STATISTICS IN TRANSITION-new series, Summer 2013
Vol. 14, No. 2, pp. 201–216

THE CLASS OF ESTIMATORS OF FINITE POPULATION MEAN USING INCOMPLETE MULTI-AUXILIARY INFORMATION

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ABSTRACT

In this paper, a class of estimators is considered for estimating the mean of the finite population utilizing available incomplete multi-auxiliary information. Some special cases of this class of estimators are considered. The approximate expressions for bias and mean square error of the suggested estimators have also been derived and theoretical results are numerically supported.

Key words: bias, mean square error, multi-auxiliary information.

1. Introduction

The use of auxiliary information for improving the precision of the estimators is well known when the variable under study Y and the auxiliary variable X are correlated. In large scale surveys, we often collect data on more than one auxiliary variable and some of these may be correlated with Y. Olkin (1958), Raj (1965), Rao and Mudholkar (1967), Srivastava (1971), Singh (1982), Agrawal and Panda (1993, 1994), Dalabehara and Sahoo (1997), Sahoo and Bala (2000), Abu-Dayyeh et. al. (2003), Ahmed (2004), Singh et. al. (2004), Kadilar and Cingi (2005), Perril (2007), Singh et. al. (2007), etc., considered some estimators which utilize information on several auxiliary variables which are correlated with the variable under study. In many situations, we may have information on several auxiliary variables but each variable may not be known for each population unit. Singh (1977) considered the concept of stratification for weighting the given incomplete auxiliary information. Srivastava and Garg (2009) suggested a class of estimators for estimating the finite population mean utilizing available incomplete multi-auxiliary information when frame is unknown for each stratum.

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The aim of the present paper is to develop a general class of weighted estimators based on incomplete multi-auxiliary information as to show that it is always better to use additional auxiliary variables which are correlated with Y. The general class of weighted estimators has been constructed when frame is known for each stratum.

Generally, the stratification is done on the basis of heterogeneity in the population with respect to the study variable Y. But we view stratification in the other way. In our case, the heterogeneity of the population is considered with respect to the unequal number of auxiliary variables. We stratify the population in terms of the information provided by the p auxiliary variables. Thus, there will be a stratum for which no auxiliary information is available, p strata for which only one auxiliary variable out of p auxiliary variables is known. Similarly, there will be ${}^{p}C_{2}$ strata for which the two auxiliary variables are known, ${}^{p}C_{3}$ strata for which three auxiliary variables are known, and so on. Ultimately, we will have a stratum for which all the p auxiliary variables are known.

It is seen that the given method, considering stratification of population on the basis of unequal number of auxiliary variable is capable of giving more precise results than simple sample mean per unit.

2. Notations

Let us consider a finite population U = (U_1, U_2, ..., U_N) of N identifiable units taking values on a study variable Y and p auxiliary variables X_1, X_2, …, X_p, which are correlated with Y. Auxiliary variables X_1, X_2, ..., X_p are known for total M_1, M_2, …, M_p units of the population, respectively. For the maximum utilization of available incomplete auxiliary information, the population is divided into different strata according to the known number of auxiliary variables and a random sample of n units is drawn from these groups with simple random sampling without replacement.

${}^{p}C_{j}$: Number of strata for which j auxiliary variables are known; j = 0, 1, 2, … , p.

N : Population size

n : Sample size

N_0 : Size of the stratum for which no auxiliary variable is known

N_{ii} : Size of the stratum for which 1 auxiliary variable X_i is known*;* i = 1,2, … , p

N_{ij} : Size of the stratum for which 2 auxiliary variable X_i and X_j are known; i < j; i, j = 1, 2, … , p

N_{ijk} : Size of the stratum for which 3 auxiliary variable X_i, X_j and X_k are known; i < j < k; i, j, k = 1,2, …, p

$N_{1,2,\ldots..,p}$: Size of the strata for which all p auxiliary variable X_1, X_2, …, X_p is known; i < j < k; i, j, k = 1,2,…, p where $N_{ii} + N_{ij} + N_{ijk} + \ldots + N_{1,2,\ldots i\ldots,p} = M_i$

