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Mutual information between Polish subindexes – the use of copula entropy around the time of the COVID-19 pandemic

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Abstract

In this paper, the copula theory is used to describe the dependence structure between variables, while the information theory provides the tools necessary to measure the uncertainty associated with these variables. What both theories have in common is copula entropy, which is strictly related to mutual information.

The findings of this study, focusing on the dependence of the (sub)indexes of the Polish stock market during the pandemic period, may prove useful not only to investors from Poland, but also from other countries, especially Central European, in making investment decisions.

The results of calculating the interdependencies between WIG, sectoral indexes and among sectoral indexes of the Polish economy using copula entropy and Pearson's correlation are quite different.

The source of the basic difference between copula entropy and Pearson's correlation is that the former enables the measurement of nonlinear interdependencies, while the latter not. The interrelations on the stock markets are nonlinear and returns are not normally distributed in general. The use of copulas is also superior in terms of ranking correlation, as it is more general and allows the examination of the structure of dependencies between extreme values.

JEL Classification: G15, G19

Key words: Polish subindexes, COVID-19 pandemic, mutual information, copula entropy.

1. Introduction

At the end of 2019, the COVID-19 pandemic broke out. According to WHO, by September 1 2020 there were 25,327,098 cases of COVID-19. In addition 848,255 deaths were registered across the world. Since the outbreak of the pandemic, governments have tried to restrict the spread of COVID-19.

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All countries have used different measures in order to protect societies from the pandemic. These measures include stopping production and quarantining people in their own homes. The implementation of these measures has had a great impact on economic development, the economic situation of enterprises and national economies. These economic and social problems are reflected in the growing number of academic contributions. These studies have documented the negative impact of the pandemic (Goodell, 2020 among others) on trade, tourism, transportation, and employment (Leduc and Liu, 2020), even at the beginning. Some contributors have started to compare the effects of the spread of COVID-19 and its consequences to those of an economic crisis (Sharif et al., 2020).

The pandemic has had a great effect on economic and social development as well as the financial markets. Some studies have documented the impact of the pandemic on the returns of financial markets (Ashraf, 2020; Zhang et al., 2020; Aslam et al., 2020 a,b,c) and/or their volatility (Albulescu, 2020; Bakas and Triantafyllou, 2020; Zaremba et al., 2020; Okorie and Lin, 2020).

The risk of contagion between financial markets was also the subject of these inquiries (Akhtaruzzaman, 2020; Goldstein and Pauzner, 2004). Baig et al. (2020) investigated the impact of the pandemic on the liquidity and volatility of the stock market. They established that the increase in confirmed cases and deaths due to the pandemic caused a lack of liquidity, stability and strong volatility on the financial market.

Rizwan et al. (2020) investigated the banking systemic risk in eight important countries. All of them were strongly affected by the pandemic. The authors found that the financial systemic risk of the countries under consideration rose significantly during the pandemic period.

Some studies have concentrated on the performance of stocks in different sectors or different countries. Mazur et al. (2020) examined the return of the healthcare, food, natural gas, and software sectors. They observed that these sectors performed well during the pandemic. However, they also found that the crude petroleum, real estate, entertainment, and hospitality sectors declined noticeably. In addition, these sectors displayed great volatility.

Shehzad et al. (2020) compared the impact of the pandemic on the stock market with that of global financial crises. They established that the American and the European stock markets were affected by the pandemic more strongly, and COVID-19 disturbed economic communication throughout the world and was the source of a financial crisis.

For investors it is very important to analyze the interdependence structure of the stock market. This is important with respect to diversifying investment and building investment portfolios during the time of the pandemic, which is essential from the point of view of risk management of the financial market taken into account by financial regulators.

In recent years, several researchers have tried to investigate the interdependence among stock markets (Sukcharoen and Leatham, 2016; Long et al., 2016; Qiao et al., 2016; Long et al.,2017a, b; Surya et al., 2018; Alomari et al., 2018; Huang et al., 2019, Kodres and Pritsker 2002, Barberis et al., 2005, Chiang and Zheng, 2010, Wang and Hui, 2018). They have used, among others, the GARCH model, Copula model, Granger causality test, DCC model, and some other models in order to detect the interdependence structure between different stock sectors in the countries under consideration.

