

IMPROVED ESTIMATORS FOR SIMPLE RANDOM SAMPLING AND STRATIFIED RANDOM SAMPLING UNDER SECOND ORDER OF APPROXIMATION

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ABSTRACT

Singh and Solanki (2012) and Koyuncu (2012) proposed estimators for estimating population mean $\bar{Y}$. Up to the first order of approximation and under optimum conditions, the minimum mean squared error of both the estimators is equal to the MSE of the regression estimator. In this paper, we have tried to find out the second order biases and mean square errors of these estimators using information on auxiliary variable based on simple random sampling. Finally, we have compared the performance of these estimators with some numerical illustration.

Key words: simple random sampling, stratified random sampling, population mean, study variable, exponential ratio type estimators, bias and MSE.

1. Introduction

Suppose n pairs (x_i, y_i) ($i=1,2,\dots,n$) observations are taken on n units sampled from N population units using simple random sampling without replacement scheme. For estimating the population mean $\bar{Y}$ of a study variable Y , let us consider X be the auxiliary variable that is correlated with the study variable Y , taking the corresponding values of the units. In sampling theory the use of suitable auxiliary information results in considerable reduction in MSE of the ratio estimators. Many authors suggested estimators using some known population parameters of an auxiliary variable. Many authors including Upadhyaya and Singh (1999), Kadilar and Cingi (2006), Khoshnevisan et al. (2007), Singh et al. (2007), Singh and Kumar (2011) suggested estimators in simple random sampling using auxiliary variables. Most of the authors discussed the properties of estimators along with their first order bias and MSE. Hossain et al. (2006) studied some estimators in second order of approximation. In this study

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we have studied the properties of some estimators under second order of approximation.

2. Some estimators in simple random sampling

For estimating the population mean $\bar{Y}$ of Y , Singh and Solanki (2012) proposed a ratio type estimator t_1 as

$$t_1 = \bar{y} \left(\frac{a\bar{X} + bc}{a\bar{x} + bc} \right)^\alpha \quad (2.1)$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ and $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$, a and c are either real numbers or function of known parameters of the auxiliary variable, and b is an integer which takes values $+1$ and -1 for designing the estimators such that $a\bar{X} + bc$ and $a\bar{x} + bc$ are non-negative. The scalar α takes values -1 , (for product-type estimator) and $+1$ (for ratio-type estimator).

Koyuncu (2012) proposed an estimator t_2 as

$$t_2 = \bar{y} \exp \left[\frac{d(\bar{X} - \bar{x})}{\bar{X}(\bar{X} + \bar{x}) + 2e} \right] \quad (2.2)$$

where d and e is either real number or a function of the known parameter associated with auxiliary information.

3. Notations used

Let us define, $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$, such that $E(e_0) = E(e_1) = 0$.

For obtaining the bias and MSE, the following lemmas will be used:

Lemma 3.1

- (i) $V(e_0) = E\{(e_0)^2\} = \frac{N-n}{N-1} \frac{1}{n} C_{02} = L_1 C_{02}$
- (ii) $V(e_1) = E\{(e_1)^2\} = \frac{N-n}{N-1} \frac{1}{n} C_{20} = L_1 C_{20}$
- (iii) $COV(e_0, e_1) = E\{(e_0 e_1)\} = \frac{N-n}{N-1} \frac{1}{n} C_{11} = L_1 C_{11}$

Lemma 3.2

$$(iv) \quad E\{(e_1^2 e_0)\} = \frac{(N-n)}{(N-1)} \frac{(N-2n)}{(N-2)} \frac{1}{n^2} C_{21} = L_2 C_{21}$$

$$(v) \quad E\{(e_1^3)\} = \frac{(N-n)}{(N-1)} \frac{(N-2n)}{(N-2)} \frac{1}{n^2} C_{30} = L_2 C_{30}$$

