

Confidence intervals for the Double XShanker distribution parameter: a simulation and application study

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Abstract

We examine four two-sided confidence intervals for parameter θ of the Double XShanker distribution: the likelihood-based, Wald-type, bootstrap- t , and the bias-corrected and accelerated (BCa) bootstrap intervals. An explicit expression for the observed Fisher information is derived, allowing the Wald interval to be computed directly from the maximum likelihood estimate. Interval performance is evaluated by empirical coverage probability (ECP) and average width (AW) through the Monte Carlo experiments over a range of sample sizes and parameter values. The likelihood-ratio and Wald intervals remain close to the nominal 0.95 level across most settings considered, whereas the bootstrap- t and BCa intervals tend to be narrower but show undercoverage in small samples. An additional sensitivity analysis using $B = 1000, 2000$, and 5000 bootstrap resamples indicates that the bootstrap-based conclusions are stable with respect to the number of resamples. The methods are also illustrated using rainfall data from the Los Angeles Civic Center, where the empirical interval estimates are consistent with the simulation findings.

Keywords: parameter estimation, confidence interval, Monte Carlo simulation, coverage probability, bootstrap.

1. Introduction

Accurate parameter estimation is central to statistical modeling, as it supports reliable inference and informed decision-making. This is particularly important in lifetime or survival analysis, where inference is based on time-to-event outcomes such as failure times, survival probabilities, and hazard rates. In practical fields including medical research, engineering, and actuarial science, imprecise or biased estimates may lead to misleading conclusions regarding system reliability, prognosis, or risk assessment.

To accommodate the diverse shapes observed in empirical lifetime data, various generalized and mixed probability distributions have been proposed to increase modeling flexibility (Nadarajah & Kotz, 2006). For example, Shukla (2018) introduced the Ram Awadh distribution as a two-component mixture combining an exponential distribution with scale parameter θ and a gamma distribution with shape parameter σ and common scale parameter θ . Messaadia and Zeghdoudi (2018) proposed the Zeghdoudi distribution, constructed from

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a mixture of $\text{Gamma}(2, \theta)$ and $\text{Gamma}(3, \theta)$ distributions. Related developments include compounded models such as the Gamma zero-truncated Poisson distribution (Niyomdecha et al., 2023), which further illustrate the continued interest in flexible lifetime distributions. More recently, Etaga et al. (2023) introduced the Double XShanker distribution by combining the exponential distribution with the XShanker distribution. This construction yields a flexible one-parameter model capable of representing a range of lifetime data patterns.

Although the Double XShanker distribution has been studied from a distributional perspective, the finite-sample behaviour of confidence intervals for its parameter has received little attention. Interval estimation in non-standard or flexible models may exhibit non-negligible small-sample effects. For instance, Srisuradetchai and Dangsupa (2022) examined likelihood-based and score-type CIs for the geometric parameter in a zero-inflated geometric model and demonstrated the importance of coverage accuracy in small samples.

Motivated by this gap, this study considers four methods for constructing two-sided CIs for the parameter of the Double XShanker distribution: the likelihood-based interval, the Wald-type interval, the bootstrap- t interval, and the bias-corrected and accelerated (BCa) bootstrap interval. Their performance is assessed through Monte Carlo simulation across a range of sample sizes and parameter values. An empirical application to rainfall data is also provided to illustrate the methods in practice.

2. Methodology

This section defines the XShanker and Double XShanker models, outlines maximum-likelihood estimation for θ , and specifies the four confidence-interval constructions used in the simulation and data analysis.

2.1. The XShanker and the Double XShanker distributions

The XShanker distribution is formulated as a mixture of the exponential and Shanker distributions (Shanker, 2015), combined using specific mixing probabilities. In this construction, the exponential component has rate parameter θ , and the Shanker component is parameterized by θ . Let X denote a random variable following the XShanker distribution with parameter θ . The probability density function (pdf) of the XShanker distribution can be derived using a mixed model comprising two components, each weighted by their respective mixing probabilities, as follows:

$$f_{\text{XShanker}}(x; \theta) = p \cdot f_{\text{Exp}}(x; \theta) + (1 - p) \cdot f_{\text{Shanker}}(x; \theta),$$

where $f_{\text{Exp}}(x; \theta) = \theta e^{-\theta x}$ is the pdf of the exponential distribution with rate θ , $f_{\text{Shanker}}(x; \theta) = \frac{\theta^2}{\theta^2 + 1} (\theta + x) e^{-\theta x}$ is the pdf of the Shanker distribution with parameter θ , and the mixing proportion is $p = \theta^2 / (\theta^2 + 1)$. The pdf of the XShanker distribution is expressed as follows:

$$f_{\text{XShanker}}(x; \theta) = \frac{\theta^2}{(\theta^2 + 1)^2} [\theta^3 + 2\theta + x] e^{-\theta x}, \quad x > 0, \theta > 0. \quad (1)$$

Figure 1 illustrates the pdf plot of the XShanker distribution for various parameter values.

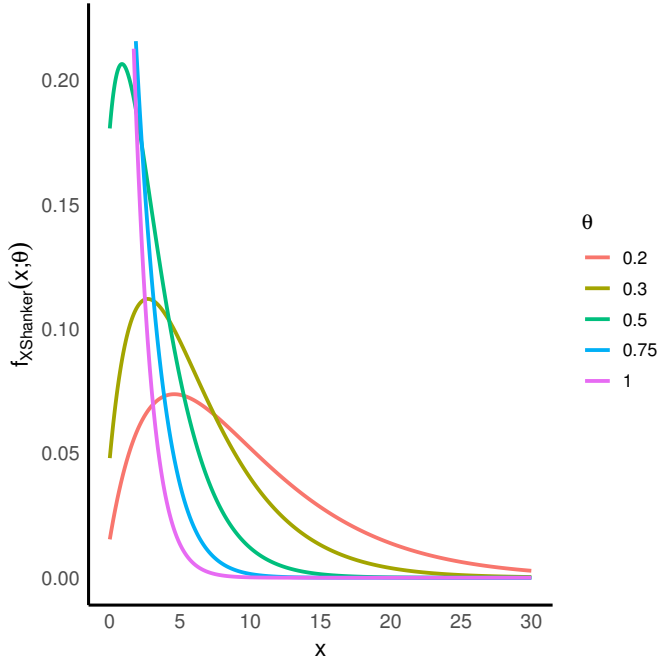


Figure 1. The pdf plot of the XShanker distribution for different parameter settings

