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Reliability for Zeghdoudi distribution with an outlier, fuzzy reliability and application

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Abstract

This study focuses on estimating reliability *P[Y<X]*, where *Y* has a Zeghdoudi distribution with parameter *a*, *X* has a Zeghdoudi distribution with one outlier present and parameter *c*, and the remaining (*n* – 1) random variables are from a Zeghdoudi distribution with parameter *b*, in order for *X* and *Y* to be independent. Several findings of a simulation study and the maximum likelihood estimate of *R* are provided. We also present some results related to fuzzy dependability. Finally, using actual data on survival durations (in days) of 72 Algerians infected with a coronavirus, we demonstrate how the Zeghdoudi distribution may be applied to other distributions in order to demonstrate its adaptability.

Key words: Zeghdoudi distribution, maximum likelihood estimator, Newton-Raphson method, outlier, fuzzy reliability

1. Introduction

Inferences regarding $R = P(Y < X)$, when *X* and *Y* are independently distributed, are of relevance in the reliability context and play a significant role in many practical domains, including engineering, medicine, and quality control. In the statistical literature, *R* estimation is a highly common practice. It calculates the likelihood that a component's stress *Y* will be greater than its random strength *X*. Additionally, R offers the likelihood that a system would malfunction if the applied stress exceeds its capacity.

The earliest research on this issue dates back to Birnbaum (1956) and Birnbaum and McCarty (1958). Kapur and Lamberson have also discussed the reliability under stress (1977). *R* was estimated for the negative binomial distribution by Sathe and Dixit

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(2001). In the presence of outliers produced by uniform distributions, Dixit and Nasiri (2001) provided an estimation of the parameters of the exponential distribution. Then, in the exponential and gamma cases, respectively, Baklizi and Dayyeh (2003) performed a shrinkage estimation of *R*, Kundu and Gupta (2005) considered an estimation of $P[Y < X]$ for a generalized exponential distribution, and Deiri (2011) performed an estimation of *R* with the presence of two outliers. Jafari (2011) obtained the moment, maximum likelihood, and mixture estimators of *R* in the Rayleigh distribution in the presence of one outlier, and Jabbari (2013) discussed the estimation of *R* in the Lindley distribution with an Outlier. Deiri (2010) considered the estimation of reliability for the exponential case in the presence of one outlier. And most recently, Lindley distributions have been used to draw conclusions on stress-strength reliability by Mutairi, Guitani, and Kundu (2013).

With one outlier generated from the same distribution, we obtain the maximum likelihood estimate of *R* for the Zeghdoudi distribution in this study. The Zeghdoudi distribution with parameter a, probability density function (pdf) is given by

$$f(y,a) = \frac{a^3}{2+a} y(1+y)e^{-ay} \quad a > 0$$

In this study, it is assumed that the random variables $(Y_1, Y_2, \dots, Y_m)$ have a Zeghdoudi distribution with parameter a, while the random variables $(X_1, X_2, \dots, X_n)$ are such that one of them comes from a Zeghdoudi distribution with parameter c and the remaining $(n-1)$ random variables come from a Zeghdoudi distribution with parameter b.

The body of the article is structured as follows. The joint distribution of $(X_1, X_2, \dots, X_n)$ in the presence of one outlier is obtained in Section 2. The MLE of *R* and the method of maximum likelihood estimators of parameters are discussed in Sections 3 and 4, respectively. The simulation studies are provided in Section 5. Section 6 discusses illustrative combinations of the Zeghdoudi distribution with other distributions to demonstrate the adaptability of this distribution. Finally, Section 7 is the conclusion of the study.

2. Joint distribution of $(X_1,\ X_2,\ ,X_n)$ with an outlier

Consider that $(X_1, X_2, \dots, X_n)$ are distributed with p.d.f $g(x,c)$ as $Zeghdoudi\ (c)$ and remaining $(n-1)$ of them are distributed with p.d.f $f(x,b)$ as $Zeghdoudi\ (b)$. The joint distribution of $(X_1, X_2, \dots, X_n)$ can be expressed as

$$f(x_1, x_2, \dots, x_n; b, c) = \frac{(n-1)!}{n!} \prod_{i=1}^{n} f(x,b) \sum_{i=1}^{n} \frac{g(x_i;c)}{f(x_i;b)}$$