Research on the interdependence structure of the stock markets has indentified which sector plays the most important role in a national economy. These studies provide new opportunities for investors to build a proper portfolio of assets (Poynter et al., 2015). However, in Europe there are not many studies concerned with the interdependence structure of the stock sectors during the selected period of the COVID-19 pandemic.

China was the first country that faced COVID-19. This country was the first in the world to implement measures to tackle the pandemic. Taking scientific investment methods into account, investors expect to obtain higher profits and/or reduce investment losses. A very well- known investment strategy is diversification. According to this strategy, assets are distributed to stocks from different sectors. Its main goal is to avoid investment losses caused by investing in closely dependent assets.

The pandemic, which began in 2019 in China, was the source of the greatest recession in economic and social development since the global financial crisis of 2008. Identifying the structure and changes in the interdependence between various sectors during the time of COVID-19 is a very useful piece of advice for investors trying to optimize their investment during the pandemic.

The copula entropy used in this contribution is a combination of copula theory and information theory. The copula function is employed to describe the dependence between variables, and mutual information is used to quantify the dependence. There is a connection between copula theory and information theory, and mutual information can be expressed in terms of copulas, as copula entropy.

One of the first and most important contributions using copula entropy is a paper by Zhao and Liu, 2011. In this research, the copula entropy model was constructed by the copula and the entropy theory. Therefore the copula entropy model combines the advantages of both of them. The used approach is not limited to measuring the linear correlation; it also can describe the nonlinear correlation. It not only measures the degree of the dependence, but considers the structure. In this paper, the contributors propose copula entropy models with two and three variables to measure dependence in stock markets, which extend the copula theory and are based on Jaynes's information criterion.The research sample is composed of 12 stocks indexes from 12 countries selected by two methods. They chosen three copula functions to represent three different economic situations: recession, boom and interim. Having completed the two experiments, they provided a comparative analysis. The authors proven that three-variable dependence changes across the three economic circles are less obvious than the two-variable dependence.

This study of the dependence of the Polish stock market during the pandemic period may be useful not only for Polish investors in making investment decisions, but also those from other countries, especially Central Europe. Our aim to study interdependencies around the time of the outbreak of COVID-19 seems to be reasonable.

The main task of this study is detection of changes in dependence around event day (13.03.2020 - the day a state of epidemic threat was introduced in Poland). We will prove the dependence of subindexes using the concept of mutual information before and after the event day. By means of mutual information based on copula entropy we aim to check whether the parameters of mutual information are greater before or after the event day. Further research question concerns behaviour of Pearson correlation with respect to the event day.

We will compare results of both measures of dependence before the event day and after the event day and explain possible differences with respect to linear and nonlinear dependence notions.

2. Copulas

Sklar (1959) introduced a new class of multivariate cumulative distribution functions, which are multivariate cumulative distributions with uniform margins. Assume that random vector (X, Y) has joint distribution function $F_{XY}(x, y)$ and density $f_{XY}(x, y)$. Let $F_X(x)$ and $F_Y(y)$ be marginal distributions, whereas $f_X(x)$ and $f_Y(y)$ are marginal density functions of X and Y, respectively. Sklar's theorem (see Nelsen, 2006) states that there exists function C (called the copula), such that $F(x, y) = C(F_X(x), F_Y(x))$. From this we see that the copula is a function that combines one-dimensional distributions into a multivariate (bivariate is a special case) distribution with uniform margins. Moreover, if marginal distributions are continuous the copula C is unique and the equation holds

$$C(u, v) = F\left(F_X^{-1}(u), F_Y^{-1}(V)\right) \tag{1}$$

where $u, v \in [0,1]$ and F_X^{-1}, F_Y^{-1} are quasi-inverses of the distribution of F_X and F_Y, respectively. The density of copula C is the mixed second derivative of C and can be expressed as

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} = \frac{f_{XY}(x,y)}{f_X(x) \cdot f_Y(y)} \tag{2}$$

The best known classes of copulas are elliptical and Archimedean copulas. The bivariate Gaussian (or normal) copula is an elliptical copula of the form

$$C_\rho^{Ga}(u_1, u_2) = \Phi_\rho\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right) \tag{3}$$

where Φ_ρ is the cumulative distrubution function of the bivariate standard normal with Pearson's correlation coefficient ρ, while Φ^{-1} is the inverse of the univariate cumulative distribution function of the standard normal.
The other example of elliptical copula is copula *t* which is based on the *t* distribution function and is given by

$$C_{\nu,\rho}^t(u_1, u_2) = t_{\nu,\rho}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)\right). \tag{4}$$

where $t_{\nu,\rho}$ is the cumulative distribution function of the bivariate *t* cumulative distribution function with linear correlation coefficient ρ and ν degrees of freedom, whereas t_ν^{-1} is the inverse of the univariate cumulative distribution function of *t* with ν degrees of freedom.