Etaga et al. (2023) introduced the Double XShanker distribution, a one-parameter probability distribution developed as a more flexible extension of the XShanker distribution. Let $X \sim \text{Double XShanker}(\theta)$. The density can be written as

$$f_{\text{Double XShanker}}(x; \theta) = p \cdot f_{\text{Exp}}(x; \theta) + (1 - p) \cdot f_{\text{XShanker}}(x; \theta),$$

where $f_{\text{Exp}}(x; \theta)$ is the pdf of the exponential distribution with rate θ , $f_{\text{XShanker}}(x; \theta)$ is the pdf of the XShanker distribution, given in Equation (1) and the mixing proportion is $p = \theta^2 / (\theta^2 + 1)$. The pdf of the Double XShanker distribution is mathematically defined in the equation below

$$f_{\text{Double XShanker}}(x; \theta) = \frac{\theta^2}{(\theta^2 + 1)^3} \left(\theta^5 + 3\theta^3 + 3\theta + x \right) e^{-\theta x}, \quad x > 0, \theta > 0.$$

Figure 2 provides a graphical representation of the pdf of the Double XShanker distribution, showcasing how the distribution behaves under a range of different parameter settings.

2.2. Point parameter estimation

We estimate θ by maximum likelihood. For a sample $x_1, \dots, x_n$ from the Double XShanker model, the likelihood and score equations are given below.

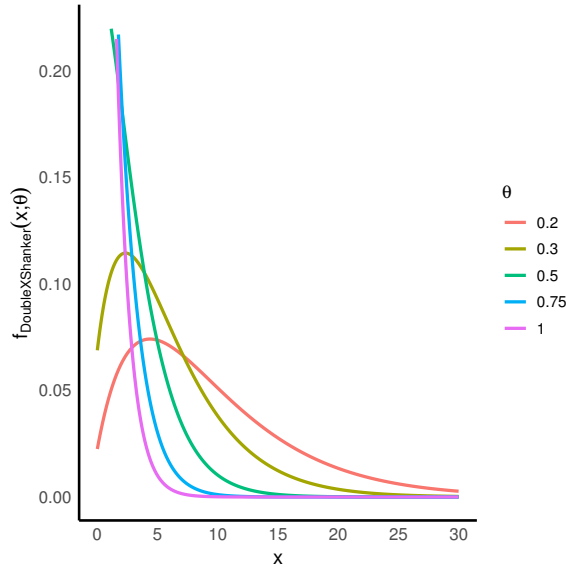


Figure 2. The pdf plot of the Double XShanker distribution for different parameter settings

Step 1: Likelihood

The likelihood function $L(\theta; x_i)$ represents the joint probability of observing a random sample drawn from the Double XShanker distribution. It is defined as:

$$L(\theta; x_i) = \prod_{i=1}^n f(x_i; \theta) = \left(\frac{\theta^2}{(\theta^2 + 1)^3} \right)^n \prod_{i=1}^n (\theta^5 + 3\theta^3 + 3\theta + x_i) \exp \left(-\theta \sum_{i=1}^n x_i \right).$$

Step 2: Log-likelihood function

Due to the mathematical complexity involved in directly differentiating the likelihood function, the log-likelihood function is employed to facilitate the differentiation process. The log-likelihood function is given by

$$\log L(\theta; \mathbf{x}) = 2n \log \theta - 3n \log(\theta^2 + 1) + \sum_{i=1}^n \log(\theta^5 + 3\theta^3 + 3\theta + x_i) - \theta \sum_{i=1}^n x_i.$$

Step 3: Score

To determine the value of θ that maximizes the log-likelihood function, the derivative of $\log L(\theta; x_i)$ with respect to θ is computed. This yields the score function $S(\theta; x_i)$, which is expressed as follows:

$$S(\theta; \mathbf{x}) = \frac{\partial}{\partial \theta} \log L(\theta; \mathbf{x}) = \frac{2n}{\theta} - \frac{6n\theta}{\theta^2 + 1} + \sum_{i=1}^n \frac{5\theta^4 + 9\theta^2 + 3}{\theta^5 + 3\theta^3 + 3\theta + x_i} - \sum_{i=1}^n x_i.$$

Step 4: MLE

The value of θ is obtained by solving the score equation $S(\theta; \mathbf{x}) = 0$.

This process identifies the critical points that may correspond to potential maxima of the log-likelihood function. Given the absence of a closed-form solution for the maximum likelihood (ML) estimator of the parameter, numerical iterative methods are employed to address the resulting non-linear optimization problem (Nwry et al., 2021). In this study, ML estimation was carried out using the Newton–Raphson method in R. For numerical stability, the iteration was initialized at $1/\bar{x}$, the reciprocal of the sample mean, which provides a natural scale-based starting value for positive lifetime data. The computations were implemented using the `maxLik` package (Henningsen & Toomet, 2011; Myung, 2003).

2.3. Confidence intervals

This section presents four approaches for constructing confidence intervals (CIs) for the parameter of the Double XShanker distribution: the likelihood-based CI, the Wald-type CI, the bootstrap- t CI, and the bias-corrected and accelerated (BCa) bootstrap CI.

Likelihood-based Confidence Interval

The likelihood-based CI is a fundamental statistical technique for parameter estimation. This approach is based on the likelihood function, which measures the probability of observing the given data under specific parameter values within a statistical model. The key idea is to determine a range of parameter values that are most consistent with the observed data, while ensuring that this range corresponds to a specified confidence level. This is typically accomplished by maximizing the log-likelihood function with respect to the parameter of interest and then identifying the parameter values that fall within the confidence region defined by the likelihood ratio criterion.

The ML estimator, denoted by $\hat{\theta}$, is obtained by solving the score equation:

$$S(\theta; \mathbf{x}) = 0.$$

The ML estimator represents the parameter value that is most consistent with the observed data under the assumed statistical model. Once the ML estimate is obtained, a likelihood-based confidence interval can be constructed around it. This method relies on the likelihood ratio, which is defined as:

$$\lambda(\theta) = \frac{L(\theta; \mathbf{x})}{L(\hat{\theta}; \mathbf{x})}.$$

Under regularity conditions, Wilks' theorem states that the statistic $-2 \log \lambda(\theta)$ asymptotically follows a chi-square distribution, with degrees of freedom equal to the number of parameters estimated—typically one in the context of this study (Severini, 2000; Casella & Berger, 2002; Kummaraka & Srisuradetchai, 2023). The likelihood-ratio interval is defined by the set of θ values that are not rejected by the likelihood-ratio test at level α :

$$-2 \log \left(\frac{L(\theta; \mathbf{x})}{L(\hat{\theta}; \mathbf{x})} \right) \leq \chi^2_{1-\alpha, 1}.$$