$$= \frac{(n-1)!}{n!} \prod_{i=1}^{n} \frac{b^3}{2+b} x(1+x)e^{-bx} \sum_{i=1}^{n} \left[\frac{\left(\frac{c^3}{2+c} x(1+x)e^{-cx}\right)}{\left(\frac{b^3}{2+b} x(1+x)e^{-bx}\right)} \right]$$

$$= \frac{(n-1)!}{n!} \frac{b^{3n}}{(2+b)^n} \prod_{i=1}^{n} x_i(1+x_i)e^{-b\sum_{i=1}^{n} x_i} \sum_{i=1}^{n} \left[\frac{\left(\frac{c^3}{2+c} x_i(1+x_i)e^{-cx_i}\right)}{\left(\frac{b^3}{2+b} x_i(1+x_i)e^{-bx_i}\right)} \right]$$

$$= \frac{(n-1)!}{n!} \frac{b^{3n-3}}{(2+b)^{n-1}} \frac{c^3}{2+c} \prod_{i=1}^{n} x_i(1+x_i)e^{-b\sum_{i=1}^{n} x_i} \sum_{i=1}^{n} x_i(1+x_i)e^{x_i(b-c)} \tag{1}$$

The marginal distribution of X is

$$f(x;b,c) = \frac{1}{n} \frac{c^3}{2+c} x_i(1+x_i)e^{-cx} + \frac{n-1}{n} \frac{b^3}{2+b} x(1+x)e^{-bx} \tag{2}$$

Using (2) to obtain $R = (Y < X)$.

3. Maximum likelihood estimators of parameters

Let $(Y_1, Y_2, \dots, Y_m)$ be a random sample for Y with pdf,

$$f(y;a) = \frac{a^3}{2+a} x(1+x)e^{-ay} \qquad x, a > 0$$

The log likelihood function is given by

$$L(a) = 3mlna(a+2) + \sum_{i=1}^{m} \ln(y_i + y_i{}^2) - a\sum_{i=1}^{m} y_i$$

The MLE of a is obtained by taking the derivative with regard to a and equating it to 0.

$$\hat{a} = \frac{1}{y}\left(-\bar{y} + \sqrt{4\bar{y} + \bar{y}^2 + 1} + 1\right) \tag{3}$$

Now, consider $X_1, X_2, \dots, X_n$ as a random sample for X with one outlier present and pdf,

$$f(x;b,c) = \frac{1}{n} \frac{c^3}{2+c} x_i(1+x_i)e^{-cx} + \frac{n-1}{n} \frac{b^3}{2+b} x(1+x)e^{-bx}$$

From (1), the log likelihood function is given by

$$L(b,c) = \ln\left(\frac{(n+1)!}{n!}\right) + (3n+3)lnb - (n-1)\ln(2+b) + 3lnc$$

$$-\ln(2+c) + \sum_{i=1}^{n} \ln\big(x_i(1+x_i)\big) - b\sum_{i=1}^{n} x_i + \ln\sum_{i=1}^{n} e^{x_i(b-c)}$$

We derive the normal equations by taking the derivative with regard to b and c and equating the results to 0.

$$\frac{\delta L(b,c)}{\delta b} = \frac{3n-3}{b} - \frac{n-1}{2+b} - \sum_{i=1}^{n} x_i + \frac{\sum_{i=1}^{n} x_i e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}} \tag{4}$$

$$\frac{\delta L(b,c)}{\delta c} = \frac{3}{c} - \frac{1}{1+c} - \frac{\sum_{i=1}^{n} x_i e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}} \tag{5}$$

This system of equations lacks a closed-form solution, so the authors use the Newton-Raphson method to iteratively find the values of $\hat{b}$and $\hat{c}$. In this instance, we will iteratively estimate $\hat{\beta} = \left(\hat{b}, \hat{c}\right)$.

$$\hat{\beta}_{i+1} = \hat{\beta}_i - K^{-1}k \tag{6}$$

where k is the vector of normal equations for which we want

$$k = [k_1, k_2]$$

with

$$k_1 = \frac{3n-3}{b} - \frac{n-1}{2+b} - \sum_{i=1}^{n} x_i + \frac{\sum_{i=1}^{n} x_i e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}}$$

$$k_2 = \frac{3}{c} - \frac{1}{1+c} - \frac{\sum_{i=1}^{n} x_i e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}}$$

and K is the matrix of second derivatives

$$K = \begin{bmatrix} \frac{dk_1}{db} & \frac{dk_1}{dc} \\ \frac{dk_2}{db} & \frac{dk_2}{dc} \end{bmatrix}$$

where

$$\frac{dk_1}{db} = \frac{3-3n}{b^2} + \frac{n-1}{(1+b)^2} + \frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}} - \left(\frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}}\right)^2$$

$$\frac{dk_2}{db} = -\frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}} + \left(\frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}}\right)^2$$

$$\frac{dk_2}{dc} = -\frac{3}{c^2} + \frac{1}{(1+c)^2} + \frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}} - \left(\frac{\sum_{i=1}^{n} {x_i}^2 e^{x_i(b-c)}}{\sum_{i=1}^{n} e^{x_i(b-c)}}\right)^2$$

