STATISTICS IN TRANSITION new series, March 2019
Vol. 20, No. 1, pp. 41–65, DOI 10.21307/stattrans-2019-003

MODELLING SENSITIVE ISSUES ON SUCCESSIVE WAVES

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ABSTRACT

This paper addresses the problem of estimation of population mean of sensitive character using non-sensitive auxiliary variable at current wave in two wave successive sampling. A general class of estimator is proposed and studied under randomized and scrambled response model. Many existing estimators have been modified to work for sensitive population mean estimation. The modified estimators became the members of proposed general class of estimators. The detail properties of all the estimators have been discussed. Their behaviour under randomized and scrambled response techniques have been elaborated. Numerical illustrations including simulation have been accompanied to judge the performance of different estimators. Finally suitable recommendations are forwarded.

Key words: Sensitive variable, Successive waves, Scrambled Response model, Class of estimators, Population mean, Bias, Mean squared error, Optimum matching fraction.

1. Introduction

The occurrence of unpleasant phenomenon are plenty and abundance in the human society. We as a part of it, are sometimes obliged to take serious notice of and spread their occurrences among the conscientious public. This phenomenon necessitates assemblage of truthful and reliably adequate and accurate date. But the usual survey practices were not enough to elicit human responses through queries about sensitive and stigmatized issues.

Some of the features like gambling habits, alcoholism, illegal drug use, tax evasion, rash driving of motorized vehicles, conjugal malpractices and domestic violence etc., people like to hide from the human communities.

Hence, to deal with sensitive issues, an alternative technique has been introduces by Warner (1965), which is to obtain responses through a randomized response (RR) survey where every sampled unit is asked to give a response through an RR device as per instruction from the investigator. One can refer to Greenberg *et al.* (1971), Barlev *et al.* (2004), Diana and Perri (2011) and Arcos *et al.* (2015), etc.

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for a comprehensive review of such RR procedure. However, there is another approach to deal with sensitive issue called scrambled response technique introduced by Pollock and Bek (1976). Many researchers such as Eichhorn and Hayre (1983), Saha (2007) and Diana and Perri (2010), etc. considered the scrambled response models to deal with sensitive issues.

There are many situations were one need to study the variable over time as they may opt of change by time. Jessen (1942) inaugurated the journey of research program related to variables which changes by time. Later Patterson (1950), Sen (1973), Feng and Zou (1997), Singh and Priyanka (2008), Priyanka and Mittal (2014, 2015a, 2015b), and Priyanka *et al.* (2015), etc. added sub-sensitive literature in this area.

However, if the variable which opt to change by time is also sensitive in nature, then their arises a need to apply randomized/scrambled response techniques on successive waves. Arnab and Singh (2013), Yu *et al.* (2015), Naeem and Shabbir (2016) and Singh *et al.* (2017) have put their efforts to deal with sensitive issues on successive waves.

In the present work a general class of estimators have been proposed for estimating sensitive population mean at current wave in two wave successive sampling using a non-sensitive auxiliary variable. The proposed estimators have been studied under both the randomized and scrambled response technique. Many existing estimators in successive sampling literature such as estimators by Jessen (1942), Singh and Priyanka (2008), Singh and Karna (2009) and Singh and Prasad (2010) when modified to work for sensitive population mean estimation, becomes the members of proposed general class of estimators. The modified estimators have also been checked for their applicability under considered randomized and scrambled response models. The proposed general class of estimators have been compared with the members of its class in terms of percent relative efficiency. Simulation study has also been carried out to show the practicability of proposed methods. Finally, suitable concluding remarks have been forwarded.