For the Double XShanker model, this set can be written explicitly as

$$\left\{ \theta : -2\log \left[\left(\frac{\theta}{\hat{\theta}} \right)^{2n} \left(\frac{\hat{\theta}^2 + 1}{\theta^2 + 1} \right)^{3n} \prod_{i=1}^n \frac{\theta^5 + 3\theta^3 + 3\theta + x_i}{\hat{\theta}^5 + 3\hat{\theta}^3 + 3\hat{\theta} + x_i} \right] \times \exp \left\{ -(\theta - \hat{\theta}) \sum_{i=1}^n x_i \right\} \right] \leq \chi_{1-\alpha,1}^2 \right\},$$

where $\chi_{1-\alpha,1}^2$ is the $(1 - \alpha)$ quantile of the chi-square distribution with one degree of freedom. In this study, the ML estimator $\hat{\theta}$ is obtained by numerically solving the score equation $S(\theta; \mathbf{x}) = 0$ using the Newton–Raphson method, as implemented in the maxLik package (Henningson and Toomet, 2011) in R. The score equation is given by

$$S(\theta; \mathbf{x}) = \frac{2n}{\theta} - \frac{6n\theta}{\theta^2 + 1} + \sum_{i=1}^n \frac{5\theta^4 + 9\theta^2 + 3}{\theta^5 + 3\theta^3 + 3\theta + x_i} - \sum_{i=1}^n x_i = 0.$$

Figure 3 illustrates the plot of the negative twice log-likelihood ratio $-2\log \lambda(\theta)$ versus θ (solid blue line), the ML estimate (dashed red line), and the 95% likelihood-based CI (solid green line) for a simulated sample of size 50 from the Double XShanker distribution with $\theta = 1$.

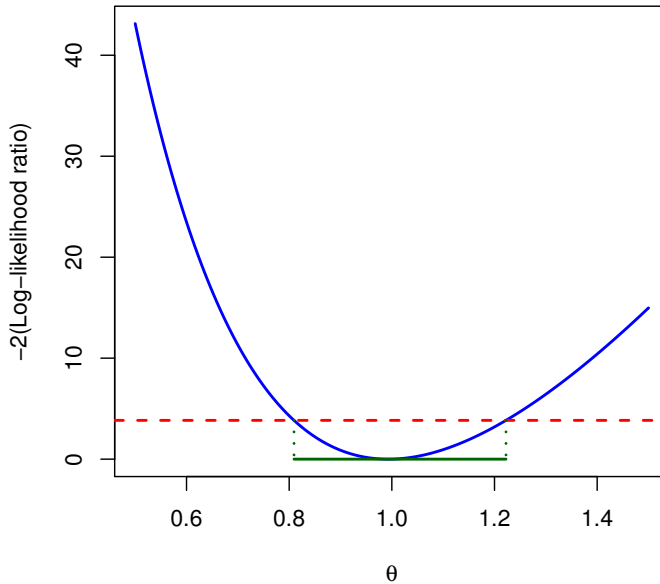


Figure 3. The plot of $-2\log \lambda(\theta)$ versus θ .

Wald-type Confidence Interval

The Wald-type CI for estimating the parameter of the Double XShanker distribution is derived from the ML estimator, denoted by $\hat{\theta}$. This CI is constructed using a second-order Taylor series expansion of the log-likelihood function $\log L(\theta; x)$ around $\hat{\theta}$. The approximation relies on the Wald statistic, which is based on the second derivative of the log-likelihood function, as the first derivative evaluates to zero at the ML estimate (Srisuradetchai & Tonprasongrat, 2022):

$$\log L(\theta; x) \approx \log L(\hat{\theta}; x) - \frac{1}{2} I(\hat{\theta})(\theta - \hat{\theta})^2,$$

where $I(\hat{\theta})$ is the observed Fisher information evaluated at $\hat{\theta}$. Under asymptotic conditions, where the sample size is sufficiently large, the Wald statistic provides an effective quadratic approximation to the likelihood ratio test (LRT) statistic (Pawitan, 2001). For the Double XShanker distribution, the first and second derivatives of the log-likelihood function are derived as follows:

$$\begin{aligned} \frac{\partial}{\partial \theta} \log L(\theta; x_i) &= \frac{2n}{\theta} - \frac{6n\theta}{\theta^2 + 1} + \sum_{i=1}^n \left(\frac{5\theta^4 + 9\theta^2 + 3}{\theta^5 + 3\theta^3 + 3\theta + x_i} \right) - \sum_{i=1}^n x_i \\ \frac{\partial^2}{\partial \theta^2} \log L(\theta; \mathbf{x}) &= -\frac{2n}{\theta^2} - \frac{6n}{\theta^2 + 1} + \frac{12n\theta^2}{(\theta^2 + 1)^2} \\ &\quad + \sum_{i=1}^n \frac{20\theta^3 + 18\theta}{\theta^5 + 3\theta^3 + 3\theta + x_i} - \sum_{i=1}^n \frac{(5\theta^4 + 9\theta^2 + 3)^2}{(\theta^5 + 3\theta^3 + 3\theta + x_i)^2}. \end{aligned} \quad (2)$$

Consequently, the observed Fisher information is estimated as:

$$\hat{I}(\hat{\theta}) = - \frac{\partial^2}{\partial \theta^2} \log L(\theta; x_i) \Big|_{\theta=\hat{\theta}}. \quad (3)$$

The Wald-type CI for θ at a confidence level of $(1 - \alpha)100\%$ is given by

$$\hat{\theta} \pm z_{1-(\alpha/2)} \sqrt{\hat{I}^{-1}(\hat{\theta})},$$

where $z_{1-(\alpha/2)}$ denotes the $(1 - (\alpha/2))^{\text{th}}$ quantile of the standard normal distribution, and $\hat{I}^{-1}(\hat{\theta})$ is the inverse of the observed Fisher information given in Equation (3).

Bootstrap-*t* Confidence Interval

The bootstrap-*t* CI is a resampling-based method that quantifies parameter uncertainty by integrating the bootstrap procedure with the *t*-distribution framework. Unlike the conventional percentile bootstrap, the bootstrap-*t* approach accounts for the estimator's variability via its standard error, thereby enhancing accuracy and robustness—particularly in small

samples or when the estimator's sampling distribution deviates from normality (Davison and Hinkley, 1997; Efron & Tibshirani, 1993).