As our estimate of b and c fluctuate by less than a permitted amount with each subsequent iteration, the Newton-Raphson method converges to $\hat{b}$ and $\hat{c}$.

4. Fuzzy reliability of Zeghdoudi distribution

Let T denote the time until a system fails, which is a continuous random variable (component). The fuzzy probability in formula can then be used to compute the fuzzy dependability.

$$R_F(t) = P(T > t) = \int_t^{\infty} \mu(x) f\ \ (x)dx, \quad 0 \le t \le x < \infty,$$

where $\mu(x)$ is a membership function that expresses how much a given universe's elements belong to a fuzzy set. Assume now that $\mu(x)$ is

$$\mu(x) = \begin{cases} 0, & x \le t_1 \\ \dfrac{x - t_1}{t_0 - t_1}, & t_1 < x < t_2,\ \ t_1 \ge 0 \\ 1, & x \ge t_2 \end{cases}$$

For $\mu(x)$, by the computational analysis of the function of fuzzy numbers, the lifetime $x(\gamma)$ can be obtained corresponds to a certain value of $\gamma - Cut, \gamma \in [0,1]$, can be obtained by $\mu(x) = \gamma \to \frac{x-t_1}{t_0-t_1} = \gamma$, then

$$\begin{cases} x(\gamma) \le t_1, & \gamma = 0 \\ x(\gamma) = t_1 + \gamma(t_2 - t_1), & 0 < \gamma < 1 \\ x(\gamma) \ge t_2, & \gamma = 1 \end{cases}$$

As a result, it is possible to determine the fuzzy reliability values for all γ values. The fuzzy reliability definition establishes the Zeghdoudi distribution's fuzzy dependability. The Zeghdoudi distribution's fuzziness dependability is defined as

$$R_F(t) = \left(\frac{x^2a^2 + a(2+a)x + a + 2}{a+2}\right)e^{-ax} - \left(\frac{x(\gamma)^2a^2 + a(2+a)x(\gamma) + a + 2}{a+2}\right)e^{-ax(\gamma)}$$

Then $R_F(t) = 0$.

4.1. Numerical values of fuzzy reliability

We compared traditional reliability and fuzzy reliability in this subsection, where traditional reliability is a survival function as $R(x) = \left(\frac{x^2a^2+a(2+a)x+a+2}{a+2}\right)e^{-ax}$

The comparison was discussed in Table 1. Based on findings, the following observations are made:

- when $\gamma - Cut$ is increased, the Fuzzy reliability increases.
- when t_2 interval of membership function is increased, the Fuzzy reliability increases.
- when t_1 is decreased, the fuzzy reliability increases, and vice versa.
- the traditional reliability with t_2 is lower than the traditional reliability with t_1.

A sequence of drawings from the Zeghdoudi distribution is produced by the fuzzy estimating procedure.

Algorithm: fuzzy estimation algorithm

- **Input**: initial values of a, interval time (t_1, t_2) and γ where $0 < \gamma < 1$.
- **Calculate**: $x(\gamma) = t_1 + \gamma(t_2 - t_1)$.
- **For** each method do

Set: i=1.

Estimate parameter as $\hat{a}$.

Calculate

$$\hat{R}_F(t) = \left(\frac{{t_1}^2a^2 + a(2+a)t_1 + a + 2}{a+2}\right)e^{-at_1} - \left(\frac{x(\gamma)^2a^2 + a(2+a)x(\gamma) + a + 2}{a+2}\right)e^{-ax(\gamma)}$$

- **End**

Table 1: Fuzzy reliability with different values of a, t_1, t_2, γ.