Lemma 3.3

$$(vi) \quad E(e_0 e_1^3) = L_3 C_{31} + 3L_4 C_{20} C_{11}$$

$$(vii) \quad E\{(e_1^4)\} = \frac{(N-n)(N^2 + N - 6nN + 6n^2)}{(N-1)(N-2)(N-3)} \frac{1}{n^3} C_{30} = L_3 C_{40} + 3L_4 C_{20}^2$$

$$(viii) \quad E(e_0^2 e_1^2) = L_3 C_{40} + 3L_4 C_{20}$$

where

$$L_3 = \frac{(N-n)(N^2 + N - 6nN + 6n^2)}{(N-1)(N-2)(N-3)} \frac{1}{n^3},$$

$$L_4 = \frac{N(N-n)(N-n-1)(n-1)}{(N-1)(N-2)(N-3)} \frac{1}{n^3}$$

$$\text{And } C_{pq} = \sum_{i=1}^N \frac{(X_i - \bar{X})^p}{\bar{X}^p} \frac{(Y_i - \bar{Y})^q}{\bar{Y}^q}$$

The proofs of these lemmas are straightforward by using SRSWOR (see Sukhatme and Sukhatme (1970)).

4. First order biases and mean squared errors

The bias expressions of the estimators t_1 and t_2 are respectively written as

$$\text{Bias}(t_1) = \bar{Y} \left[\frac{1}{2} \alpha(\alpha+1) A^2 L_1 C_{20} - \alpha A L_1 C_{11} \right] \quad (4.1)$$

$$\text{Bias}(t_2) = \bar{Y} \left[\frac{3}{2} B^2 L_1 C_{20} - B L_1 C_{11} \right] \quad (4.2)$$

$$\text{where } A = \frac{a\bar{X}}{a\bar{X} + bc} \text{ and } B = \frac{d\bar{X}}{2e + 2d\bar{X}}.$$

The MSE expressions of the estimators t_1 and t_2 are respectively given by

$$\text{MSE}(t_1) = \bar{Y}^2 [L_1 C_{02} + A^2 \alpha^2 L_1 C_{20} - 2A\alpha L_1 C_{11}] \quad (4.3)$$

$$\text{MSE}(t_2) = \bar{Y}^2 [L_1 C_{02} + B^2 L_1 C_{20} - 2BL_1 C_{11}] \quad (4.4)$$

Under optimum conditions $\text{MSE}(t_1) = \text{MSE}(t_2)$, and is the same as that of MSE of usual regression estimator. In search of the best estimator, we have extended our study to the second order of approximation.

5. Second order biases and mean squared errors

Expressing estimator t_1 in terms of e 's ($i=0,1$), we get

$$t_1 = \bar{Y}(1 + e_0)(1 + Ae_1)^{-\alpha}$$

Or

$$t_1 - \bar{Y} = \bar{Y} \{ e_0 - A\alpha e_1 + MA^2 e_1^2 - A\alpha e_0 e_1 + MA^2 e_0 e_1^2 - NA^3 e_1^3 - NA^3 e_0 e_1^3 + OA^4 e^4 \} \quad (5.1)$$

Taking expectations and using lemmas, we get the bias of the estimator t_1 to the second order of approximation, given by

$$\text{Bias}_2(t_1) = \bar{Y} [MA^2 L_1 C_{20} - A\alpha L_1 C_{11} + MA^2 L_2 C_{21} - NA^3 L_2 C_{30} - NA^3 (L_3 C_{31} + 3L_4 C_{20} C_{11}) + OA^2 (L_3 C_{40} + 3L_4 C_{20}^2)] \quad (5.2)$$

Similarly, we get the bias of the estimator t_2 to the second order of approximation as

$$\text{Bias}_2(t_2) = \bar{Y} \left[\frac{3}{2} A^2 L_1 C_{20} - AL_1 C_{11} + \frac{3}{2} A^2 L_2 C_{21} - \frac{7}{6} A^3 L_2 C_{30} - \frac{7}{6} A^3 (L_3 C_{31} + 3L_4 C_{20} C_{11}) + \frac{25}{24} A^4 (L_3 C_{40} + 3L_4 C_{20}^2) \right] \quad (5.3)$$

$$\text{where, } M = \frac{\alpha(\alpha+1)}{2} \quad N = \frac{\alpha(\alpha+1)(\alpha+2)}{6} \quad O = \frac{\alpha(\alpha+1)(\alpha+2)(\alpha+3)}{24}.$$