2^p : Total number of strata, i.e. $\sum_{i=0}^{p} {}^pC_i = 2^p$

N_i : Population size of the i^{th} stratum; $i = 1, 2, ..., 2^p$

such that $\sum_{i=1}^{2^p} N_i = N$

n_i : Sample size of the i^{th} stratum; $i = 1, 2, \ldots, 2^p$ such that $\sum_{i=1}^{2^p} n_i = n$

Y_{ik} : Value of the k^{th} observation on variable under study in i^{th} stratum;

$i = 1, 2, \ldots, {}^pC_j$; $j = 0, 1, 2, \ldots, p$; $k = 1, 2, ..., N_i$

X_{ijk} : Value of the k^{th} observation on j^{th} auxiliary variable in i^{th} stratum

$i = 1, 2, \ldots, {}^pC_j$; $j = 0, 1, 2, \ldots, p$; $k = 1, 2, ..., N_i$

$\overline{Y}_i$: Population mean of the Y variable in i^{th} stratum

$\overline{y}_i$: Sample mean of the Y variable in i^{th} stratum

$\overline{X}_{ij}$: Population mean of the j^{th} auxiliary variable in i^{th} stratum

$\overline{x}_{ij}$: Sample mean of the j^{th} auxiliary variable in i^{th} stratum

S_i^2 : Population mean square error of *Y* variable in i^{th} stratum

S_{ij}^2 : Population mean square error of the j^{th} auxiliary variable in i^{th} stratum

C_i^2 : Coefficient of variation of the variable under study Y in i^{th} stratum, i.e. $C_i^2 = \dfrac{S_i^2}{\overline{Y}_i^2}$

C_{ij}^2 : Coefficient of variation of the j^{th} auxiliary variable in i^{th} stratum, i.e. $C_{ij}^2 = \dfrac{S_{ij}^2}{\overline{X}_{ij}^2}$

ρ_{ij} : Correlation coefficient between the variables Y, variable under study and X_{ij}, the j^{th} auxiliary variable in the i^{th} stratum

ρ_{ijh} : Correlation coefficient between the variables X_j and X_h ($j \neq h$) in the i^{th} stratum.

b_{ij} : Regression coefficient of the variables Y, variable under study and X_{ij}, the j^{th} auxiliary variable in the i^{th} stratum

W_i : Proportion of units in the i^{th} stratum, i.e. $W_i = \frac{N_i}{N}$

f : Sampling fraction, i.e. $f = \frac{n}{N}$

f_i : Sampling fraction in the i^{th} stratum, i.e. $f_i = \frac{n_i}{N_i}$

α_{ij} : Weights, attached to the j^{th} auxiliary variable in the i^{th} stratum adding up to

$$K_{ij} = \rho_{ij}\frac{C_i}{C_{ij}}; \quad j=1,2, \ldots, p$$

The following figure shows the construction of strata ($2^3 = 8$) on the basis of available incomplete three-auxiliary information (X_1, X_2 and X_3):