The other class of copulas is Archimedean copulas, whose construction is based on a special convex and strictly decreasing continuous function called generator (see Nelsen (2006) for details). In Table 1 we present the definitions of the selected copulas and the range of parameters

Table 1: Some families of Archimedean copulas

name	$C(u,v)$	Range of parameter
Frank	$-\frac{1}{\theta}\log\left[1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1}\right]$	$(-\infty, \infty)\backslash\{1\}$
Clayton	$\max([u^{-\theta} + v^{-\theta} - 1]^{-\frac{1}{\theta}}, 0)$	$[-1, \infty)\backslash\{0\}$
Gumbel	$\exp(-[(-lnu)^\theta + (-lnv)^\theta]^{\frac{1}{\theta}})$	$[1, \infty)$
BB1	$\left\{1 + \left[(u_1{}^{-\theta} - 1)^\delta + (u_2{}^{-\theta} - 1)^\delta\right]^{1/\delta}\right\}^{-1/\theta}$	$\theta \in (0, \infty), \delta \in [1, \infty)$

The number of copulas can be easily extended using rotations. In applications, the most frequently used copulas are those rotated 180 degrees, called survival copulas of the form

$$\hat{C}(u,v) = u + v - 1 + C(1-u, 1-v). \tag{5}$$

The main use of copulas is to model dependence. Such concordance measures as Kendall's τ or Spearman's ρ can be expressed in terms of copulas, but the dependencies between extreme values which can be investigated with copulas are often more interesting. Upper- and lower-tail dependence coeefficients are defined as

$$\lambda_U = \lim_{u \to 1^-} \frac{1-2u+C(u,u)}{1-u} \quad \text{and} \quad \lambda_L = \lim_{u \to 0^+} \frac{C(u,u)}{u} \qquad (6).$$

The copula C has upper-tail dependence if $\lambda_U \in (0,1]$ and upper-tail independence if $\lambda_U = 0$. The definitions for lower-tail coefficients are analogous. The upper- (lower-) tail dependence coefficients of survival copula are equal to lower- (upper-) tail dependence coefficients.

The Gaussian copula exhibits tail independence for both tails and for t copula $\lambda_U = \lambda_L = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(\rho-1)}{1+\rho}}\right)$. The Archimedean copulas include non-symmetrical cases. The tail-dependence coefficients of the families selected are presented in the table below.

Table 2: Tail dependence coefficients of some Archimedean copulas

copula	λ_U	λ_L
Frank	0	0
Clayton	0	$2^{-1/\theta}$
Gumbel	$2-2^{1/\theta}$	0
BB1	$2-2^{1/\delta}$	$2^{-1/(\delta\theta)}$

The objective of some research, such as that of Ma and Sun (2011), is to present a copula entropy approach based on entropy theory and copula theory to measure the dependence relationship between the financial variables with practical applications.

3. Linear correlation vs. mutual information

Most research methods that are concerned with the dependence of the stock markets are based on linear assumptions. Some of them refer to a specific model and parameters. The best known Pearson correlation coefficient can only measure the linear relationship between variables, instead of effectively measuring the nonlinear relationship. Pearson's correlation coefficient is based on the multivariate ellipticity assumption, which does not always hold. This measure will not estimate the dependence between two variables properly when the sample size is not large enough or the dependence relationship is nonlinear.

The rank correlation coefficient can be used in order to estimate the nonlinear dependence relationship between two variables. It has no restriction regarding the distribution of variables. The rank correlation coefficient primarily includes the

Kendall correlation coefficient and the Spearman correlation coefficient, which are examples of concordance measures.

Due to the development of entropy theory and its application, different methods, such as mutual information, have been used in a number of pieces of financial market research (Fiedor, 2014; Fiedor, 2015; Yang et al., 2013; Yang et al., 2014; Kwon and Yang, 2008; Wang and Hui, 2017, Wang et al., 2017 Khan et al., 2007). We briefly present the fundamental concepts of information theory, such as entropy and mutual information.

