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A chain ratio-type exponential estimator for population mean in double sampling

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Abstract

In this paper we have proposed an efficient ratio-type exponential estimator for estimating the population mean of the study variable, by incorporating two auxiliary variables in two-phase (double) sampling. The bias and the mean square error of the proposed estimator have been obtained up to the first order of approximation. The newly proposed estimator offers more precision in comparison to other competing estimators, theoretically as well as empirically, by considering a known value of some population parameter.

Key words: two-phase sampling, auxiliary variables, study variable, bias, mean square error, percent relative efficiency.

1. Introduction

Consider a finite population $U = (U_1, U_2, \ldots\ldots\ldots\ldots\ldots . . U_N)$ of N units. Let $\bar{X}, \bar{Y}$ and $\bar{Z}$ denote the population mean, C_x, C_y and C_z denote the coefficient of variation, ρ_{yx}, ρ_{yz} and ρ_{xz} denote the correlation coefficient. Let Y be the study variable and X and Z be the auxiliary variables with corresponding value y_i, x_i, z_i $(i = 1,2, \ldots\ldots\ldots N)$. The problem is to estimate $\bar{Y}$ in the presence of two auxiliary variable x and z.

Let $S_y^2 = \sum_{i=1}^{n}(y_i - \bar{Y})^2/(N-1)$ and $S_x^2 = \sum_{i=1}^{n}(x_i - \bar{X})^2/(N-1) S_z^2 = \sum_{i=1}^{n}(z_i - \bar{Z})^2/(N-1)$ and let $C_y = S_y/\bar{Y}$ and $C_x = S_x/\bar{X}\, C_z = S_z/\bar{Z}$ be the coefficients of variation of y, x and z respectively. $f_1 = \left(\frac{1}{n} - \frac{1}{N}\right) = \left(\frac{(1-f)}{n}\right)$, $f_2 = \left(\frac{1}{n'} - \frac{1}{N}\right) = \left(\frac{(1-f')}{n'}\right)$, $f_3 = f_1 - f_2 = \left(\frac{(1-f'')}{n}\right)$

where $f = \frac{n}{N}$, $f' = \frac{n'}{N}$ and $f'' = \frac{n}{n'}$

$$v(\bar{y}) = f_1 \bar{Y}^2 C_y^2$$

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2. Estimator with single auxiliary variable

When the population mean of the auxiliary variable x is not known, Sukhatme (1962) defined the two-phase sampling ratio estimator for population mean $\bar{Y}$ as

$$t_1 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{1}$$

$$MSE(t_1) = \bar{Y}^2\left[f_1 C_y^2 + f_3\left(C_x^2 - 2\rho_{yx}C_y C_x\right)\right] \tag{2}$$

The usual regression estimator in two-phase sampling is defined as

$$t_2 = \bar{y} + b_{yx}(n)(\bar{x}' - \bar{x}) \tag{3}$$

$$MSE(t_2) = \bar{Y}^2 C_y^2\left[f_1\left(1 - \rho_{yx}^2\right) + f_2\rho_{yx}^2\right] \tag{4}$$

Singh and Vishwakarma (2007) suggested exponential ratio and product type estimator for $\bar{Y}$as

$$t_3 = \bar{y}exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \tag{5}$$

$$t_4 = \bar{y}exp\left(\frac{x - \bar{x}'}{x + \bar{x}'}\right) \tag{6}$$

The MSEs of the estimators t_3 and t_4 respectively are

$$MSE(t_3) = \bar{Y}^2\left[f_1 C_y^2 + \frac{f_3}{4}\left(C_x^2 - 4\rho_{yx}C_y C_x\right)\right] \tag{7}$$

$$MSE(t_4) = \bar{Y}^2\left[f_1 C_y^2 + \frac{f_3}{4}\left(C_x^2 + 4\rho_{yx}C_y C_x\right)\right] \tag{8}$$

3. Estimator with two auxiliary variables

Chand (1975) suggested a chain ratio-type estimator for the population mean $\bar{Y}$ defined as

$$t_5 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right);\ \bar{x} \neq 0\,, \bar{z} \neq 0 \tag{9}$$

$$MSE(t_5) = \bar{Y}^2\left[f_1 C_y^2 + f_3\left(C_x^2 - 2\rho_{yx}C_y C_x\right) + f_2\left(C_z^2 - 2\rho_{yz}C_y C_z\right)\right] \tag{10}$$

Kiregyera (1980, 1984) suggested some modification of Chand (1975) and proposed ratio to regression estimator of population mean given as

$$t_6 = \bar{y} + b_{yx}(n)\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z} - \bar{x}\right) \tag{11}$$

where $b_{yx}(n)$: Sample regresssion coefficient of y on x based on s (Sub-sample)

$\rho_{yx}, \rho_{yz}, \rho_{xz}$: Population correlation coefficient between the variables.