Squaring equation (5.1) and then taking expectations and using lemmas we get MSE of t_1 up to the second order of approximation as

$$\begin{aligned} \text{MSE}_2(t_1) = \bar{Y}^2 & \left[L_1 C_{02} + A^2 \alpha^2 L_1 C_{20} - 2A\alpha L_1 C_{11} - 2M\alpha A^3 L_2 C_{30} \right. \\ & - 2A\alpha L_2 C_{12} + A^2 (\alpha^2 + 2M) (L_3 C_{22} + 3L_4 (C_{20} C_{02} + C_{11}^2)) \\ & + 2(M + \alpha^2) A^2 L_2 C_{21} + (M^2 + 2\alpha N) (L_3 C_{40} + 3L_4 C_{20}^2) \\ & \left. + (2N - 4M\alpha) A^3 (L_3 C_{31} + 3L_4 C_{20} C_{11}) \right] \quad (5.4) \end{aligned}$$

Similarly, the MSE of the estimator t_2 up to the second order of approximation is given by

$$\begin{aligned} \text{MSE}_2(t_2) = \bar{Y}^2 & \left[L_1 C_{02} + B^2 L_1 C_{20} - 2BL_1 C_{11} - 3B^3 L_2 C_{30} + 5B^2 L_2 C_{21} - 2BL_2 C_{12} \right. \\ & - \frac{25}{3} B^3 (L_3 C_{31} + 3L_4 C_{20} C_{11}) + 4B^2 (L_3 C_{22} + 3L_4 (C_{20} C_{02} + C_{11}^2)) \\ & \left. + \frac{55}{12} B^4 (L_3 C_{40} + 3L_4 C_{20}^2) \right] \quad (5.5) \end{aligned}$$

6. Numerical illustration

For natural population data, we calculate the bias and the mean square error of the estimators and compares biases and MSE of the estimators under the first and second order of approximation.

Data Set

The data for the empirical analysis are taken from 1981, Uttar Pradesh District Census Handbook, Aligar. The population consists of 340 villages under koil police station, with Y =Number of agricultural workers in 1981 and X =Area of the villages (in acre) in 1981. The following values are obtained:

$$\begin{aligned} \bar{Y} &= 73.76765, \quad \bar{X} = 2419.04, \quad N = 340, \quad n=70, \quad C_{02}=0.7614, \quad C_{11}=0.2667, \\ C_{12} &= 0.0747, \\ C_{03} &= 2.6942, \quad C_{12}=0.1589, \quad C_{30}=0.7877, C_{13}=0.1321, \quad C_{31}=0.8851, \quad C_{04}=17.4275, \\ C_{40} &= 1.3051 \\ C_{22} &= 0.8424, \end{aligned}$$

Table 6.1. Bias and MSE of estimators

Estimators	Bias		MSE	
	First order	Second order	First order	Second order
t_1	0.214866	0.214695	47.142303	48.69108
t_2	0.4297319	0.428945	47.142303	47.86158

In the Table 6.1 the bias and MSE of the estimators t_1 and t_2 are written under the first order and the second order of approximations. From the Table 6.1 it is observed that the biases of the estimators t_1 and t_2 decrease and the mean squared errors increase for the second order of approximation. MSEs up to the first order of approximation on their optimum values are equal for both the estimators, which prompt us to the study of the estimators up to the second order of approximation, and on the basis of the study up to the second order of approximation we conclude that the estimator t_2 is better than t_1 for the given data set.

7. Estimators under stratified random sampling

While planning surveys, stratified random sampling has often proved useful in improving the precision of unstratified sampling strategies to estimate the finite

population mean of the study variable $\bar{Y} = \frac{1}{N} \sum_{h=1}^L \sum_{i=1}^{N_h} y_{hi}$. Assume that the

population U consists of L strata as $U=U_1, U_2, \dots, U_L$. Here, the size of the stratum U_h is N_h , and the size of the simple random sample in stratum U_h is n_h , where $h=1, 2, \dots, L$. In this study, we consider our proposed estimators from section (2) to estimate $\bar{Y}$ under stratified random sampling without replacement scheme, given respectively by

$$t'_1 = \bar{y}_{st} \left(\frac{a\bar{X} + bc}{a\bar{x}_{st} + bc} \right)^\alpha \quad (7.1)$$

$$t'_2 = \bar{y}_{st} \exp \left[\frac{d(\bar{X} - \bar{x}_{st})}{X(\bar{X} + \bar{x}_{st}) + 2e} \right] \quad (7.2)$$

$$\text{where } \bar{y}_{st} = \sum_{h=1}^L w_h \bar{y}_h, \quad \bar{x}_{st} = \sum_{h=1}^L w_h \bar{x}_h,$$

$$\text{and } \bar{X} = \sum_{h=1}^L w_h \bar{X}_h.$$

$$\text{Here, } \bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}, \quad \text{and } \bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}.$$