Entropy is the average amount of information. For discrete random variable X with support set $\mathcal{X}$ is given by

$$H(X) = -\sum_{x\in\mathcal{X}} p(x)\log_m p(x) \tag{7}$$

If the logarithm base is equal to 2, the unit of entropy is bit. For $m = e$ and $m = 10$ we get nat and dit respectively (we omit this parameter in the following formulas).

In the case of a pair of random variables X and Y one can compute conditional entropy $H(Y|X)$, which measures the entropy of variable Y when the values of X are known. This is given by

$$H(Y|X) = -\sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p(x,y)\log\frac{p(x,y)}{p(x)} \tag{8}$$

where $p(x,y) = P(X = x, Y = y)$ and $p(y) = P(Y = y)$. Defining joint entropy by

$$H(X,Y) = -\sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p(x,y)\log p(x,y) \tag{9}$$

conditional entropy can be expressed as $H(Y|X) = H(X,Y) - H(X)$. In this paper, we use mutual information (MI) to measure the dependence between variables. Mutual information is given by $MI(X,Y) = H(X) - H(Y|X)$ and for the two given variables X and Y, assuming that their respective marginal probability distributions and joint probability distribution are known, and they are, respectively, $p(x)$, $p(y)$, and $p(x, y)$, mutual information can be expressed as

$$MI(X,Y) = \sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)} \tag{10}$$

Mutual information measures the amount of information of X contained in Y and, conversely, the amount of information of Y contained in X. In other words, mutual information measures the uncertainty of one variable given knowledge of the other.

This measure has the following properties:

1. $MI(X,Y) \geq 0$
2. $MI(X,X) = H(X)$
3. $MI(X,Y) \leq \min(H(X), H(Y))$.

Although MI is bounded by the entropies of each variables, it is not normalized. Following Joe (1989) we can normalize mutual information using formula $\delta = \sqrt{1-\exp(-2MI)}$. In this way we obtain a normalized index, which is contained in interval [0,1].

For continuous random variables, the definitions above can be reformulated in terms of integrals. In this case, the Shannon entropy is called differential entropy, which unfortunately does not have all the desired properties, such as a discrete version (for example non-negativity). Mutual information is given by

$$MI(X,Y) = \int_{\mathcal{Y}} \int_{\mathcal{X}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} dxdy. \tag{11}$$

In the literature (for example Jenison and Reale 2004, Ma and Sun 2011, Zhao and Liu, 2011), we can also find the term copula entropy. Using copula entropy, association information and dependence structure information can be measured simultaneously. In addition, copula entropy does not impose constraints on the dimension of multiple variables. Copula entropy can also be used to measure multivariate dependence in different branches of the economy. In general, for N-dimensional copula with density $c(\boldsymbol{u})$, where $\boldsymbol{u} = (u_1, u_2, \dots, u_N)$ copula entropy is defined as

$$H_C(\boldsymbol{u}) = -\int_{[0,1]^N} c(\boldsymbol{u}) \log c(\boldsymbol{u})\, d\boldsymbol{u}, \tag{12}$$

which in the bivariate case takes the form

$$H_C(U,V) = -E[\log c(u,v)] = -\int_0^1 \int_0^1 c(u,v) \log c(u,v)\, dudv. \tag{13}$$

Ma and Sun (2011) show in their paper that mutual information is copula entropy and more specifically $MI = -H_C$. They also implement a method for estimating mutual information in a non-parametric way. Given the density of the copula, one can use numerical integration to obtain H_C, or simply as $-\frac{1}{n}\sum_{t=1}^{n} \log c(u_t, v_t)$. This is a general approach, but for some families of copulas there are explicit formulas. For example, in the case of the Gaussian copula with parameter ρ, mutual information is equal to $-\frac{1}{2}\ln(1-\rho^2)$.

To summarize, dependence measurement using mutual information expressed in terms of copulas has many advantages. It is not limited to measuring linear correlations; it can also capture a nonlinear correlation. It not only measures the degree of the dependence, but also considers the dependence structure which is more than correlation. Moreover, there is no assumption about the ellipticity of marginal and joint distribution. It even allows the dependence of variables with different cumulative distribution functions to be modelled. Although in this paper we consider bivariate copulas, extension to the multidimensional case is obvious.