2. Survey Strategies and Analysis

2.1. Background

Let P be finite population of N units which has been considered for two successive waves. The sensitive study variable be named as x at the first wave and y at second wave. Whereas z is assumed to be non-sensitive auxiliary variable which is available at the both the successive waves. A simple random sample without replacement of size n is drawn at the first wave and at the second wave two independent samples are drawn by considering the partial overlapping case, one is matched sample of size $m = n\lambda$ drawn as sub sample from the sample of size n from first wave and another is unmatched simple random sample of size $u = (n-m) = n\mu$ drawn afresh at the second (current) wave so that the sample size at both the wave is n. On first(second) wave the sensitive variables $x(y)$ are switched to $x'(y')$ with the aid of scrambling variables W_1, W_2 and W_3. The scrambling variable are considered such that they may follow any distribution. The following notations to be considered further are presented below:

$\bar{X},\ \bar{Y},\ \bar{Z},\ \bar{X}_i',\ \bar{Y}_i',\ \bar{W}_1,\ \bar{W}_2,\ \bar{W}_3$	: Population means of the variables x, y, z, x_i', y_i', W_1, W_2 and W_3 respectively where $i =$ 1 and 2 corresponds to randomized and scrambled response models respectively.
$\bar{x}_{ui}',\ \bar{y}_{mi}',\ \bar{x}_{mi}',\ \bar{y}_{ni}'$	: Sample mean of the variate based on sample sizes shown in suffices under i^{th} model.
$\bar{z}_u,\ \bar{z}_m,\ \bar{z}_n$	: Sample mean of the non-sensitive auxiliary variate based on sample sizes shown in suffice.
$\rho_{yx},\ \rho_{xz},\ \rho_{yz},\ (\rho_{x'y'})_i,\ (\rho_{y'z})_i,\ (\rho_{x'z})_i$	: Correlation coefficient between the variables shown in suffices and $'i'$ denote the scrambled and randomized response model.
$C_x,\ C_y,\ C_z$	: Coefficient of variation of variables shown in suffices.
$S_x^2,\ S_y^2,\ S_z^2,\ S_{W_1}^2,\ S_{W_2}^2,\ S_{W_3}^2$	: Population mean squared error of variables x, y, z, W_1, W_2 and W_3 respectively.

2.2. Randomized Response Techniques on successive waves

A unified approach for randomized response technique has been proposed by Arcos *et al.*(2015). Their technique say M_{AR} is modified to be applied on successive wave for estimation of population mean of sensitive variable. Each respondent on first(second) wave is asked to rotate a spinner bearing the following statements
- Report the real value of variable $x_i[y_i]$
- Report the scrambled response $(x_iW_1 + W_2)$ $[(y_iW_1 + W_2)]$
- Report a value of variable W_3

with corresponding probabilities p_1, p_2 and $(1-p_1-p_2)$ respectively on first [second] waves. Using above randomization devise, response given by j^{th} respondent on first and second wave respectively are described as

$$X_{1j}' = \begin{cases} x_j & \text{with probability } p_1 \\ x_{jW_1+W_2} & \text{with probability } p_2 \\ W_3 & \text{otherwise} \end{cases},\ Y_{1j}' = \begin{cases} y_j & \text{with probability } p_1 \\ y_{jW_1+W_2} & \text{with probability } p_2 \\ W_3 & \text{otherwise} \end{cases}$$

Therefore applying M_{AR} on two successive waves, the sensitive variable $x(y)$ are perturbed to $x^{'}(y^{'})$ and are given by

$$X_1^{'} = Xp_1 + (XW_1 + W_2)p_2 + W_3(1 - p_1 - p_2)$$

and

$$Y_1^{'} = Yp_1 + (YW_1 + W_2)p_2 + W_3(1 - p_1 - p_2)$$

such that

$$(\bar{Y})_1 = \frac{\bar{Y}_1^{'} - p_2\bar{W}_2 - (1 - p_1 - p_2)\bar{W}_3}{p_1 + p_2\bar{W}_1} \quad (1)$$

$$\left(\rho_{y'x'}\right)_1 = \frac{p_1^2\rho_{yx}S_yS_x + 2p_1p_2\rho_{yx}S_yS_x\bar{W}_1 + p_2^2(\rho_{yx}S_yS_xS_{W_1}^2 + \rho_{yx}S_yS_x\bar{W}_1^2 + \bar{X}\bar{Y}S_{W_1}^2) + (1-p_1-p_2)^2S_{W_3}^2}{\sqrt{I_1}\sqrt{I_2}},$$

$$\left(\rho_{y'z}\right)_1 = \frac{(p_1 + p_2\bar{W}_1)\rho_{yz}S_y}{\sqrt{I_2}}, \left(\rho_{x'z}\right)_1 = \frac{(p_1 + p_2\bar{W}_1)\rho_{xz}S_x}{\sqrt{I_1}}$$