Notations used under stratified random sampling

$$\text{Let us define } e_0 = \frac{\bar{y}_{st} - \bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x}_{st} - \bar{X}}{\bar{X}}, \text{ then } E(e_0) = E(e_1) = 0.$$

To obtain the bias and MSE of the proposed estimators, we use the following notations in the rest of the article:

$$V_{rs} = \sum_{h=1}^L w_h^{r+s} \frac{1}{\bar{Y}^r \bar{X}^s} E \left[(\bar{y}_h - \bar{Y}_h)^r (\bar{x}_h - \bar{X}_h)^s \right]$$

where $\bar{y}_h$ and $\bar{Y}_h$ are the sample and population means of the study variable in the stratum h , respectively. Similar expressions for X and Z can also be defined.

We also have,

$$E(e_0^2) = \frac{\sum_{h=1}^L w_h^2 \gamma_h S_{yh}^2}{\bar{Y}^2} = V_{20}$$

$$E(e_1^2) = \frac{\sum_{h=1}^L w_h^2 \gamma_h S_{xh}^2}{\bar{X}^2} = V_{02}$$

$$E(e_0 e_1) = \frac{\sum_{h=1}^L w_h^2 \gamma_h S_{xyh}^2}{\bar{X} \bar{Y}} = V_{11}$$

$$\text{where } S_{yh}^2 = \frac{\sum_{i=1}^{N_h} (\bar{y}_h - \bar{Y}_h)^2}{N_h - 1}, S_{xh}^2 = \frac{\sum_{i=1}^{N_h} (\bar{x}_h - \bar{X}_h)^2}{N_h - 1},$$

$$S_{xyh} = \frac{\sum_{i=1}^{N_h} (\bar{x}_h - \bar{X}_h)(\bar{y}_h - \bar{Y}_h)}{N_h - 1},$$

$$\gamma_h = \frac{1 - f_h}{n_h}, f_h = \frac{n_h}{N_h}, w_h = \frac{N_h}{N}.$$

Some additional notations used for the second order of approximation are

$$C_{rs(h)} = \frac{1}{N_h} \sum_{i=1}^{N_h} \left[(\bar{y}_h - \bar{Y}_h)^s (\bar{x}_h - \bar{X}_h)^r \right]$$

$$V_{12} = \sum_{h=1}^L W_h^3 \frac{k_{1(h)} C_{12(h)}}{\bar{Y} \bar{X}^2},$$

$$V_{21} = \sum_{h=1}^L W_h^3 \frac{k_{1(h)} C_{21(h)}}{\bar{Y}^2 \bar{X}},$$

$$V_{30} = \sum_{h=1}^L W_h^3 \frac{k_{1(h)} C_{30(h)}}{\bar{Y}^3},$$

$$V_{03} = \sum_{h=1}^L W_h^3 \frac{k_{1(h)} C_{03(h)}}{\bar{X}^3},$$

$$V_{13} = \sum_{h=1}^L W_h^4 \frac{k_{2(h)} C_{13(h)} + 3k_{3(h)} C_{01(h)} C_{02(h)}}{\bar{Y} \bar{X}^3},$$

$$V_{04} = \sum_{h=1}^L W_h^4 \frac{k_{2(h)} C_{04(h)} + 3k_{3(h)} C_{02(h)}^2}{\bar{X}^4},$$

$$V_{22} = \sum_{h=1}^L W_h^4 \frac{k_{2(h)} C_{22(h)} + k_{3(h)} (C_{01(h)} C_{02(h)} + 2C_{11(h)}^2)}{\bar{Y}^2 \bar{X}^2},$$

$$\text{where } k_{1(h)} = \frac{(N_h - n_h)(N_h - 2n_h)}{n^2(N_h - 1)(N_h - 2)},$$

$$k_{2(h)} = \frac{(N_h - n_h)(N_h + 1)N_h - 6n_h(N_h - n_h)}{n^3(N_h - 1)(N_h - 2)(N_h - 3)},$$

$$k_{3(h)} = \frac{(N_h - n_h)N_h(N_h - n_h - 1)(n_h - 1)}{n^3(N_h - 1)(N_h - 2)(N_h - 3)}.$$