4. Data and empirical results

We consider the closing prices of 14 sectoral indexes between January 3, 2018 and May 13, 2022 (the day the state of epidemic in Poland was lifted). In addition, the largest index from the Warsaw Stock Exchange WIG is considered. On March 11, 2020 the World Health Organization (WHO) announced the COVID-19 pandemic. We divide the time series of logarithmic returns with the date 13 March, 2020. On that day a state of epidemic threat was introduced in Poland (we refer to this as the event day). For all the series we computed descriptive statistics (mean, standard deviation, kurtosis and skewness given in Table 3).

We conducted a Ljung-Box test of lack of autocorrelation and Jarque-Bera test of normality. To save space we present their quartiles (all results of the computations are available upon request).

Table 3: Descriptive statistics of returns

before event				
Specification	mean	sd	kurtosis	skewness
min	-0.21	0.92	5.31	-2.89
1st quartile	-0.17	1.23	7.83	-2.02
median	-0.13	1.63	12.14	-1.37
3rd quartile	-0.03	1.90	23.68	-0.77
max	0.14	2.58	29.75	-0.52
after event				
	mean	sd	kurtosis	skewness
min	-0.08	1.26	3.78	-5.48
1st quartile	0.05	1.97	6.25	-0.30
median	0.10	2.26	6.49	0.02
3rd quarile	0.14	2.85	9.04	0.38
max	0.19	3.50	88.23	1.08

The results of the first test are mixed, but we reject null of normality for all the series. This is also the case for the WIG series.

Main index – subindex dependence

To investigate the dependence between the WIG index and the sectoral indexes we compute mutual information for all pairs main index - subindex before and after the event day using copula entropy.

First, we filter our time series using Vector Autoregression models for conditional means, GARCH type models for conditional variance and skew t for conditional distribution. Given the estimated models we computed the probability integral transform. Tenzer and Elidan (2016) established a monotonic relationship between the mutual information and the copula dependence parameter. We limit the set of potential copulas to selected families, but allow their rotated versions (survival copulas). Using the Bayesian information criterion, we choose bivariate copulas that fit the best.

In most cases, the fitted copulas belong to a class of asymmetric (BB1, survival Gubmel, survival BB1) copulas with a different structure in the upper and lower tails. The computed tail dependence coefficients are not smaller after the event for 9 and 12 subindexes, respectively. Given the densities of the estimated copulas we compute mutual information and the corresponding parameter δ for all pairs under investigation. We present this in Figure 1 (red before, green after).

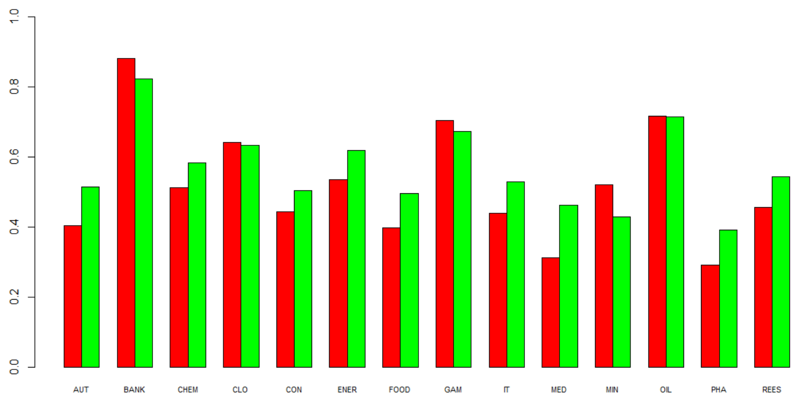

Figure 1: Parameter δ for pairs index-subindex

Both before and after the event the weakest and strongest dependence is observed for PHA and BANK, respectively.

In 9 cases out of 14 delta values are higher after the event (the exceptions are the sectors BANK, CLO, GAM, MIN, OIL). The three largest percentage increases in the delta parameter are observed for MED (32.5%), PHA (25.3%) and AUT (21.5%), the largest decreases for MIN (-21.3%), BAN (-7.1%) and GAM (-4.6).

For the purpose of comparison we computed linear correlation coefficients. Before the event, the weakest and strongest dependence is observed for MED and BANK, whereas after PHA and BANK. Only in 3 cases (AUD, IT, MED) does the correlation increase after the event, with the highest percentage increase for IT (14.1%) and largest decrease for MIN(-27.3%).