This method is particularly useful when the analytical form of the sampling distribution is complex or unknown. The confidence interval is constructed from the empirical distribution of a standardized statistic—referred to as the bootstrap- t statistic—which serves as an approximation to the pivotal quantity used in parametric inference.

The procedure for constructing a bootstrap- t CI involves the following steps (Panichkitkosolkul & Srisuradetchai, 2022):

Step 1: Initial Estimation

Draw a random sample $X = (X_1, \dots, X_n)$ from the population. Compute the point estimate $\hat{\theta}$ of the parameter of interest, such as the ML estimate.

Step 2: Bootstrap Sampling

Generate B bootstrap samples, X_b^* , $b = 1, 2, \dots, B$, by sampling with replacement from the original dataset X .

Step 3: Parameter Estimation for Each Sample

For each bootstrap sample X_b^* , compute the bootstrap replicate of the estimator, denoted as $\hat{\theta}_b^*$.

Step 4: Construct the bootstrap- t Statistic

For each bootstrap replicate, calculate the standardized statistic (bootstrap- t) as:

$$t_b^* = \frac{\hat{\theta}_b^* - \hat{\theta}}{\widehat{\text{SE}}_b},$$

where $\widehat{\text{SE}}_b = \sqrt{\hat{I}^{-1}(\hat{\theta}_b^*)}$ is the bootstrap estimate of the standard error of $\hat{\theta}_b^*$, often obtained via the observed Fisher information (see Equation (3)), with the parameter substituted by $\hat{\theta}_b^*$.

Step 5: Construct the Empirical Distribution

Repeat Steps 2–4 a total of B times to obtain the empirical distribution of the t_b^* values. This distribution approximates the sampling distribution of the standardized estimator.

Step 6: Quantile Calculation

Determine the lower and upper quantiles $t_{\alpha/2}^*$ and $t_{1-\alpha/2}^*$ corresponding to the $\alpha/2$ and $1 - (\alpha/2)$ levels of the empirical bootstrap- t distribution. That is,

$$t_{\alpha/2}^* = \inf \left\{ t : \frac{1}{B} \sum_{b=1}^B \mathbf{1}_{[t_b^* \leq t]} \geq \alpha/2 \right\}$$

and

$$t_{1-(\alpha/2)}^* = \inf \left\{ t : \frac{1}{B} \sum_{b=1}^B \mathbf{1}_{[t_b^* \leq t]} \geq 1 - (\alpha/2) \right\},$$

where $\mathbf{1}_{[\cdot]}$ is the indicator function.

Step 7: Construct the bootstrap- t Confidence Interval

The two-sided $(1 - \alpha)100\%$ bootstrap- t confidence interval is given by

$$\left[\hat{\theta} - t_{1-\alpha/2}^* \widehat{\text{SE}}(\hat{\theta}), \hat{\theta} - t_{\alpha/2}^* \widehat{\text{SE}}(\hat{\theta}) \right],$$

where $\widehat{\text{SE}}(\hat{\theta}) = \sqrt{\hat{I}^{-1}(\hat{\theta})}$ is the standard error of the original estimator $\hat{\theta}$.

Bias-corrected and accelerated (BCa) bootstrap confidence interval

The bias-corrected and accelerated (BCa) bootstrap CI is an advanced extension of the basic bootstrap method, designed to correct for both bias and skewness in the empirical distribution of the estimator. This approach is particularly effective in small-sample scenarios or when the estimator's distribution deviates substantially from normality. The BCa bootstrap method enhances the accuracy of interval estimates by incorporating two key adjustments: (1) a bias-correction factor that accounts for asymmetry in the proportion of bootstrap replicates less than the original estimate, and (2) an acceleration parameter that adjusts for the skewness in the estimator's sampling distribution (Bittmann, 2021; Efron, 1987; Panichkitkosolkul & Srisuradetachai, 2022). Unlike conventional methods that rely on normality or symmetry assumptions, the BCa bootstrap approach is nonparametric and distribution-free, offering improved robustness and broader applicability.

The construction of the BCa bootstrap CI proceeds through the following steps:

Step 1: Initial Estimation

Draw a random sample $X = (X_1, \dots, X_n)$ from the population. Estimate the parameter of interest $\hat{\theta}$ using an appropriate method, such as maximum likelihood.

Step 2: Bootstrap Resampling

Generate B bootstrap samples X_b^* , $b = 1, 2, \dots, B$, by sampling with replacement from the original dataset X .

Step 3: Bootstrap Estimation

For each bootstrap sample, compute the bootstrap estimator $\hat{\theta}_b^*$, resulting in a distribution of bootstrap estimates $\{\hat{\theta}_1^*, \hat{\theta}_2^*, \dots, \hat{\theta}_B^*\}$.

Step 4: Bias Correction Factor (z_0)

Compute the proportion $\hat{p}$ of bootstrap estimates that are less than the original estimate $\hat{\theta}$:

$$\hat{p} = \frac{1}{B} \sum_{b=1}^B \mathbf{1}_{[\hat{\theta}_b^* < \hat{\theta}]}$$

The bias correction factor z_0 is then obtained as the standard normal quantile:

$$z_0 = \Phi^{-1}(\hat{p}),$$

where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function.

Step 5: Acceleration Constant (a)

Estimate the acceleration constant a , which reflects the rate of change of the standard error of the estimator with respect to the data. It is typically obtained using the jackknife method:

$$a = \frac{\sum_{i=1}^n (\bar{\theta}_{(\cdot)} - \hat{\theta}_{(i)})^3}{6 \left[\sum_{i=1}^n (\bar{\theta}_{(\cdot)} - \hat{\theta}_{(i)})^2 \right]^{3/2}},$$

where $\hat{\theta}_{(i)}$ is the leave-one-out estimate, and $\bar{\theta}_{(\cdot)}$ is their average.

Step 6: Adjusted Percentiles

The corrected quantiles α_1 and α_2 for the confidence interval are calculated as:

$$\alpha_1 = \Phi \left(z_0 + \frac{z_0 + z_{\alpha/2}}{1 - a(z_0 + z_{\alpha/2})} \right) \quad \text{and} \quad \alpha_2 = \Phi \left(z_0 + \frac{z_0 + z_{1-(\alpha/2)}}{1 - a(z_0 + z_{1-(\alpha/2)})} \right),$$

where $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are the $(\alpha/2)^{\text{th}}$ and $(1 - (\alpha/2))^{\text{th}}$ quantiles of the standard normal distribution, respectively.