8. First order biases and mean squared errors under stratified random sampling

The bias of the estimators t'_1 and t'_2 under stratified random sampling is respectively written as

$$\text{Bias}(t'_1) = \bar{Y} \left[\frac{1}{2} \alpha(\alpha + 1) A^2 V_{02} - \alpha A V_{11} \right] \quad (8.1)$$

$$\text{Bias}(t'_2) = \bar{Y} \left[\frac{3}{2} B^2 V_{02} - B V_{11} \right] \quad (8.2)$$

The MSE of the estimators t'_1 and t'_2 under stratified random sampling is given by

$$\text{MSE}(t'_1) = \bar{Y}^2 [V_{20} + A^2 \alpha^2 V_{02} - 2A\alpha V_{11}] \quad (8.3)$$

$$\text{MSE}(t'_2) = \bar{Y}^2 [V_{20} + B^2 V_{02} - 2B V_{11}] \quad (8.4)$$

Note that the optimum value of A (for $\alpha=1$) and B is obtained as $A_{\text{opt}} = B_{\text{opt}} = \frac{V_{11}}{V_{02}}$. Using these optimal values, the minimum MSEs of the estimators t'_1 and t'_2 are given by

$$\text{MSE}_{\min}(t'_1) = \left(V_{20} - \frac{V_{11}^2}{V_{02}} \right) = \sum_{h=1}^L W_h^2 \gamma_h S_{yh}^2 (1 - \rho^2) = \text{MSE}(\text{RE}_s) \quad (8.5)$$

assuming $\alpha=1$.

Similarly, for the estimator t'_2 , the minimum MSE is given by

$$\text{MSE}_{\min}(t'_2) = \left(V_{20} - \frac{V_{11}^2}{V_{02}^2} \right) = \sum_{h=1}^L W_h^2 \gamma_h S_{yh}^2 (1 - \rho^2) = \text{MSE}(\text{RE}_s) \quad (8.6)$$

The minimum MSE of the estimators t'_1 and t'_2 is equal to the MSE of the regression estimator under stratified random sampling. To find the most efficient estimator among t'_1 and t'_2 , it is useful to find their MSE equations up to the second order of approximation.

9. Second order biases and mean squared errors under stratified random sampling

Expressing the estimator t_1 in terms of e 's ($i=0,1$), we get

$$t'_1 = \bar{Y}(1 + e_0)(1 + Ae_1)^{-\alpha}$$

or

$$t'_1 - \bar{Y} = \bar{Y} \left\{ e_0 - A\alpha e_1 + MA^2 e_1^2 - A\alpha e_0 e_1 + MA^2 e_0 e_1^2 - NA^3 e_1^3 - NA^3 e_0 e_1^3 + OA^4 e^4 \right\} \quad (9.1)$$

Taking expectations and using lemmas we get the bias of the estimator t'_1 up to the second order of approximation given by

$$\text{Bias}_2(t'_1) = \bar{Y} \left[MA^2 V_{02} - A\alpha V_{11} + MA^2 V_{12} - NA^3 V_{03} - NA^3 V_{13} + OA^4 V_{04} \right] \quad (9.2)$$

Similarly, we get the bias of the estimator t'_2 up to the second order of approximation as

$$\text{Bias}_2(t'_2) = \bar{Y} \left[\frac{3}{2} A^2 V_{02} - AV_{11} + \frac{3}{2} A^2 V_{12} - \frac{7}{6} A^3 V_{03} - \frac{7}{6} A^3 V_{13} + \frac{25}{24} A^4 V_{04} \right] \quad (9.3)$$

Squaring equation (9.1) and then taking expectations and using lemmas we get the MSE of t'_1 up to the second order of approximation as

$$\begin{aligned} \text{MSE}_2(t'_1) = \bar{Y}^2 & \left[V_{20} + A^2 \alpha^2 V_{02} - 2A\alpha V_{11} - 2M\alpha A^3 V_{03} + 2(M + \alpha^2) A^2 V_{12} \right. \\ & - 2A\alpha V_{21} + A^2 (\alpha^2 + 2M) V_{22} + \\ & \left. + (2N - 4M\alpha) A^3 V_{13} + (M^2 + 2\alpha N) V_{04} \right] \quad (9.4) \end{aligned}$$