Dependence of subindexes

We repeat the procedure for all pairs of subindexes. In most cases the copula that fits the best is a rotated version of Gumbel (63 cases before and 50 cases after the event), which is dependent in the lower tail and independent in the upper one. The number of symmetric copulas increases, and most of them are Gaussian copulas with tail independence. However, in 50 cases the number of estimated lower-tail dependence coefficients increases. Upper-tail dependence coefficients remain the same in 72 cases because of elliptical copulas and Archimedean copulas with upper-tail independence. For the subsectors GAM, MED and MIN, the lower-tail dependence coefficients do not increase with the other sectors in most cases, whereas for FOOD and OIL they do. In the left diagram of Figure 2, we present pairs of sectors for which lower-tail dependence coefficients do not decrease (yellow) and decrease (grey). On the right, we present pairs of sectors for which upper-tail dependence increases (yellow) and does not increase (grey).

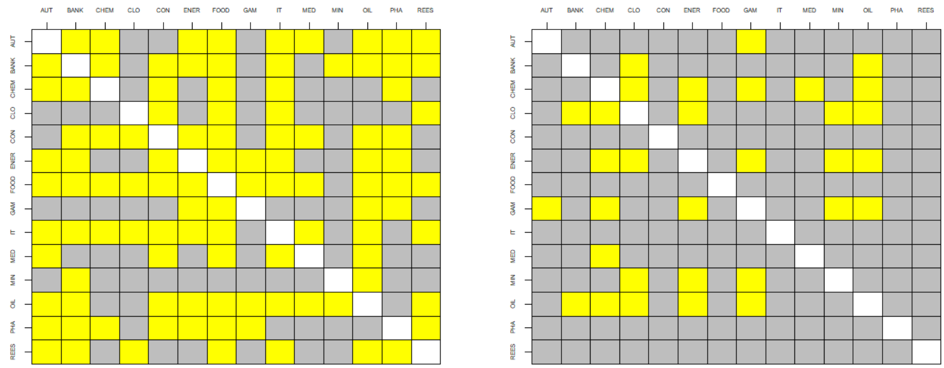

Figure 2: Designation of pairs with tail-dependence coefficient changes

Despite the lack of economic justification, we compute mutual information and cooresponding paramerer δ for all pairs and for both subperiods (see Figure 3).

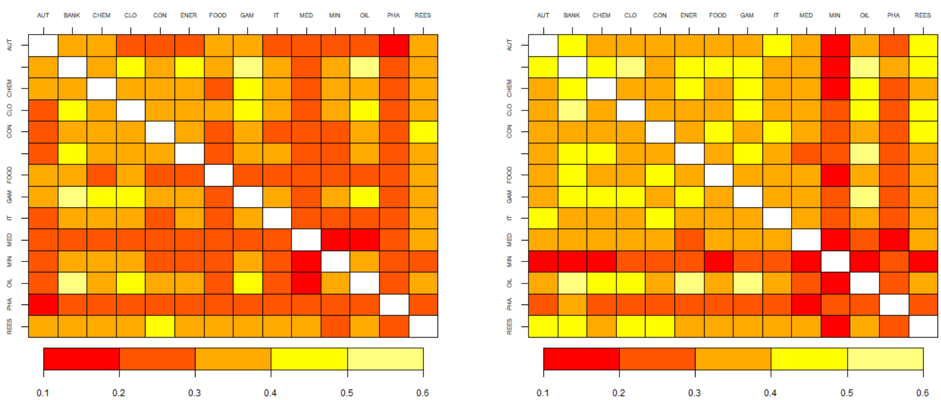

Figure 3: Heat map of parameters delta before (left) and after (right) the event

In Table 4 we present the 3 weakest and 3 strongest relationships before and after the event.

Table 4: Selected weakest and strongest relationships

before event		after event	
pair	**value**	**pair**	**value**
MIN - MED	0.1911	REES - MIN	0.1640
OIL - MED	0.1928	MIN - CHEM	0.1659
PHA - AUT	0.1990	MIN - BANK	0.1716
…		…	
CLO - BANK	0.5000	CLO - BANK	0.5149
OIL - BANK	0.5078	OIL - GAM	0.5308
GAM - BANK	0.5606	OIL - BANK	0.5909

In 69 cases dependence increases, which accounts for over 75% of all cases. In Figure 4, we present the heat map of the percentage changes of delta coefficients.