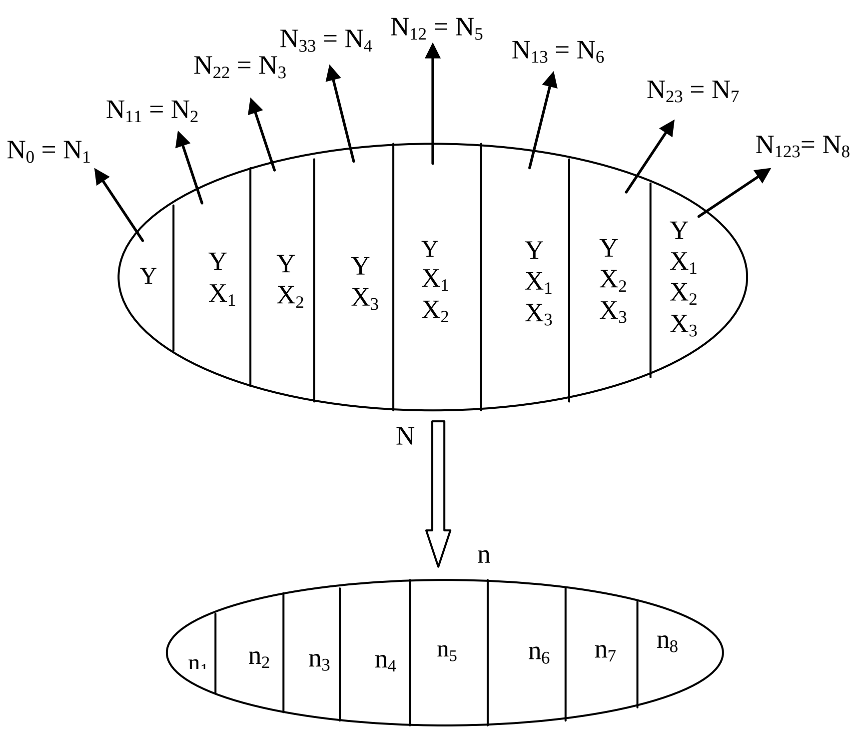

Figure 2.1. Construction of strata.

3. Suggested general class of estimators

Now, we can suggest the following general class of estimators using incomplete multi-auxiliary information.

$$\bar{y}_{pr} = \sum_{i=1}^{2^p} W_i g_i(y_i, x_i) \tag{1}$$

where $g_i(y_i, x_i) = \sum_{j=1}^{p} \alpha_{ij} g_{ij}(y_i, x_{ij})$ and $g_{11}(y_1, x_{11}) = \bar{y}_1$

$g_{ij}(y_i, x_{ij})$ is the function of $y_i = (y_{ik}; k = 1,2,..., N_i)$ and $x_{ij} = (x_{ijk}; k = 1,2,..., N_i)$

The bias and mean squared error of $\bar{y}_p$ are as follows

$$E(\bar{y}_{pr}) = \sum_{i=1}^{2^p} W_i E g_i(y_i, x_i) = \sum_{i=1}^{2^p} W_i \sum_{j=1}^{p} \alpha_{ij} E g_{ij}(y_i, x_{ij}) \tag{2}$$

$$MSE(\bar{y}_{pr}) = \sum_{i=1}^{2^p} W_i^2 MSE(g_i(y_i, x_i)) \tag{3}$$

$MSE(g_i(y_i, x_i))$ can easily be obtained for different values of the function g by generalizing the procedure used by Olkin (1958). Thus,

$$MSE(g_i(y_i, x_i)) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right) \sum_{j=1}^{p} \sum_{h=1}^{p} \alpha_{ij} \alpha_{ih} v_{ijh} \tag{4}$$

where $\left(\frac{1}{n_i} - \frac{1}{N_i}\right) v_{ijh} = Cov(g_{ij}, g_{ih})$

In matrix notation,

$$MSE(g_i(y_i, x_i)) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right) \underset{\sim}{\alpha_i} V_i \underset{\sim}{\alpha_i}'$$

where the matrix $V_i=(v_{ijh})$ and $\underset{\sim}{\alpha_i} = (\alpha_{i1}, \alpha_{i2}, ..., \alpha_{ip})$, $\underset{\sim}{\alpha_i}'$ being the transpose of $\underset{\sim}{\alpha_i}$.

3.1. Optimum values of α_{ij} for j = 1, 2, …, p

It is fairly simple to establish that the optimum α_{ij} is given by

$$\alpha_{ij} = \frac{\text{Sum of the elements of the } j^{th} \text{ column of } V_i^{-1}}{\text{Sum of all the } p^2 \text{ elements in } V_i^{-1}}$$

where V_i^{-1} is the matrix inverse to V_i. using the optimum weights, the mean square error is found to be

$$MSE(g_i(y_i, x_i)) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right) \Big/ \text{Sum of all the } p^2 \text{ elements in } V_i^{-1}$$

4. Some special cases of the suggested class of estimators

Case I. When each X_j (j = 1, 2, …, p) is positively correlated with Y in each stratum, our estimator will convert into weighted ratio estimator given as

$$\bar{y}_{pr.rat} = \sum_{i=1}^{2^p} W_i \sum_{j=1}^{p} \alpha_{ij} g_{ij.rat}(y_i, x_{ij}) \tag{5}$$

where $g_{ij.rat}(y_i, x_{ij}) = \dfrac{\bar{y}_i}{\bar{x}_{ij}} \bar{X}_{ij}$

$$\begin{aligned} Bias(\bar{y}_{pr.rat}) &= E(\bar{y}_{pr.rat}) - \bar{Y} \\ &= \sum_{i=1}^{2^p} W_i \frac{1-f_i}{n_i} \left[\bar{Y}_i \sum_{j=1}^{p} \alpha_{ij} \left\{ C_{ij}^2 (1 - K_{ij}) \right\} \right] \end{aligned} \tag{6}$$