The MSE of estimator t_6 is as follows:

$$MSE(t_6) = \bar{Y}^2 C_y^2 \left[f_1\{1 - \rho_{yx}^2\} + f_2\left\{\rho_{yx}^2 + \rho_{yx}^2 \frac{C_z^2}{C_x^2} - 2\rho_{yx}\rho_{yz}\frac{C_z}{C_x}\right\}\right] \tag{12}$$

Singh and Khalid (2015) suggested the following estimator:

$$t_7 = \bar{y}exp\left(\frac{\bar{x}'\left(\frac{\bar{z}^*}{\bar{Z}}\right) - \bar{x}}{\bar{x}'\left(\frac{\bar{z}^*}{\bar{Z}}\right) + \bar{x}}\right) \tag{13}$$

where

$$\bar{z}^* = \frac{(N\bar{Z} - n'\bar{z}')}{(N - n')} \text{ and } k = \frac{n'}{(N - n')}$$

The required mean square error of the estimator t_7 is

$$MSE(t_7) = \bar{Y}^2\left[f_1 C_y^2 + f_2\left(\frac{k^2}{4}C_z^2 - k\rho_{yz}C_yC_z\right) + \frac{f_3}{4}\left(C_x^2 - 4\rho_{yx}C_yC_x\right)\right] \tag{14}$$

Singh and Choudhury (2012) developed the following exponential chain-type ratio estimators of $\bar{Y}$ under double sampling as

$$t_8 = \bar{y}exp\left(\frac{\left(\frac{\bar{x}'}{\bar{z}'}\right)\bar{Z} - \bar{x}}{\left(\frac{\bar{x}'}{\bar{z}'}\right)\bar{Z} + \bar{x}}\right) \tag{15}$$

The MSE of the estimator t_8 is

$$MSE(t_8) = \bar{Y}^2\left[f_1 C_y^2 + \frac{1}{4}(f_3 C_x^2 + f_2 C_z^2) - \left(f_3\rho_{yx}C_yC_x + f_2\rho_{yz}C_yC_z\right)\right] \tag{16}$$

Motivated by Singh and Vishwakarma (2007), Yadav, Singh and Chatterjee (2013) suggested a class of chain ratio exponential type estimator for population mean $\bar{Y}$ using information on two auxiliary variables x and z in two phase or double sampling as

$$t_9 = \bar{y} exp\left[\frac{\hat{\hat{X}}_{rd} - \bar{x}}{\hat{\hat{X}}_{rd} + \bar{x}}\right] \tag{17}$$

where

$$\hat{\hat{X}}_{rd} = \frac{\bar{x}'}{(a\bar{z}' + b)}(a\bar{Z} + b)$$

Thus, the above estimator can be expressed as

$$t_9 = \bar{y} exp\left[\frac{\frac{\bar{x}'}{(a\bar{z}'+b)}(a\bar{Z}+b) - \bar{x}}{\frac{\bar{x}'}{(a\bar{z}'+b)}(a\bar{Z}+b) + \bar{x}}\right] \tag{18}$$

The mean square error of the estimator t_9 is

$$MSE\ (t_9) = \bar{Y}^2\left[f_1 C_y^2 + \frac{f_3}{4}\left(C_x^2 - 4\rho_{yx} C_y C_x\right) - f_2 \rho_{yz}^2 C_y^2\right] \tag{19}$$

4. The suggested class of estimator

Following the previously discussed estimation procedures for two-phase sampling, we have proposed an efficient ratio-type exponential estimator

$$t_{10} = \bar{y} exp\left[k_1\left\{\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right\} + k_2\left\{\frac{\bar{z}' - \bar{Z}}{\bar{z}' + \bar{Z}}\right\}\right] \tag{20}$$

Where k_1 and k_2 are unknown constants. The values of k_1 and k_2 can be determined by the principle of optimality conditions.