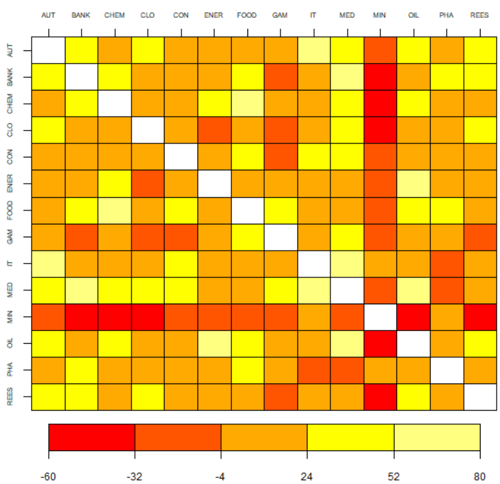

Figure 4: Heat map of percentage changes of δ

From Figure 4 we notice that mining is the sector with the lowest percentage changes in dependencies. Table 5 contains the three smallest and three largest percentage changes.

Table 5: Selected smallest and largest percentage changes of δ

pair	percentage change
MIN - BANK	-53.7
MIN - CHEM	-52.1
MIN - REES	-40.6
....	
MED - IT	59.7
FOOD - CHEM	68.1
IT - AUT	74.8

Given δ_{ij}, the dependence parameter between i and j subindexes, we compute $S_i = \sum_j \delta_{ij}$, which reflects the sum of parameters δ of certain subindex with all of the other subindexes.

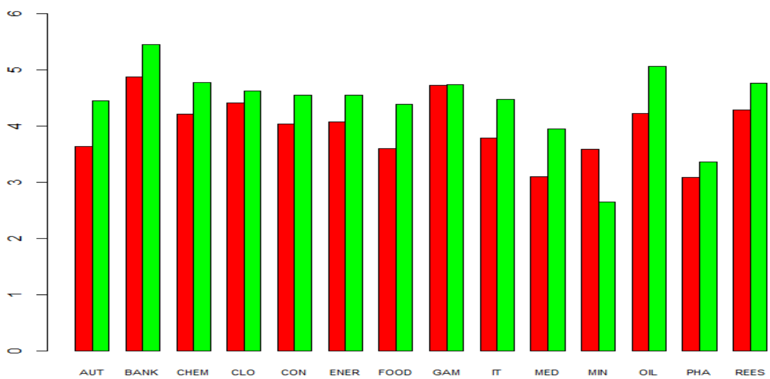

Figure 5: S_i of each sector before (red) and after the event (green)

We can see from Figure 5 that only in the case of the mining sector S_i is greater before the event than after with a drop of about 47%. The increase in the gaming sector is small (about 0.25%) while the greatest change is observed for the media sector (about 71%).

Again, for the purposes of comparison we computed the linear correlation coefficients between the returns of the subindexes. In 24 cases the dependence after the event is greater than dependence before the event. This is contrary to the results by mutual information. However, the correlation coefficient was calculated for returns. Moreover, the smallest value of correlation coefficient before the event is greater than the smallest one after, and the maximum value of the correlation coefficient is greater before the event than after. These results are presented in the heat maps (Figure 6):

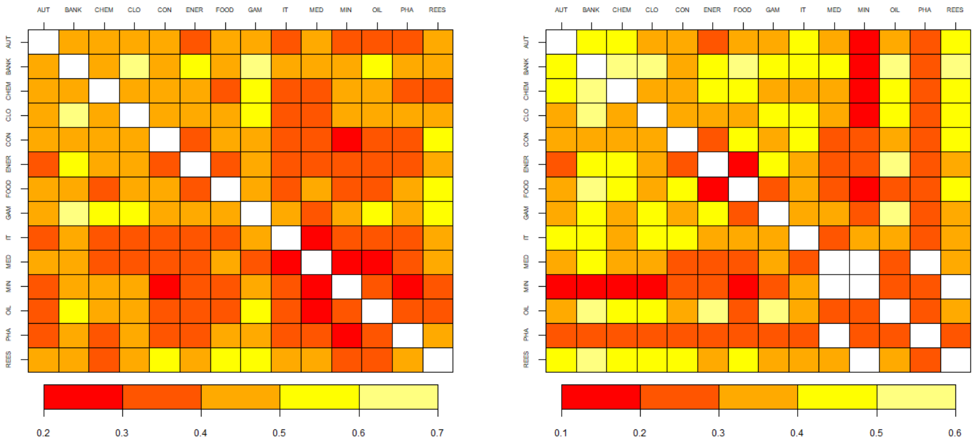

Figure 6: Heat maps of correlation coefficients before (left) and after (right) the event