where,

$$I_1 = p_1^2S_x^2 + p_2^2(S_x^2S_{W_1}^2 + S_x^2\bar{W}_1^2 + S_{W_1}^2\bar{X}^2 + S_{W_2}^2) + (1 - p_1 - p_2)^2S_{W_3}^2 + 2p_1p_2\bar{W}_1S_x^2$$

and

$$I_2 = p_1^2S_y^2 + p_2^2(S_y^2S_{W_1}^2 + S_y^2\bar{W}_1^2 + S_{W_1}^2\bar{Y}^2 + S_{W_2}^2) + (1 - p_1 - p_2)^2S_{W_3}^2 + 2p_1p_1\bar{W}_1S_y^2$$

Many other randomised response models such as Greenberg *et al.* (1971) (M_G), Barlev *et al.* (2004) (M_B), Diana and Perri (2010) (M_{DP1}) and scrambled response models by Pollock and Bek (1976) (M_{PB}), Eichhorn and Hayre (1983) (M_{EH}), Saha (2007) (M_{SH}) and Diana and Perri (2010) (M_{DP2}) can be viewed as particular cases of above described techniques and are presented in Table 1 .

Table 1. Particular cases

Name of the Model	p_1	p_2	W_1	W_2	W_3
M_G	p	$1-p$	0	W_2	0
M_{PB}	0	1	1	W_2	0
M_{EH}	0	1	W_1	0	0
M_B	p	$1-p$	W_1	0	0
M_{SH}	0	1	W_1	W_1W_2	0
M_{DP1}	p	$1-p$	W_1	W_1W_2	0
M_{DP2}	0	1	$(1-\chi^*)W_1$	$\chi^*W_1W_2$	0

Note:$\chi^* \ \varepsilon \ [0,1]$ and $0 \leq p \leq 1$

2.3. Scrambled Response Techniques on successive waves

Considering a convex combination of the multiplicative and additive scrambled response model, Diana and Perri (2010) proposed scrambled response model. Their underlying idea was to combine the two models giving them a different weight according to the problem at hand. Therefore, their model say M_{DP} is modified to be applied on two successive waves, the sensitive variable $x(y)$ is perturbed to $x^{'}(y^{'})$ in the light of this model as:

$$X_2^{'} = \varphi_x^*(X + W_2) + (1 - \varphi_x^*)W_1X \ ; \ where \ \varphi_x^* \ \varepsilon \ [0,1]$$

and

$$Y_2^{'} = \varphi_y^*(Y + W_2) + (1 - \varphi_y^*)W_1 Y \; ; \; where \; \varphi_y^* \; \varepsilon \; [0,1]$$

such that

$$(\bar{Y})_2 = \frac{\bar{Y}_1^{'} - \varphi_y^* \bar{W}_1}{\varphi_y^* + (1 - \varphi_y^*)\bar{W}_2}, \tag{2}$$

$\left(\rho_{y'x'}\right)_2 = \frac{\varphi_x^* \varphi_y^* [I_3] + [I_4]\left(\varphi_x^* + \varphi_y^*\right) + I_5}{\sqrt{I_6 \sqrt{I_7}}}$, $\left(\rho_{y'z}\right)_2 = \frac{\rho_{yz} S_y \left[\varphi_y^*(1 - \bar{W}_1) + \bar{W}_1\right]}{\sqrt{I_6}}$,

$(\rho_{x'z})_2 = \frac{\rho_{xz} S_x [\varphi_x^*(1 - \bar{W}_1) + \bar{W}_1]}{\sqrt{I_7}}$, $\left(C^2_{x'}\right)_2 = \frac{I_7}{\bar{X}'^2}$ and $\left(C^2_{y'}\right)_2 = \frac{I_6}{\bar{Y}'^2}$

where,

$$I_3 = \rho_{yx} S_y S_x + S^2_{W_2} - 2\rho_{yx} \bar{W}_1 S_y S_x + S^2_{W_1}\left[\rho_{yx} S_y S_x + \bar{X}\bar{Y}\right] + \rho_{yx} \bar{W}_1^2 S_y S_x,$$