To obtain the mean square error of the proposed estimator t_{10}, we consider

$$\bar{y} = \bar{Y}(1 + e_0), \bar{x} = \bar{X}(1 + e_1)\ ,\quad \bar{x}' = \bar{X}(1 + e_2)\ ,\ \bar{z}' = \bar{Z}(1 + e_3)$$

Such that

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0$$

$$E(e_0^2) = \left(\frac{1-f}{n}\right) C_y^2 \ , \ E(e_1^2) = \left(\frac{1-f}{n}\right) C_x^2$$

$$E(e_2^2) = \left(\frac{1-f'}{n'}\right) C_x^2, \ E(e_3^2) = \left(\frac{1-f'}{n'}\right) C_z^2$$

$$E(e_0 e_1) = \left(\frac{1-f}{n}\right) \rho_{yx} C_y C_x, \ E(e_0 e_2) = \left(\frac{1-f'}{n'}\right) \rho_{yx} C_y C_x$$

$$E(e_0 e_3) = \left(\frac{1-f'}{n'}\right) \rho_{yz} C_y C_z, \ E(e_1 e_2) = \left(\frac{1-f'}{n'}\right) C_x^2$$

$$E(e_1 e_3) = \left(\frac{1-f'}{n'}\right) \rho_{xz} C_x C_z, \ E(e_2 e_3) = \left(\frac{1-f'}{n'}\right) \rho_{xz} C_x C_z$$

The mean square error of the proposed estimator t_{10} is as follows:

$$t_{10} = \bar{y} exp\left[k_1\left\{\frac{e_2-e_1}{2+e_1+e_2}\right\} + k_2\left\{\frac{e_3}{2+e_3}\right\}\right]$$
$$= \bar{y} exp\left[k_1\{e_2-e_1\}\frac{1}{2}\left(1-\frac{e_1}{2}-\frac{e_2}{2}+\frac{e_1^2}{4}+\frac{e_2^2}{4}\right) + k_2\left\{e_3\frac{1}{2}\left(1-\frac{e_3}{2}+\frac{e_3^2}{4}\right)\right\}\right]$$
$$= \bar{y} exp\left[k_1\left\{\frac{e_2}{2}-\frac{e_1e_2}{4}-\frac{e_2^2}{4}-\frac{e_1}{2}+\frac{e_1^2}{4}+\frac{e_1e_2}{4}\right\} + k_2\left\{\frac{e_3}{2}-\frac{e_3^2}{4}\right\}\right]$$

$$= \bar{Y}(1+e_0)\left[1 + k_1\left\{\frac{e_2}{2}-\frac{e_1e_2}{4}-\frac{e_2^2}{4}-\frac{e_1}{2}+\frac{e_1^2}{4}+\frac{e_1e_2}{4}\right\} + k_2\left\{\frac{e_3}{2}-\frac{e_3^2}{4}\right\} + \left\{k_1\left(\frac{e_2}{2}-\frac{e_1e_2}{4}-\frac{e_2^2}{4}-\frac{e_1}{2}+\frac{e_1^2}{4}+\frac{e_1e_2}{4}\right)\right\}^2 + \left\{k_2\left(\frac{e_3}{2}-\frac{e_3^2}{4}\right)\right\}^2\right]$$

Neglecting the term having power greater than two, we have

$$t_{10} = \bar{Y}\left[1 + k_1\frac{e_2}{2} - k_1\frac{e_2^2}{4} - k_1\frac{e_1}{2} + k_1\frac{e_1^2}{4} + k_2\frac{e_3}{2} - k_2\frac{e_3^2}{4} + k_1^2\frac{e_2^2}{8} + k_1^2\frac{e_1^2}{8} + k_2^2\frac{e_3^2}{8} + e_0 + k_1\frac{e_0e_2}{2} - k_1\frac{e_0e_1}{2} + k_2\frac{e_0e_3}{2}\right]$$

or

$$t_{10} - \bar{Y} = \bar{Y}\left[k_1\frac{e_2}{2} - k_1\frac{e_2^2}{4} - k_1\frac{e_1}{2} + k_1\frac{e_1^2}{4} + k_2\frac{e_3}{2} - k_2\frac{e_3^2}{4} + k_1^2\frac{e_2^2}{8} + k_1^2\frac{e_1^2}{8} + k_2^2\frac{e_3^2}{8} + e_0 + k_1\frac{e_0e_2}{2} - k_1\frac{e_0e_1}{2} + k_2\frac{e_0e_3}{2}\right] \quad (21)$$