Similarly, the MSE of the estimator t'_2 is given by

$$\text{MSE}_2(t'_2) = \bar{Y}^2 \left[V_{20} + B^2 V_{02} - 2BV_{11} - 3B^3 V_{03} + 5B^2 V_{12} - 2BV_{21} - \frac{25}{3} B^3 V_{13} + 4B^2 V_{22} + \frac{55}{12} B^4 V_{04} \right] \quad (9.5)$$

10. Numerical illustration

For the natural population data, we calculate the bias and the mean square error of the estimators for the first and second order of approximations.

Data Set

To illustrate the performance of the above estimators, we have considered the natural data given in Singh and Chaudhary (1986, p.162). The data were collected in a pilot survey for estimating the extent of cultivation and production of fresh fruits in three districts of Uttar-Pradesh in the years 1976-1977.

$$\bar{Y} = 443.53, \bar{X} = 8.8, V_{02}=0.062801, V_{20}=0.050034, V_{11}=0.054013,$$

$$V_{13(2)}=0.000236$$

$$V_{03(2)}=0.000741, V_{11(2)}=0.000602, V_{02(2)}=2.7784\text{E-}07, V_{20(2)}=0.000554,$$

$$V_{04(2)}=0.000277, V_{12(2)}=0.000624, V_{21(2)}=0.000524, V_{22(2)}=0.000204.$$

Table 10.1. Biases and MSEs of the estimators

Estimators	Bias		MSE	
	First order	Second order	First Order	Second order
t'_1	-3.07761E-15	-0.00391163	704.0452321	897.926434
t'_2	10.30211607	10.35740036	704.0452321	711.5662075

From Table 10.1 we observe that the MSEs of the estimators t'_1 and t'_2 are the same up to the first order of approximation but the biases are different. The MSE of the estimator t'_2 is less than t'_1 under the second order of approximation. Thus, on the basis of the second order of approximation we conclude that the estimator t'_2 is better than the estimator t'_1 for this data set.

11. Conclusion

In this study we have considered two estimators motivated by Singh and Solanki (2012) and Koyuncu (2012). The MSEs of these estimators are the same up to the first order of approximation. We have extended the study to the second order of approximation to search for best estimator in the case of the minimum variance. The properties of the estimators are studied under simple random sampling without replacement and stratified random sampling. We have observed from Table 6.1 and Table 10.1 that the behavior of the estimators changes dramatically when we consider the terms up to the second order of approximation.

REFERENCES

- HOSSAIN, M. I., RAHMAN, M. I., TAREQ, M., (2006). Second order biases and mean squared errors of some estimators using auxiliary variable. SSRN.
- KADILAR, C., CINGI, H., (2006). Ratio estimators in stratified random sampling. *Biom. Jour.*, 45, 2, 218–225.
- KHOSHNEVISAN, M., SINGH, R., CHAUHAN, P., SAWAN, N., SMARANDACHE, F., (2007). A general family of estimators for estimating population mean using known value of some population parameter(s), *Far East Journal of Theoretical Statistics* 22, 181–191.
- KOYUNCU, N., (2012). Efficient estimators of population mean using auxiliary attributes. *Appl. Math. Comput.*
- SINGH, D., CHUDHARY, F. S., (1986). *Theory and analysis of sample survey designs*. Wiley Eastern Limited, New Delhi.
- SINGH, H. P., SOLANKI, R. S., (2012). Improved estimation of population mean in simple random sampling using information on auxiliary attribute. *Appl. Math. Comput.*, 218, 7798–7812.
- SINGH, R., CAUHAN, P., SAWAN, N., SMARANDACHE, F., (2007). *Auxiliary Information and A Priori Values in Construction of Improved Estimators*. Renaissance High Press.
- SINGH, R., KUMAR, M., (2011). A note on transformations on auxiliary variable in survey sampling. *MASA*, 6:1, 17–19.
- SUKHATME, P. V., SUKHATME, B. V., (1970). *Sampling theory of surveys with applications*. Iowa State University Press, Ames, U.S.A.
- UPADHYAYA, L. N., SINGH, H. P., (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biom. Jour.*, 41, 627–636.