$$\begin{aligned} MSE(\bar{y}_{pr.rat}) = \sum_{i=1}^{2^p} W_i^2 \bar{Y}_i^2 \frac{1-f_i}{n_i} \Big[C_i^2 + \sum_{j=1}^{p} \sum_{h=1}^{p} \alpha_{ij} \alpha_{ih} \big(\rho_{ijh} C_{ij} C_{ih} \\ - \rho_{ij} C_i C_{ij} - \rho_{ih} C_i C_{ih} \big) \Big] \end{aligned} \tag{7}$$

Case II. When each X_j (j = 1, 2,……, p) is positively correlated with Y in each stratum, we can also use weighted regression estimator given as

$$\bar{y}_{pr.reg} = \sum_{i=1}^{2^p} W_i \sum_{j=1}^{p} \alpha_{ij} g_{ij.reg}(y_i, x_{ij}) \tag{8}$$

where $g_{ij.reg}(y_i, x_{ij}) = \bar{y}_i + b_{ij}(\bar{X}_{ij} - \bar{x}_{ij})$

$$\because \text{Bias}\left(\bar{y}_{pr.reg}\right) = E\left(\bar{y}_{pr.reg}\right) - \bar{Y} = 0 \tag{9}$$

$$\text{MSE}\left(\bar{y}_{pr.reg}\right) = \sum_{i=1}^{2^p} W_i^2 \frac{1-f_i}{n_i}\Big[S_i^2 + \sum_{j=1}^{p}\sum_{h=1}^{p}\alpha_{ij}\alpha_{ih}\big(b_{ij}b_{ih}\rho_{ijh}S_{ij}S_{ih} - b_{ij}\rho_{ij}S_iS_{ij} - b_{ih}\rho_{ih}S_iS_{ih}\big)\Big] \tag{10}$$

Case III. If each X_j (j = 1, 2,……, p) is negatively correlated with Y in each stratum then our estimator will convert into weighted product estimator given as

$$\bar{y}_{pr.prod} = \sum_{i=1}^{2^p} W_i \sum_{j=1}^{p} \alpha_{ij} g_{ij.prod}\left(y_i, x_{ij}\right) \tag{11}$$

where $g_{ij.prod}(y_i, x_{ij}) = \dfrac{\bar{y}_i}{\bar{X}_{ij}}\bar{x}_{ij}$

$$\text{Bias}\left(\bar{y}_{pr.prod}\right) = \sum_{i=1}^{2^p} W_i \frac{1-f_i}{n_i}\left[\bar{Y}_i \sum_{j=1}^{p} \alpha_{ij} C_{ij}^2 K_{ij}\right] \tag{12}$$

$$\text{MSE}\left(\bar{y}_{pr.prod}\right) = \sum_{i=1}^{2^p} W_i^2 \bar{Y}_i^2 \frac{1-f_i}{n_i}\Big[C_i^2 + \sum_{j=1}^{p}\sum_{h=1}^{p}\alpha_{ij}\alpha_{ih}\big(\rho_{ijh}C_{ij}C_{ih} + \rho_{ij}C_iC_{ij} + \rho_{ih}C_iC_{ih}\big)\Big] \tag{13}$$

5. Empirical study

The above theoretical developments are applied on two artificially constructed populations. In data set one, the population consists of N = 70 observations. For example, the yield rates of the crop can be taken as a study variable Y and no of shoots / canes, average height of shoots / cane and average width of the 3rd leaf can be considered as three auxiliary variables X_1, X_2 and X_3, respectively. For the purpose of computation, a sample of size n = 25 has been drawn by SRSWOR from the first population and a sample of n = 43 units has been drawn by SRSWOR from the second data set of size N=90. Both the populations are given in the Appendix.

In both the data sets, the information on all the three auxiliary variables was missing for different population units and hence there will be 8 strata. The computed sample size for each stratum by proportional allocation and the observed statistics about the population in case of incomplete auxiliary information are given in the following tables.