From these pictures we notice that after the event the smallest values of correlation coefficients are when we consider the PHA or MIN sectors as a member of a pair, which is partly in line with the results from mutual information. In Table 6 we again present the 3 weakest and 3 strongest dependencies. These results are similar to those from Table 4.

Table 6: Selected weakest and strongest relationships measured with linear correlation coefficient.

before event		after event	
pair	**value**	**pair**	**value**
OIL - MED	0.2253	MIN - MED	0.1320
MED - IT	0.2476	REES - MIN	0.1468
MIN - MED	0.2702	PHA - MED	0.1487
…		…	
GAM - CLO	0.5509	OIL - GAM	0.5297
CLO - BANK	0.6253	OIL - BANK	0.5501
GAM - BANK	0.6671	CLO - BANK	0.5595

The last confirmation (Figure 7) of difference between mutual information and linear correlation results is the sum of the correlation coefficients of certain subindex with all of the other subindexes (red before event, green after event). The largest drop in these values is observed for MIN (about 43%) and PHA (about 37%). The only positive change is observed for the IT sector with the value of 3.7%.

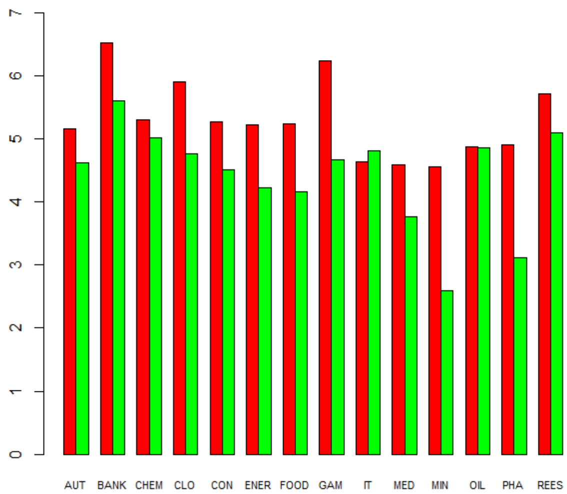

Figure 7: Sum of correlation coefficients of each sector before and after the event

5. Conclusions

The main goal of this contribution was to detect changes in dependence around the event day. The event day was 13.03.2020. On that day a state of epidemic threat was introduced in Poland. In the first part of this empirical study we examined the dependence between the WIG index and (14) sectoral subindexes. We calculated mutual information and their normalized value for all pairs main index – subindex before and after the event day using copula entropy. In most cases dependence parameters were higher after the event day than before this day.

Then we checked the dependence of subindexes using the concept of mutual information before and after the event day. In all cases, except for the subsector, mining dependence parameters were greater after the event day than before the event day. A quite different picture emerges from a linear correlation analysis. In almost all subsectors, the sum of the Pearson correlation coefficients before the event day is larger than after the event day.

The COVID-19 pandemic was reflected not only in a crisis in health systems, but also in an economic crisis, among other things. Past experiences, e.g. the Global Financial Crisis of 2007 and 2008, have convinced us that in times of crisis dependence between economic variables, especially variables from stock exchanges become stronger. The results before and after the event day, which are based on mutual information, are in line with our expectations formulated by observing crises in the past. On the contrary, Pearson's correlation delivers different results. This may be caused by an increase in nonlinear dependencies between subsectors of the Warsaw Stock Exchange after the event day. Nonlinearities after the event day may be incorrectly categorized as independent by Pearson' correlation.

An interesting question in future research will be a comparison of pandemic outbreak of the mutual entropy and correlation results for subindexes of developed capital markets and a comparison of these dependence measures with results for Polish subindexes (and/or) subindexes of other emerging markets.

The second research question will be the impact of the size of subsectors on the dependence measures under study.

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