Step 7: BCa Bootstrap Confidence Interval Construction

Finally, the two-sided $(1 - \alpha)100\%$ BCa bootstrap CI is constructed using the adjusted percentiles α_1 and α_2 as:

$$\left[\hat{\theta}_{(\alpha_1)}^*, \hat{\theta}_{(\alpha_2)}^* \right],$$

where $\hat{\theta}_{(\alpha_1)}^*$ and $\hat{\theta}_{(\alpha_2)}^*$ are $(\alpha_1)^{\text{th}}$ and $(\alpha_2)^{\text{th}}$ quantiles of sorted bootstrap estimates $\hat{\theta}_b^*$.

3. Simulation studies and results

This study evaluates four two-sided CIs for the parameter of the Double XShanker distribution: likelihood-based, Wald-type, bootstrap- t , and bias-corrected and accelerated (BCa) bootstrap intervals. The performance of these methods was assessed through a Monte Carlo simulation study (Robert & Casella, 2010). For each simulated sample, the maximum likelihood estimate was obtained numerically by solving the score equation. To improve the stability of the numerical optimization, the initial value of θ was chosen as the reciprocal of the sample mean, $1/\bar{x}$, which provides a natural scale-based starting value for positive lifetime data.

The performance of the proposed confidence intervals was evaluated using two standard measures: empirical coverage probability (ECP) and average width (AW). These measures were examined across a range of sample sizes and true parameter values. Specifically, sample sizes $n = 10, 20, 30, 50, 100$, and 200 were used, with true parameter values $\theta = 0.20, 0.30, 0.50, 0.75, 1.00, 1.50, 2.00$, and 2.50 . For each combination of n and θ , the simulation was replicated 2,000 times. In the main simulation study, the bootstrap- t and BCa bootstrap intervals were constructed using $B = 1000$ bootstrap resamples.

To assess the sensitivity of the bootstrap-based intervals to the number of bootstrap resamples, an additional analysis was conducted for selected representative cases. The settings $(n, \theta) = (10, 0.5), (10, 2.5), (50, 0.5), (50, 2.5), (200, 0.5)$, and $(200, 2.5)$ were considered, covering small, moderate, and large sample sizes, as well as moderate and relatively large parameter values. For each setting, the bootstrap- t and BCa intervals were recomputed using $B = 1000, 2000$, and 5000 bootstrap resamples. To ensure a fair comparison across bootstrap sizes, 5000 bootstrap resamples were generated once for each Monte Carlo sample, and the results for $B = 1000$ and $B = 2000$ were obtained from nested subsets of these resamples. Since the likelihood-based and Wald-type intervals do not depend on B , they were not included in this sensitivity comparison.

The simulation results, summarized in Table 1 and visualized in Figures 4 and 5, compare the ECPs and AWs of the four 95% two-sided CI methods. Across most parameter values and sample sizes, the likelihood-based and Wald-type intervals maintained coverage close to the nominal level of 0.95. Their performance was especially stable for moderate and large sample sizes.

By contrast, the bootstrap- t and BCa bootstrap intervals showed lower ECPs in small samples, particularly when $n = 10, 20$, or 30 . For example, at $\theta = 0.2$ and $n = 10$, the ECPs of the bootstrap- t and BCa bootstrap intervals were 0.8850 and 0.8915, respectively, which are below the nominal level. Similar undercoverage was also observed for other parameter values in small samples. As the sample size increased, however, the coverage of the bootstrap-based intervals improved. For instance, when $\theta = 0.2$ and $n = 200$, the ECPs increased to 0.9445 for the bootstrap- t interval and 0.9500 for the BCa bootstrap interval.

With respect to AW, all methods showed the expected decrease in interval width as the sample size increased. The bootstrap- t and BCa bootstrap intervals were generally narrower than the likelihood-based and Wald-type intervals, especially in small samples. This narrower width was accompanied by lower coverage, indicating a trade-off between interval precision and coverage accuracy.

Overall, the likelihood-based and Wald-type methods provided more reliable coverage in small to moderate samples. The bootstrap- t and BCa bootstrap intervals were often shorter, but their coverage was less stable when n was small. These findings suggest that likelihood-based and Wald-type intervals are preferable when coverage accuracy is the main criterion, whereas bootstrap-based intervals may be useful in larger samples when shorter intervals are desired.

The sensitivity results for the number of bootstrap resamples are presented in Table 2. Increasing B from 1000 to 2000 and 5000 produced only small changes in both ECP and AW. The qualitative conclusions remained unchanged: the bootstrap- t and BCa bootstrap intervals continued to show undercoverage when $n = 10$, while their coverage was closer to the nominal level for $n = 50$ and $n = 200$. These results indicate that the main conclusions based on $B = 1000$ are stable with respect to the number of bootstrap resamples considered in this sensitivity analysis.

Table 1. Empirical coverage probability and average width of the 95% two-sided CIs for the parameter of the Double XShanker distribution