The bias of the estimator t_{10} can be obtained by taking expectation on both sides of equation (21) and is given by

$$\boldsymbol{B(t_{10}) = \bar{Y}^2\left[k_1\frac{e_1^2}{4} - k_1\frac{e_2^2}{4} - k_2\frac{e_3^2}{4} + k_1^2\frac{e_2^2}{8} + k_1^2\frac{e_1^2}{8} + k_2^2\frac{e_3^2}{8} + k_1\frac{e_0e_2}{2} - k_1\frac{e_0e_1}{2} + k_2\frac{e_0e_3}{2}\right]} \quad (22)$$

By squaring and taking expectation on both sides of equation (21), we get the mean square error of t_{10} to the first degree of approximation.

$$E(t_{10}-\bar{Y})^2 = \bar{Y}^2 E\left[k_1^2\frac{e_2^2}{4} + k_1^2\frac{e_1^2}{4} + k_2^2\frac{e_3^2}{4} + e_0^2 - k_1^2\frac{e_1e_2}{2} + k_1k_2\frac{e_2e_3}{2} + \quad k_1e_0e_2 - k_1k_2\frac{e_1e_3}{2} - k_1e_0e_1 + \ k_2e_0e_3\right]$$

$$= \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + \frac{k_1^2}{4}\left(\frac{1-f''}{n}\right)C_x^2 + \frac{k_2^2}{4}\left(\frac{1-f'}{n'}\right)C_z^2 - k_1\left(\frac{1-f''}{n}\right)\rho_{yx}C_yC_x + \quad k_2\left(\frac{1-f'}{n'}\right)\rho_{yz}C_yC_z\right]$$

$$MSE(t_{10}) = \bar{Y}^2\left[f_1C_y^2 + \frac{k_1^2}{4}f_3C_x^2 + \frac{k_2^2}{4}f_2C_z^2 - k_1f_3\rho_{yx}C_yC_x + k_2f_2\rho_{yz}C_yC_z\right] \tag{23}$$

Now, we have to find out the optimum values of k_1 and k_2

$$\frac{\partial MSE(t_{10})}{\partial k_1} = 0$$

$$\Rightarrow k_{1_{opt}} = \frac{2\rho_{yx}C_yC_x}{C_x^2}$$

$$\Rightarrow k_{1_{opt}} = 2\rho_{yx}\left(\frac{C_y}{C_x}\right)$$

Now, for k_2

$$\frac{\partial\ MSE(t_{10})}{\partial k_2} = 0$$

$$\Rightarrow k_{2_{opt}} = \frac{2\rho_{yz}C_yC_z}{C_z^2}$$

$$\Rightarrow k_{2_{opt}} = -2\rho_{yz}\left(\frac{C_y}{C_z}\right)$$

By substituting above $k_{1_{opt}}$ and $k_{2_{opt}}$ in equation (23) we obtained the minimum mean square error of the estimator t_{10} as

$$MSE(t_{10})_{opt} = \bar{Y}^2\left[f_1C_y^2 + \left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right)f_3\rho_{yx}C_yC_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right)f_2\rho_{yz}C_yC_z\right] \tag{24}$$

5. Theoretical Comparison

The proposed estimator t_{10} under its optimality condition is more efficient than the existing estimators $t_i (i = 1,2 \dots\dots 9)$ if and only if the following conditions hold.