Data Set I

Table 1. Population parameters

Without Stratification	$\bar{Y}$ = 33.95903, S_y^2 = 4.543188
Stratum	
I	$N_1 = 5$, $n_1 = 2$, $\bar{Y}_1$=27.90000 S_1^2=46.70625
II	$N_2 = 9$, $n_2 = 3$ $\bar{Y}_2$=36.27778 $\bar{X}_{21}$=63.66667 K_{21}=0.77911 S_2^2=124.50694 ρ_{21}=0.79522 b_{21}=0.44394 S_{21}^2=399.50000
III	$N_3 = 13$, $n_3 = 5$ $\bar{Y}_3$=37.63846 $\bar{X}_{32}$=1.13538 K_{32}=1.25426 S_3^2=206.57090 ρ_{32}=0.70908 b_{32}=41.57907 S_{32}^2=0.06008
IV	$N_4 = 6$, $n_4 = 2$ $\bar{Y}_4$=43.25000 $\bar{X}_{43}$=3.70000 K_{43}=2.25222 S_4^2=181.87500 ρ_{43}=0.96175 b_{43}=26.32663 S_{43}^2=0.24272
V	$N_5 = 11$, $n_5 = 4$ $\bar{Y}_5$=26.49382 $\bar{X}_{51}$=55.00000 $\bar{X}_{52}$=1.11455 S_5^2=88.52406 ρ_{51}=0.85455 ρ_{512}=0.65101 S_{51}^2=602.00000 ρ_{52}=0.60681 K_{51}=0.68028 S_{52}^2=0.12349 K_{52}=0.68348 b_{51}=0.32770 b_{52}=16.24694
VI	$N_6 = 6$, $n_6 = 2$ $\bar{Y}_6$=45.83333 $\bar{X}_{61}$=87.83333 $\bar{X}_{63}$=3.75333

	S_6^2=32.96667 ρ_{61}=0.79500 ρ_{613}=0.63680 S_{61}^2=236.96667 ρ_{63}=0.83236 K_{61}=0.56825 S_{63}^2=0.05499 K_{63}=1.66899 b_{61}=0.29653 b_{63}=20.38070
VII	N_7 = 3, n_7 = 1 $\bar{Y}_7$=20.50000 $\bar{X}_{72}$ =0.97000 $\bar{X}_{73}$=3.14667 S_7^2=12.25000 ρ_{72}=0.53995 ρ_{723}=0.96961 S_{72}^2=0.03430 ρ_{73}=0.31761 K_{72}=0.48283 S_{73}^2=0.27693 K_{73}=0.32425 b_{72}=10.20408 b_{73}=2.11242
VIII	N_8=17, n_8 = 6 $\bar{Y}_8$=33.99412 $\bar{X}_{81}$=60.82353 $\bar{X}_{82}$ =1.18529 $\bar{X}_{83}$=3.65176 S_8^2=238.57309 ρ_{81}= 0.82364 ρ_{812}=0.63381 ρ_{813}=0.14220 S_{81}^2=644.40441 ρ_{82}=0.54901 ρ_{832}=0.52914 ρ_{83}=0.22148 S_{82}^2=0.04304 K_{81}=0.89667 K_{82}=1.42522 K_{83}=0.87872 S_{83}^2=0.17490 b_{81}=0.50115 b_{82}=40.87506 b_{83}=8.17996

Data Set II

Table 2. The Population parameters

Without Stratification	$\bar{Y}$ = 48.14444, S_y^2 = 228.01261,
I Stratum	N_1 = 16, n_1 = 8, $\bar{Y}_1$= 45.81250, S_1^2=207.22917
II Stratum	N_2 = 13, n_2 = 6 $\bar{Y}_2$=48.53846 $\bar{X}_{21}$=51.69231 K_{21}=0.57466 S_2^2=284.26923 ρ_{21}=0.85959 b_{21}=0.53960 S_{21}^2 = 721.39744