θ	n	Empirical Coverage Probability (ECP)				Average Width (AW)			
		Likelihood	Wald	Boot- t	BCa	Likelihood	Wald	Boot- t	BCa
0.20	10	0.9475	0.9510	0.8850	0.8915	0.1720	0.1717	0.1527	0.1584
	20	0.9415	0.9405	0.9025	0.9065	0.1203	0.1202	0.1121	0.1137
	30	0.9390	0.9370	0.9160	0.9155	0.0973	0.0973	0.0936	0.0942
	50	0.9520	0.9520	0.9400	0.9385	0.0752	0.0752	0.0729	0.0732
	100	0.9485	0.9460	0.9365	0.9360	0.0529	0.0529	0.0519	0.0521
	200	0.9545	0.9525	0.9445	0.9500	0.0374	0.0374	0.0368	0.0369
0.30	10	0.9505	0.9495	0.8930	0.8920	0.2490	0.2479	0.2197	0.2319
	20	0.9530	0.9535	0.9215	0.9310	0.1717	0.1714	0.1606	0.1632
	30	0.9490	0.9440	0.9345	0.9305	0.1398	0.1397	0.1341	0.1357
	50	0.9485	0.9490	0.9355	0.9325	0.1077	0.1076	0.1052	0.1058
	100	0.9410	0.9435	0.9360	0.9350	0.0763	0.0762	0.0747	0.0751
	200	0.9555	0.9545	0.9465	0.9470	0.0538	0.0538	0.0531	0.0533
0.50	10	0.9480	0.9570	0.8860	0.8950	0.4118	0.4049	0.3521	0.3928
	20	0.9525	0.9535	0.9110	0.9155	0.2811	0.2790	0.2603	0.2710
	30	0.9455	0.9460	0.9245	0.9325	0.2263	0.2252	0.2146	0.2203
	50	0.9540	0.9570	0.9385	0.9430	0.1744	0.1739	0.1687	0.1716
	100	0.9555	0.9590	0.9480	0.9485	0.1225	0.1223	0.1205	0.1217
	200	0.9540	0.9555	0.9505	0.9450	0.0865	0.0864	0.0855	0.0861
0.75	10	0.9385	0.9520	0.8955	0.8915	0.7074	0.6842	0.5882	0.6931
	20	0.9440	0.9520	0.9085	0.9160	0.4662	0.4581	0.4206	0.4531
	30	0.9550	0.9585	0.9305	0.9335	0.3695	0.3652	0.3439	0.3605
	50	0.9425	0.9440	0.9295	0.9315	0.2816	0.2796	0.2700	0.2780
	100	0.9465	0.9520	0.9395	0.9415	0.1965	0.1958	0.1916	0.1946
	200	0.9510	0.9510	0.9455	0.9465	0.1376	0.1373	0.1356	0.1371
1.00	10	0.9475	0.9490	0.8930	0.8840	1.0843	1.0517	0.9026	1.0514
	20	0.9505	0.9570	0.9230	0.9155	0.7112	0.6978	0.6411	0.6863
	30	0.9470	0.9550	0.9305	0.9275	0.5622	0.5545	0.5227	0.5469
	50	0.9460	0.9505	0.9280	0.9305	0.4259	0.4222	0.4073	0.4189
	100	0.9445	0.9490	0.9365	0.9405	0.2956	0.2942	0.2872	0.2921
	200	0.9505	0.9535	0.9430	0.9425	0.2076	0.2072	0.2047	0.2070
1.50	10	0.9470	0.9465	0.8975	0.8830	1.8961	1.8741	1.6492	1.8381
	20	0.9485	0.9505	0.9165	0.9055	1.2639	1.2570	1.1681	1.2029
	30	0.9535	0.9570	0.9385	0.9285	1.0018	0.9979	0.9523	0.9659
	50	0.9505	0.9480	0.9310	0.9335	0.7654	0.7636	0.7438	0.7484
	100	0.9435	0.9475	0.9370	0.9405	0.5311	0.5304	0.5211	0.5232
	200	0.9530	0.9560	0.9480	0.9495	0.3752	0.3750	0.3699	0.3720
2.00	10	0.9495	0.9375	0.8915	0.8875	2.6205	2.6064	2.2883	2.5190
	20	0.9470	0.9535	0.9195	0.9085	1.7674	1.7660	1.6602	1.6939
	30	0.9505	0.9470	0.9315	0.9280	1.4236	1.4235	1.3650	1.3699
	50	0.9545	0.9490	0.9380	0.9400	1.0880	1.0883	1.0608	1.0591
	100	0.9470	0.9455	0.9375	0.9380	0.7594	0.7597	0.7483	0.7486
	200	0.9480	0.9475	0.9430	0.9455	0.5395	0.5397	0.5360	0.5367
2.50	10	0.9400	0.9500	0.8935	0.8865	3.4087	3.3907	2.9752	3.2834
	20	0.9430	0.9470	0.9135	0.9155	2.2831	2.2802	2.1321	2.1834
	30	0.9560	0.9525	0.9315	0.9270	1.8347	1.8335	1.7537	1.7665
	50	0.9475	0.9545	0.9315	0.9300	1.3984	1.3980	1.3509	1.3583
	100	0.9415	0.9430	0.9375	0.9340	0.9754	0.9754	0.9583	0.9603
	200	0.9515	0.9505	0.9425	0.9415	0.6847	0.6847	0.6741	0.6763

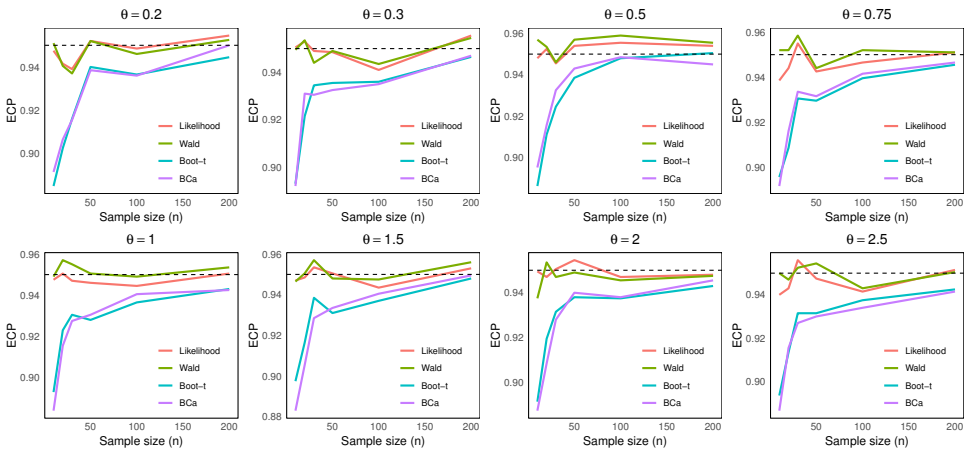


Figure 4. Plots of the ECPs of the CIs for the parameter of the Double XShanker distribution

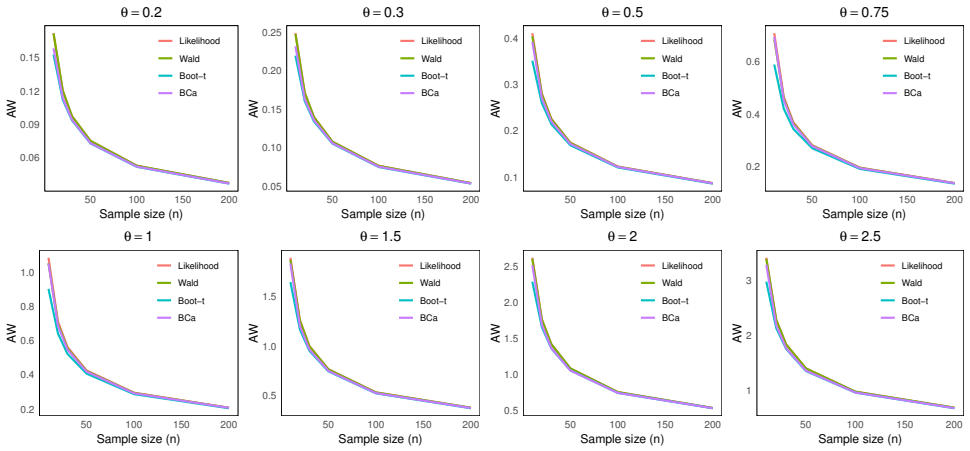


Figure 5. Plots of the AWs of the CIs for the parameter of the Double XShanker distribution