a	t_1	t_2	$R(t_1)$	$R(t_2)$	R_F		
					0.25	0.5	0.9
0.2	0.01	1	1	0.99736	0.00013	0.00057	0.00208
0.5	0.5	2	0.99297	0.88291	0.01542	0.03953	0.09393
1	0.1	3	0.99834	0.34851	0.58082	0.88103	0.96981
3	0.2	1	0.91761	0.28876	0.16824	0.34766	0.58321
5	0.1	1.5	0.93146	0.00914	0.51268	0.79802	0.91541

4.2. The maximum likelihood estimator of *R*

Let $Y \sim Zeghdoudi(a)$ with pdf $h(y;a)$ and X be distributed with pdf $f(x;b,c)$ given in (2). The parameter R is estimated as

$$R = P(Y < X) = \int_0^{\infty}\int_0^{x} h(y;a) f(x;b,c)\,dy\,dx$$
$$= \frac{1}{n}\int_0^{\infty}\int_0^{x} \frac{a^3}{2+a} x(1+x)e^{-ay}\frac{c^3}{2+c}x(1+x)e^{-cx}dydx + \frac{n-1}{n}\int_0^{\infty}\int_0^{x} \frac{a^3}{2+a} x(1+x)e^{-ay}\frac{b^3}{2+b}x(1+x)e^{-bx}dydx$$

$$= \frac{1}{n}\left[1 - \frac{c^3}{(a+2)(c+2)}\left(\frac{a+2}{(a+c)^2} + \frac{(2a^2+6a+4)}{(a+c)^3} + \frac{12a(a+2)}{(a+c)^4} + \frac{24a^2}{(a+c)^5}\right)\right] + \frac{n-1}{n}\left[1 - \frac{b^3}{(a+2)(b+2)}\left(\frac{a+2}{(a+b)^2} + \frac{(2a^2+6a+4)}{(a+b)^3} + \frac{12a(a+2)}{(a+b)^4} + \frac{24a^2}{(a+b)^5}\right)\right] \tag{7}$$

Thus, by invariant property for MLEs, the MLE of R is

$$\hat{R} = \frac{1}{n}\left[1 - \frac{\hat{c}^3}{(\hat{a}+2)(\hat{c}+2)}\left(\frac{\hat{a}+2}{(\hat{a}+\hat{c})^2} + \frac{(2\hat{a}^2+6\hat{a}+4)}{(\hat{a}+\hat{c})^3} + \frac{12\hat{a}(\hat{a}+2)}{(\hat{a}+\hat{c})^4} + \frac{24\hat{a}^2}{(\hat{a}+\hat{c})^5}\right)\right] + \frac{n-1}{n}\left[1 - \frac{\hat{b}^3}{(\hat{a}+2)(\hat{b}+2)}\left(\frac{\hat{a}+2}{\left(\hat{a}+\hat{b}\right)^2} + \frac{(2\hat{a}^2+6\hat{a}+4)}{\left(\hat{a}+\hat{b}\right)^3} + \frac{12\hat{a}(\hat{a}+2)}{\left(\hat{a}+\hat{b}\right)^4} + \frac{24\hat{a}^2}{\left(\hat{a}+\hat{b}\right)^5}\right)\right]$$

Where $\hat{a}, \hat{b}$ and $\hat{c}$ can be obtained from (3) and (6).

5. Simulation study

In this section, using the accept-reject approach and Maple software, we generate random numbers from the Zeghdoudi distribution (both with and without an outlier). We obtain the maximum likelihood estimators of the parameters a, b and c using these samples and the Newton-Raphson technique. The MLE of R is then calculated using these parameters. The values of biases and MSEs of these estimates are presented in Table 2, for $a = 1$, $b = 2$ and $c = 1.3,\ 1.4,\ 1.5,\ 1.6,\ 1.9,\ 2.0,\ 2.1,\ 2.6,\ 2.7,\ 2.9,\ 3.1,\ 3.5,\ 6.0$. All the results are based on 100 replications.

Table 2: Biases and (MSE)s of the MLEs of *R*, for *a=1*, *b=2*, and different values of *c*