III Stratum	$N_3 = 7$, $n_3 = 3$ $\bar{Y}_3 = 47.85714$ $\bar{X}_{32} = 53.00000$ $K_{32}=0.63427$ $S_3^2=285.14286$ $\rho_{32} =0.83425$ $b_{32}=0.57273$ $S_{32}^2 = 605.00000$
IV Stratum	$N_4 = 10$, $n_4 = 5$ $\bar{Y}_4 =50.30000$ $\bar{X}_{43} = 46.60000$ $K_{43}=0.64864$ $S_4^2=312.23333$ $\rho_{43} =0.93639$ $b_{43}=0.70014$ $S_{43}^2 = 558.48889$
V Stratum	$N_5 = 12$, $n_5 = 6$ $\bar{Y}_5=50.16667$ $\bar{X}_{51} = 56.50000$ $\bar{X}_{52} = 46.41667$ $S_5^2=191.60606$ $\rho_{51} =0.85239$ $\rho_{512} = 0.54232$ $S_{51}^2 = 619.72727$ $\rho_{52} =0.65961$ $K_{51}=0.53380$ $S_{52}^2 = 247.53788$ $K_{52}=0.53694$ $b_{51}=0.47396$ $b_{52}= 0.58032$
VI Stratum	$N_6 = 8$, $n_6 = 4$ $\bar{Y}_6 = 41.00000$ $\bar{X}_{61} = 46.62500$ $\bar{X}_{63} = 39.87500$ $S_6^2=208.00000$ $\rho_{61}=0.77808$ $\rho_{613} = 0.71440$ $S_{61}^2 = 399.98214$ $\rho_{63}=0.91753$ $K_{61}=0.63808$ $S_{63}^2 = 496.98214$ $K_{63}=0.57730$ $b_{61}=0.56110$ $b_{63}=0.59358$
VII Stratum	$N_7 = 9$, $n_7 = 4$ $\bar{Y}_7 =47.88889$ $\bar{X}_{72} = 50.77778$ $\bar{X}_{73} = 44.00000$ $S_7^2=252.61111$ $\rho_{72}=0.86535$ $\rho_{723} = 0.87401$ $S_{72}^2 = 647.94444$ $\rho_{73}=0.92570$ $K_{72}=0.57291$ $S_{73}^2 = 384.50000$ $K_{73}=0.68939$ $b_{72}=0.54032$ $b_{73}=0.75033$
VIII Stratum	$N_8 =15$, $n_8 = 7$ $\bar{Y}_8=51.33333$ $\bar{X}_{81} = 50.66667$ $\bar{X}_{82} = 51.33333$ $\bar{X}_{83} = 27.40000$ $S_8^2=208.66667$ $\rho_{81} = 0.89878$ $\rho_{812} = 0.58511$ $\rho_{813} = 0.56521$ $S_{81}^2 = 619.66667$

	$\rho_{82} = 0.71264$	$\rho_{832} = 0.60341$	$\rho_{83} = 0.56918$
	$S_{82}^2 = 361.80952$	K_{81}=0.51478	K_{82}=0.54120
	K_{83}=0.53627	$S_{83}^2 = 66.97143$	b_{81}=0.52156
	b_{82}=0.54120	b_{83}=1.00469	

The above data set is used to compute the biases and mean square errors of the estimators as discussed in section 4 and 5 and these estimators are compared (i) with each other in accordance with their MSE values (ii) with respect to mean per unit.

Table 3. The Biases, Mean Square Errors and the Relative Efficiencies of data set I

Estimators	***Bias***	***MSE***	***Relative Efficiency***
$\bar{y}$	-	4.54319	100
$\bar{y}_{\mathbf{pr.rat}}$	0.6732	1.49930	303.02
$\bar{y}_{\mathbf{pr.reg}}$	-	1.31585	345.27

Table 4. The Biases, Mean Square Errors and the Relative Efficiencies of data set II

Estimators	***Bias***	***MSE***	***Relative Efficiency***
$\bar{y}$	-	2.76915	100
$\bar{y}_{\mathbf{pr.rat}}$	0.36221	1.48509	186.46
$\bar{y}_{\mathbf{pr.reg}}$	-	0.90499	305.99

6. Discussion and conclusion

The proposed class of estimators using incomplete multi-auxiliary information has been compared with simple mean per unit estimator in which auxiliary information has not been used. It is seen that all the proposed estimators are more efficient for mean estimation for both the data set taken.

(i) A critical review of table 3 and 4 reveals that, though the ratio estimator of suggested class is biased ($\bar{y}_{pr.reg}$ is unbiased because we have considered it for pre-assigned value of b_{ij}), the amount of bias is almost negligible for $\bar{y}_{pr.rat}$.