Table 2. Sensitivity of bootstrap-based confidence intervals to the number of bootstrap resamples

<i>n</i>	ϑ	<i>B</i>	Bootstrap- <i>t</i>		BCa bootstrap	
			ECP	AW	ECP	AW
10	0.5	1000	0.8790	0.3530	0.8925	0.3926
		2000	0.8850	0.3539	0.8940	0.3935
		5000	0.8845	0.3545	0.8940	0.3944
	2.5	1000	0.8950	2.9629	0.8825	3.2431
		2000	0.8960	2.9693	0.8840	3.2543
		5000	0.8950	2.9724	0.8835	3.2602
50	0.5	1000	0.9400	0.1689	0.9370	0.1706
		2000	0.9405	0.1693	0.9365	0.1712
		5000	0.9400	0.1695	0.9400	0.1714
	2.5	1000	0.9455	1.3570	0.9425	1.3571
		2000	0.9460	1.3595	0.9460	1.3598
		5000	0.9440	1.3618	0.9455	1.3611
200	0.5	1000	0.9385	0.0853	0.9390	0.0855
		2000	0.9380	0.0855	0.9390	0.0857
		5000	0.9395	0.0857	0.9385	0.0859
	2.5	1000	0.9410	0.6785	0.9405	0.6774
		2000	0.9385	0.6804	0.9420	0.6794
		5000	0.9390	0.6813	0.9415	0.6804

Note. The $B = 1000$ entries provide the baseline within this separate sensitivity analysis. For each Monte Carlo sample, the results for $B = 1000$, $B = 2000$, and $B = 5000$ were obtained from nested bootstrap resamples.

4. Real data application

We apply the fitted Double XShanker model and the four confidence intervals to a rainfall dataset. For model-comparison purposes, the Double XShanker distribution was compared with several one-parameter lifetime distributions commonly used for positively skewed data. The competing models are listed below.

1. Akshaya distribution (Shanker, 2017a):

$$f(x;\theta) = \frac{\theta^4}{\theta^3 + 3\theta^2 + 6\theta + 6} (1+x)^3 e^{-\theta x},$$

2. Amarendra distribution (Shanker, 2016a):

$$f(x;\theta) = \frac{\theta^4}{\theta^3 + \theta^2 + 2\theta + 6} (1+x+x^2+x^3) e^{-\theta x},$$

3. Devya distribution (Shanker, 2016b):

$$f(x; \theta) = \frac{\theta^5}{\theta^4 + \theta^3 + 2\theta^2 + 6\theta + 24} (1 + x + x^2 + x^3 + x^4) e^{-\theta x},$$

4. Iwueze distribution (Elechi et al., 2022):

$$f(x; \theta) = \frac{\theta^5}{\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24} (1 + x + x^2)^2 e^{-\theta x},$$

5. Prakaamy distribution (Shukla, 2018a):

$$f(x; \theta) = \frac{\theta^6}{\theta^5 + 120} (1 + x^5) e^{-\theta x},$$

6. Suja distribution (Shanker, 2017b):

$$f(x; \theta) = \frac{\theta^5}{\theta^4 + 24} (1 + x^4) e^{-\theta x},$$

7. Iwok-Nwike distribution (Iwok and Nwike, 2021):

$$f(x; \theta) = \frac{\theta^3}{\theta + 2} (x^2 + x) e^{-\theta x},$$

8. Nwike distribution (Nwike et al., 2021):

$$f(x; \theta) = \frac{\theta^3}{12(\theta^2 + 10)} (\theta^3 x^5 + 12) e^{-\theta x},$$

9. Ola distribution (Al-Ta'ani and Gharaibeh, 2023):

$$f(x; \theta) = \frac{\theta^8}{\theta^7 + 6\theta^4 + 5040} (x^7 + x^3 + 1) e^{-\theta x},$$

10. Ram Awadh distribution (Shukla, 2018b):

$$f(x; \theta) = \frac{\theta^6}{\theta^6 + 120} (\theta + x^5) e^{-\theta x}.$$

The dataset consists of 72 March rainfall measurements (in inches) recorded at the Los Angeles Civic Center during the period 1943–2018:

4.55, 2.47, 3.43, 3.66, 0.79, 3.07, 1.40, 0.87, 0.44, 6.14, 0.48, 2.99, 0.56, 1.02, 5.30, 0.31, 0.57, 1.10, 2.78, 1.79, 2.49, 0.53, 2.50, 3.34, 1.49, 2.36, 0.53, 2.70, 3.78, 4.83, 1.81, 1.89, 8.02, 5.85, 4.79, 4.10, 3.54, 8.37, 0.28, 1.29, 5.27, 0.95, 0.26, 0.81, 0.17, 5.92, 7.12, 2.74, 1.86, 6.98, 2.16, 4.06, 1.24, 2.82, 1.17, 0.32, 4.32, 1.47, 2.14, 2.87, 0.05, 0.01, 0.35, 0.48, 3.96, 1.75, 0.54, 1.18, 0.87, 1.60, 0.09, 2.69.

Descriptive statistics for this dataset are summarized in Table 3, while Figure 6 provides visual representations, including a histogram, a box-and-whisker plot, a kernel density plot, and a violin plot, highlighting the positive skewness of the data.

Table 3. Descriptive statistics for the rainfall data at the Los Angeles Civic Center

Sample Size	Minimum	Q1	Median	Mean	Q3	Maximum	St.Dev
72	0.010	0.805	1.875	2.450	3.570	8.370	2.055

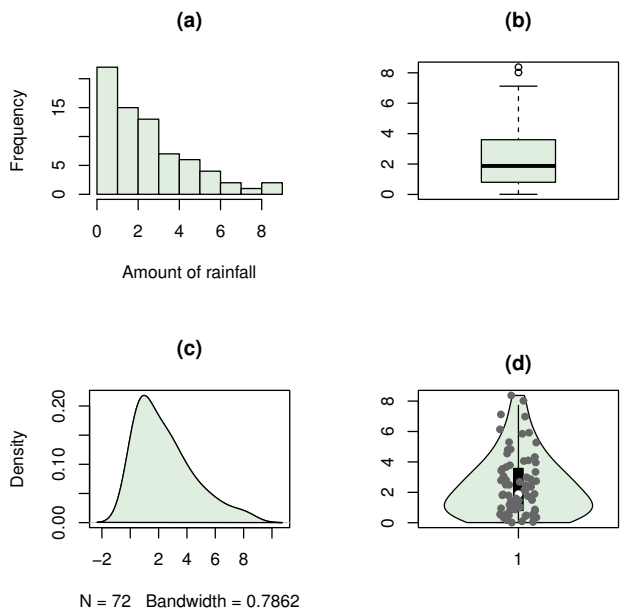


Figure 6. Visual representations of the rainfall data: (a) Histogram, (b) Box-and-whisker plot, (c) Kernel density plot, and (d) Violin plot