$n \longrightarrow$ $c \downarrow$	n=10	n=20	n=30	n=50	n=60	n= 80	n=90
1.3	0.0013611	0.0012772	0.0012493	0.0012269	0.0012213	0.0012143	0.001212
1.4	0.0013274	0.0012604	0.0012380	0.0012202	0.0012157	0.0012101	0.001208
1.5	0.0012977	0.0012455	0.0012281	0.0012142	0.0012107	0.0012064	0.001205
1.6	0.0012714	0.0012324	0.0012194	0.0012090	0.0012064	0.0012031	0.001202
1.9	0.0012095	0.0012014	0.0011987	0.0011966	0.0011961	0.0011954	0.0011952
2.0	0.0011934	0.0011934	0.0011934	0.0011934	0.0011934	0.0011934	0.0011934
2.1	0.0011791	0.0011862	0.0011886	0.0011905	0.0011910	0.0011916	0.0011918
2.6	0.0011281	0.0011607	0.0011716	0.0011803	0.0011825	0.0011852	0.0011861
2.7	0.0011209	0.0011572	0.0011692	0.0011789	0.0011813	0.0011843	0.0011853
2.9	0.0011088	0.0011511	0.0011652	0.0011765	0.0011793	0.0011828	0.0011840
3.1	0.0010991	0.0011462	0.0011619	0.0011745	0.0011777	0.0011816	0.0011829
3.5	0.0010850	0.0011392	0.0011572	0.0011717	0.0011753	0.0011798	0.0011813
6	0.0010638	0.0011269	0.0011490	0.0011668	0.0011712	0.0011712	0.0011786

6. Illustrative application

To demonstrate the adaptability of the Zeghdoudi distribution, we offer an example application of it with other distributions in this section. Therefore, we examine the Lindley, exponential, and Zeghdoudi distributions using real data on the survival times (in days) of 72 Algerians who had contracted a coronavirus (https://www.who.int/fr/news/item), Table 3.

Table 3: Comparison between Lindley, exponential and Zeghdoudi distributions

Survival time $m = 3.2$	Obsfreq	Lindley $\widehat{\theta} = 0.50$	Exp $\widehat{\theta} = 0.30$	Zeghdoudi $\widehat{\theta} = 0.6$
[0, 2]	**34**	**27.30**	**33.90**	**30.05**
[2, 4]	**17**	**22.10**	**20.50**	**19.81**
[4, 6]	**11**	**12.15**	**7.43**	**10.05**
[6, 8]	**7**	**7.28**	**6.67**	**7.02**
[8, 10]	**3**	**3.17**	**3.50**	**3.07**
Total	**92**	**92**	**92**	**92**
χ^2		2.946	**2.400**	**1.0095**

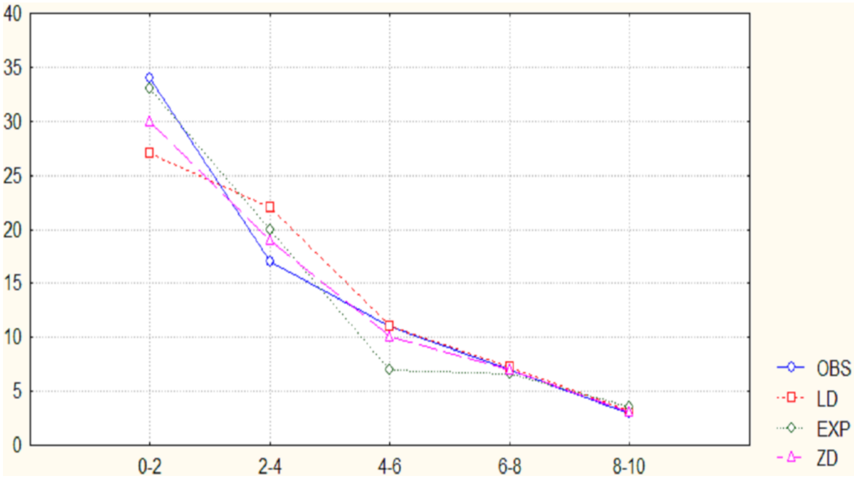

Figure 1: Comparison between distributions

As shown in Table 2 and Figure 1, the Zeghdoudi distribution offers the smallest χ^2 value in comparison to the Lindley and exponential distributions, and as a result, best fits the data of all the distributions taken into consideration.

7. Conclusion

The challenge of estimating $P(Y < X)$ for the Zeghdoudi distribution in the presence of one outlier has been addressed in this study. Studies have been done on the maximum likelihood estimator for R and fuzzy dependability.

Table 1 contains all of the results, which were based on 100 replications. According to the simulation's findings, biases and MSEs frequently hover around zero when parameters b and c are close to one another, and they rise when the difference between b and c approaches one.

In order to demonstrate the adaptability of the Zeghdoudi distribution, the authors suggested an exemplary application using real data on the survival times (in days) of 72 Algerian people who were infected with coronaviruses, and then compared the outcomes with those of other distributions.

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