by Singh et al. (2009a), when there is a strong negative correlation between the study and auxiliary characters, while the estimator t_1 is preferred over the estimator t_2 when there is a strong positive correlation between the study and auxiliary variables. The numerical results also confirmed that under the optimum conditions (3.11) and (3.12), the modified exponential estimator t_3, which is a linear function of the estimators t_1 and t_2, is more efficient than the sample mean $\bar{y}$ and the exponential estimators t_1 and t_2.

6. Conclusion

We have extended the work carried out by Singh et al. (2009a) by developing a class of product-type exponential estimators of the population mean in SRSWOR scheme, using known population parameters of an auxiliary character. The proposed class of product-type exponential estimators, under certain efficiency conditions, is shown to be more efficient than the usual sample mean estimator $\bar{y}$, in terms of having a mean square error smaller than the variance of $\bar{y}$. Also, numerical illustrations confirmed that when there is a strong negative correlation between the study and auxiliary variables, the proposed product-type exponential estimator is preferred over the ratio-type exponential estimator proposed by Singh et al. (2009a). Furthermore, in the present study we have developed a modified exponential estimator, which is a linear function or combination of both the ratio-type and product-type exponential estimators. By using the optimal weighting fractions in the proposed modified exponential estimators, the modified exponential estimator is found to be more efficient than the usual sample mean estimator $\bar{y}$, the ratio-type exponential estimators t_1 proposed by Singh et al. (2009a), and the product-type exponential estimator t_2 proposed in the present study. In practice, therefore, we suggest that the weighting fractions in the modified exponential estimator be chosen very close to their optimal choices in (3.11) and (3.12) in order to realize the full benefits of using the proposed modified or improved exponential estimators of $\bar{Y}$ in SRSWOR scheme.

REFERENCES

ABD-ELFATTAH, A. M., EL-SHERPIENY, E. A., MOHAMED, S. M., ABDOU, O. F., (2010). Improvement in estimating the population mean in simple random sampling using information on auxiliary attribute. Applied Mathematics and Computation, Vol. 215, 4198–4202.

BAHL, S., TUTEJA, R. K., (1991). Ratio and Product type exponential estimator. Information and Optimization Sciences, Vol. XII, I, 159–163.

COCHRAN, W.G., (1977). Sampling techniques. 3rd edition. John Wiley and Sons, New York.

GROVER, L. K., KAUR, P., (2011). An improved exponential estimator of finite population mean in simple random sampling using an auxiliary attribute. Applied Mathematics and Computation, Vol. 218, No.7, 3093–3097.

KADILAR, C., CINGI, H., (2006). Improvement in estimating the population mean in simple random sampling. Applied Mathematics Letters, 19, 75–79.

KHOSHNEVISAN, M., SINGH, R., CHAUHAN, P., SAWAN, N., SMARANDACHE, F., (2007). A general family of estimators for estimating population mean using known value of some population parameter(s), Far East Journal of Theoretical Statistics, 22, 181–191.

ONYEKA, A. C., (2012). Estimation of population mean in post-stratified sampling using known value of some population parameter(s). Statistics in Transition-new series, 13(1), 65–78.

SHABBIR, J., GUPTA, S., (2010). Estimation of Finite Population Mean in Two Phase Sampling When Auxiliary Variables Are Attributes. Hacettepe Journal of Mathematics and Statistics, Vol. 39, No. 1, 121–129.

SHARMA, B., TAILOR, R., (2010). A New Ratio-Cum-Dual to Ratio Estimator of Finite Population Mean in Simple Random Sampling. Global Journal of Science Frontier Research, Vol. 10, Issue 1 (Ver. 1.0), 27–31.

SINGH, R., CHAUHAN, P., SAWAN, N., SMARANDACHE, F., (2009a). Improvement in estimating the population mean using exponential estimator in simple random sampling. Bulletin of Statistics & Economics, Vol. 3, A09, 13–18.

SINGH, R., CHAUHAN, P., SAWAN, N., SMARANDACHE, F., (2011). Improved exponential estimator for population variance using two auxiliary variables. Italian Journal of Pure and Applied Mathematics, No. 28, 101–108.

SINGH, R., KUMAR, M., CHAUDHARY, M. K., KADILAR, C., (2009b). Improved exponential estimator in stratified random sampling. Pakistan Journal of Statistics and Operations Research, Vol. V, No. 2, 67–82.

SINGH, H. P., TAILOR, R., (2005). Estimation of finite population mean using known correlation coefficient between auxiliary characters. Statistica, Anno LXV, 4, 407–418.

SINGH, H. P., VISHWAKAMA, G. K., (2007). Modified exponential ratio and product estimators for finite population mean in double sampling. Austrian Journal of Statistics, Vol. 36, No. 3, 217–225.

SUKHATME, P. V., SUKHATME, B. V., (1970). Sampling theory of surveys with applications. Iowa State University Press, Ames, USA.

TAILOR, R., SHARMA, B. K., (2009). A Modified Ratio-Cum-Product Estimator of Finite Population Mean Using Known Coefficient of Variation and Coefficient of Kurtosis. Statistics in Transition-new series, Jul-09, Vol. 10, No. 1, 15–24.

TAILOR, R., TAILOR, R., PARMAR, R., KUMAR, M., (2012). Dual to ratio-cum-product estimator using known parameters of auxiliary variables. Journal of Reliability and Statistical Studies, Vol. 5, Issue 1, 65–71.