A comprehensive evaluation of model performance was conducted using multiple statistical criteria, namely the maximized log-likelihood ($\log(\hat{L})$), the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC), also known as the Schwarz Information Criterion. These metrics provide a basis for assessing model fit of competing distributions. Specifically, AIC and BIC offer adjusted likelihood-based measures that balance model fit against model complexity, thereby supporting more reliable model selection. The AIC and BIC are formally defined as follows:

$$\text{AIC} = 2k - 2\log(\hat{L}) \quad \text{and} \quad \text{BIC} = k\log(n) - 2\log(\hat{L}),$$

where k represents the number of estimated parameters in the model and $\hat{L}$ denotes the maximized value of the likelihood function for the model. A distribution with lower AIC

and BIC values is generally preferred, as these statistics indicate a better balance between model fit and complexity. Table 4 provides the parameter estimates and goodness-of-fit measures for the dataset under investigation.

Table 4. Comparative analysis of model fit statistics for different distributions applied to rainfall data at the Los Angeles Civic Center

Distribution	Estimate	LogLik	AIC	BIC
Double XShanker	0.5780	-135.681	273.362	275.639
Akshaya	1.1897	-141.282	284.564	286.841
Amarendra	1.2446	-139.365	280.730	283.006
Devya	1.5727	-142.642	287.283	289.560
Iwueze	1.5155	-144.230	290.460	292.737
Prakaamy	2.0103	-147.667	297.333	299.610
Suja	1.6531	-142.474	286.949	289.225
Iwok-Nwike	1.0811	-149.447	300.894	303.170
Nwike	1.8141	-143.623	289.245	291.522
Ola	2.7082	-158.132	318.263	320.540
Ram Awadh	1.9007	-143.549	289.097	291.374

Note. Bold values indicate the best-fitting model based on the lowest AIC and BIC values.

Table 4 presents the log-likelihood values, AIC (Akaike, 1974), and BIC (Schwarz, 1978) for the Double XShanker distribution and ten alternative distributions fitted to the rainfall data from the Los Angeles Civic Center. Among these distributions, the Double XShanker distribution provided the best overall fit, indicated by the lowest AIC and BIC values, suggesting strong compatibility with the empirical data. The maximum likelihood estimate of the Double XShanker parameter was 0.5780.

To further assess uncertainty associated with the parameter estimate, Table 5 presents the 95% two-sided CIs for the Double XShanker distribution parameter, constructed using four methods: likelihood-based, Wald-type, bootstrap-*t*, and BCa bootstrap. The likelihood-based method produced a CI of (0.4989, 0.6664), with a width of 0.1675. The Wald-type method yielded a similar interval (0.4945, 0.6615) with a slightly narrower width of 0.1670. In contrast, the bootstrap-*t* and BCa bootstrap methods produced narrower intervals—(0.5066, 0.6610) and (0.5045, 0.6598), respectively—in agreement with the simulation results. Compared to a similar simulation scenario with $\theta = 0.5$ and $n = 50$ (close to our dataset size of $n = 72$), the likelihood-based and Wald-type intervals had widths of 0.1744 and 0.1739, respectively, whereas the bootstrap-*t* and BCa bootstrap intervals were narrower at 0.1687 and 0.1716. These results support the use of the Double XShanker distribution for the rainfall data and show consistency between the simulation and empirical findings in a moderate sample-size setting.

Table 5. The 95% two-sided CIs and corresponding widths based on rainfall data at the Los Angeles Civic Center

CI Method	Confidence Interval	Width
Likelihood-based	(0.4989, 0.6664)	0.1675
Wald-type	(0.4945, 0.6615)	0.1670
bootstrap- <i>t</i>	(0.5066, 0.6610)	0.1544
BCa bootstrap	(0.5045, 0.6598)	0.1553

5. Conclusions and discussions

We compared four 95% two-sided confidence intervals for the Double XShanker parameter θ : likelihood-based, Wald-type, bootstrap-*t*, and BCa bootstrap intervals. An explicit expression for the observed Fisher information was derived, allowing direct computation of the Wald interval once the maximum likelihood estimate is obtained. Performance was evaluated through Monte Carlo simulations over a range of sample sizes and parameter values, using empirical coverage probability (ECP) and average width (AW) as criteria.

The simulation results show that the likelihood-based and Wald-type intervals remain close to the nominal 0.95 level across most settings considered. In contrast, the bootstrap-*t* and BCa bootstrap intervals are typically narrower but exhibit undercoverage in small samples. Their coverage improves as the sample size increases, approaching the nominal level in moderate to large samples. The additional sensitivity analysis using $B = 1000, 2000$, and 5000 bootstrap resamples showed only small changes in ECP and AW, indicating that the main conclusions are stable with respect to the number of bootstrap resamples considered.

The methods were further illustrated using rainfall data from the Los Angeles Civic Center. Based on the maximized log-likelihood, AIC (Akaike, 1974), and BIC (Schwarz, 1978), the Double XShanker distribution provided the best fit among the competing one-parameter lifetime models considered. The interval estimates from the data analysis were consistent with the simulation findings: bootstrap-based intervals were slightly narrower, whereas likelihood-based and Wald-type intervals aligned more closely with nominal coverage behavior.

Bootstrap procedures, particularly the bootstrap-*t* and BCa methods, require repeated estimation and, for BCa bootstrap, additional jackknife calculations. This increases computational cost in large-scale resampling settings. In practice, parallel computation and a careful choice of the number of resamples can reduce this burden. In R, packages such as *boot* (Canty & Ripley, 2025) and *bootstrap* (Kostyshak, 2019) provide standard implementations.

Overall, for small to moderate samples, the likelihood-ratio and Wald intervals are preferable when coverage accuracy is the primary concern. For larger samples, bootstrap-*t* and BCa bootstrap intervals may be considered when narrower intervals are desired and computational resources are available. When computation is limited, the Wald-type interval remains attractive due to its simple form and low cost.

Acknowledgements

We thank the reviewers for helpful comments and suggestions. This study was supported by the Thammasat University Research Unit in Mathematical Sciences and Applications.

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