(ii) When we compare the V ($\bar{y}$) with both of the proposed estimators, we find that V($\bar{y}$) = 4.54319, which is considerably higher than the MSE ($\bar{y}_{pr.rat}$) = 1.49930 and MSE ($\bar{y}_{pr.reg}$) = 1.31585 for data set I and V($\bar{y}$) = 2.76915, which is also higher than the MSE ($\bar{y}_{pr.rat}$) = 1.48509 and MSE ($\bar{y}_{pr.reg}$) = 0.90499 for data set II.

(iii) The trend becomes more clear when we compare the % relative efficiency from table 3 and 4, i.e. the gain in % relative efficiency of the estimators of suggested class is substantially higher as compared to usual per unit estimator.

It is evident from the above results that the proposed estimators establish the supremacy over $\bar{y}$ in the estimation of mean using stratification on the basis of available incomplete auxiliary information. Thus, we see that the maximum use of available incomplete multi-auxiliary information can increase the efficiency of the estimators.

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APPENDIX

Data Set I

Stratum I
Y 28.8 32.3 34.0 28.0 16.5

Stratum II
Y 50 29 26.5 46 25.5 38 45 46.5 20
X_1 87 56 50 56 36 60 96 82 50

Stratum III
Y 39 26.5 13 66 53.5 58 28.5 24.7 36.3 36.4 34.5 34.4 38.5
X_2 1.01 1.02 0.61 1.35 1.55 1.29 1.21 1.04 0.87 1.12 1.10 1.43 1.16

Stratum IV
Y 16.5 43.5 48 52 47.5 52
X_3 2.78 3.78 3.64 4.14 3.76 4.1

Stratum V
Y 20 27 26 28.5 34.1 35.1 23.5 19 39.5 33 5.7
X_1 42 48 46 64 104 69 39 32 86 56 19
X_2 0.85 0.75 0.87 0.74 1.57 1.64 0.89 1.01 1.45 1.51 0.98

Stratum VI
Y 35.5 48.5 43.5 48 52 47.5
X_1 69 110 77 89 101 81
X_3 3.42 3.78 3.78 3.64 4.14 3.76

Stratum VII
Y 18 24.5 19
X_2 1.04 1.11 0.76
X_3 3.48 3.42 2.54

Stratum VIII
Y 64.5 10 37.3 36.8 33.5 31 16.5 12.8 43.5 32.5 25 62 32.5 51 38.5 32.5 18
X_1 87 22 67 72 68 59 34 26 60 59 44 76 56 129 72 67 36
X_2 1.18 0.71.25 1.24 1.35 1.47 0.86 0.94 0.98 1.36 1.22 1.36 1.13 1.33 1.37 1.27 1.14
X_3 3.74 3.83.98 3.98 4.08 3.92.92 2.66 3.34 4.12 3.83.78 3.46 3.12 3.72 3.98 3.7

Data Set II

Stratum I

Y 36 72 54 35 47 67 38 45 57 30 26 44 49 67 27 39

Stratum II

Y 30 26 44 72 40 67 72 43 36 72 54 35 40
X_1 18 16 39 95 30 69 80 50 53 97 28 48 49

Stratum III

Y 43 36 72 54 35 28 67
X_2 25 55 97 53 42 30 69

Stratum IV

Y 47 67 38 30 26 44 72 40 67 72
X_3 29 77 28 20 19 54 65 30 69 75

Stratum V

Y 72 43 36 72 54 35 40 34 48 52 49 67
X_1 75 19 37 97 53 26 49 44 53 56 72 92
X_2 80 41 46 47 27 31 33 27 55 55 57 58

Stratum VI

Y 27 39 57 28 36 67 44 30
X_1 18 53 60 34 65 69 54 20
X_3 23 29 60 12 46 82 34 33

Stratum VII

Y 28 57 39 27 67 49 52 40 72
X_2 25 60 28 15 55 72 56 49 97
X_3 12 62 30 23 58 57 55 33 66

Stratum VIII

Y 47 67 38 29 57 44 72 40 63 69 43 36 72 54 39
X_1 48 51 24 19 42 54 95 38 69 75 31 25 97 63 29
X_2 47 79 53 31 29 34 75 31 79 75 41 46 66 31 53
X_3 29 29 16 18 18 20 34 18 31 39 32 21 35 31 40