(i) $MSE(t_1) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[f_3\left(C_x^2 - 2\rho_{yx} C_y C_x\right)\right] \tag{25}$$

(ii) $MSE(t_2) - MSE(t_{10})_{opt} > 0$

$$\left[f_1 C_y^2 + \left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < C_y^2\left[f_1\left(1 - \rho_{yx}^2\right) + f_2 \rho_{yx}^2\right] \tag{26}$$

(iii) $MSE(t_3) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[\frac{f_3}{4}\left(C_x^2 - 4\rho_{yx} C_y C_x\right)\right] \tag{27}$$

(iv) $MSE(t_4) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[\frac{f_3}{4}\left(C_x^2 + 4\rho_{yx} C_y C_x\right)\right] \tag{28}$$

(v) $MSE(t_5) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[f_3\left(C_x^2 - 2\rho_{yx} C_y C_x\right) + f_2\left(C_z^2 - 2\rho_{yz} C_y C_z\right)\right] \quad (29)$$

(vi) $MSE(t_6) - MSE(t_{10})_{opt} > 0$

$$\left[f_1 C_y^2 + \left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < C_y^2\left[f_1\{1 - \rho_{yx}^2\} + f_2\left\{\rho_{yx}^2 + \rho_{yx}^2 \frac{C_z^2}{C_x^2} - 2\rho_{yx}\rho_{yz}\frac{C_z}{C_x}\right\}\right] \quad (30)$$

(vii) $MSE(t_7) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[f_2\left(\frac{k^2}{4} C_z^2 - k\rho_{yz} C_y C_z\right) + \frac{f_3}{4}\left(C_x^2 - 4\rho_{yx} C_y C_x\right)\right] \quad (31)$$

(viii) $MSE(t_8) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[\frac{1}{4}\left(f_3 C_x^2 + f_2 C_z^2\right) - \left(f_3 \rho_{yx} C_y C_x + f_2 \rho_{yz} C_y C_z\right)\right] \quad (32)$$

(ix) $MSE(t_9) - MSE(t_{10})_{opt} > 0$

$$\left[\left(\rho_{yx}\left(\frac{C_y}{C_x}\right)\right)^2 f_3 C_x^2 + \left(\rho_{yz}\left(\frac{C_y}{C_z}\right)\right)^2 f_2 C_z^2 - 2\rho_{yx}\left(\frac{C_y}{C_x}\right) f_3 \rho_{yx} C_y C_x - 2\rho_{yz}\left(\frac{C_y}{C_z}\right) f_2 \rho_{yz} C_y C_z\right] < \left[\frac{f_3}{4}\left(C_x^2 - 4\rho_{yx} C_y C_x\right) - f_2 \rho_{yz}^2 C_y^2\right] \quad (33)$$

6. Empirical Study

To judge the superiority of our newly proposed estimator over the competing estimators we have taken the following numerical values of different population parameters from two different population data sets.
Population-1 [Source: Singh (1967)]

The variables are

y: Number of females employed

x: Number of females in services

z: Number of educated females

$N = 61 n' = 20 n = 10 \bar{Y} = 7.46$

$\bar{X} = 5.31 \bar{Z} = 179.00 C_y = 0.7103 C_x = 0.7587$

$C_z = 0.2515 \rho_{yx} = 0.7737 \rho_{yz} = -0.2070 \rho_{xz} = -0.0033$

Population-2 [Source: Murthy (1967) pp. 226]

The variables are

y: Output

x: Number of workers

z: Fixed capital

$N = 61 n' = 20 n = 10 \bar{Y} = 7.46$

$\bar{X} = 5.31 \bar{Z} = 179.00 C_y = 0.7103 C_x = 0.7587$

$C_z = 0.2515 \rho_{yx} = 0.7737 \rho_{yz} = -0.2070 \rho_{xz} = -0.0033$

To compare the efficiency of the proposed estimator and the considered existing estimators, we have computed the percent relative efficiencies (PREs).

The formula for calculating the percent relative efficiency is given by

$$PRE(t_i) = \left[\frac{var(\bar{y})}{MSE(t_i)}\right] \times 100 ; \quad (34)$$

where $i = 1,2,3,4,5,6,7,8,9,10$

Findings are given in Table 1.

Table 1:

Estimators	Population-1	Population-2
t_1	144.1214	39.1380
t_2	155.8702	233.9572
t_3	147.6438	180.6875
t_4	60.1505	25.6294
t_5	124.5640	36.8749
t_6	140.9066	607.6388
t_7	144.1214	285.2960
t_8	139.3606	361.375
t_9	151.3525	373.8362
$\boldsymbol{t_{10}}$	**159.9441**	**677.5781**

7. Conclusion

On account of the percent relative efficiencies of the estimators as shown in Table 1, it has been observed that the performance of the proposed estimator is better than the other existing estimators. Hence, it is recommended for use in practice.

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