$$I_4 = \bar{W}_1 \rho_{yx} S_y S_x - S^2_{W_1}\left[\rho_{yx} S_y S_x + \bar{X}\bar{Y}\right] - \bar{W}_1^2 \rho_{yx} S_y S_x,$$

$$I_5 = S^2_{W_1}\left[\rho_{yx} S_y S_x + \bar{X}\bar{Y}\right] + \bar{W}_1^2 \rho_{yx} S_y S_x,$$

$$I_6 = \left(\varphi_y^*\right)^2 \left[S_y^2 + S^2_{W_2}\right] + \left(1 - \varphi_y^*\right)\left[S^2_{W_1}(1 + \bar{Y}^2) + S_y^2(1 + \bar{W}_1^2)\right] + 2\varphi_y^*\left(1 - \varphi_y^*\right)\bar{W}_1 S_y^2,$$

$$I_7 = (\varphi_x^*)^2 \left[S_x^2 + S^2_{W_2}\right] + (1 - \varphi_x^*)\left[S^2_{W_1}(1 + \bar{X}^2) + S_x^2(1 + \bar{W}_1^2)\right] + 2\varphi_x^*(1 - \varphi_x^*)\bar{W}_1 S_x^2.$$

Remark 1. *If $\varphi_x^*(\varphi_y^*) = 0$, then the model M_{DP} reduces to multiplicative model and if $\varphi_x^*(\varphi_y^*) = 1$, it reduces to additive scramble response model.*

Remark 2. *The scrambling variables W_1, W_2 and W_3 are such that $E(W_1) = \bar{W}_1$, $E(W_2) = \bar{W}_2$, $E(W_3) = \bar{W}_3$, $V(W_1) = S^2_{W_1}$, $V(W_2) = S^2_{W_2}$, $V(W_3) = S^2_{W_3}$, $S^2_{y_i'} = \left(\frac{C^2_{y_i'}}{\bar{Y}_i'^2}\right)$, $S^2_{x_i'} = \left(\frac{C^2_{x_i'}}{\bar{X}_i'^2}\right)$.*

Remark 3. *$(\bar{Y})_i$, $i = 1$ and 2 denote population mean of sensitive variable y at current wave under i^{th} model in two wave successive sampling.*

Remark 4. *Suitable estimator of population mean of coded response variable $\bar{Y}_i^{'}$ need to be investigated and replaced in equation 1 and 2 respectively in order to obtain appropriate estimator of sensitive population mean at current wave under five different models in two wave successive sampling.*

2.4. General Class of estimators on Successive waves

For estimating the population mean of coded response variable at current wave in two wave successive sampling under randomized as well as scrambled response models described in section 2.2 and 2.3 respectively, two classes of estimators have been proposed based on sample of size u and m respectively. The final estimators is the general class of estimator formulated by considering convex linear combination of two classes of estimators based on sample size u and m respectively under two consider models.

2.4.1 Class of Estimators based on unmatched sample on the second wave

The literature on successive sampling reveals that in general difference, regression, ratio, product, exponential ratio or product type estimator can be modified for the estimation of population mean of coded response variable. Some of them can be seen as:
$L_{u1i} = \bar{y}'_{ui}$, if no additional non-sensitive auxiliary information is used at any wave.
$L_{u2i} = \bar{y}'_{ui} + k(\bar{z}_u - \bar{Z})$,
$L_{3i} = \bar{y}'_{ui} + \beta_{qiz}(\bar{z}_u - \bar{Z})$,
$L_{u4i} = \bar{y}'_{ui} \frac{\bar{Z}}{\bar{z}_u}$,
$L_{u5i} = \bar{y}'_{ui} \frac{\bar{z}_u}{\bar{Z}}$,
$L_{u6i} = \bar{y}'_{ui} (\frac{\bar{z}_u}{\bar{Z}})^{\theta_1}$,
$L_{u7i} = \bar{y}'_{ui}[2-(\frac{\bar{z}_u}{\bar{Z}})^{\theta_2}]$,
$L_{u8i} = \bar{y}'_{ui}\ exp\ (\frac{\bar{Z} - \bar{z}_u}{\bar{Z} + \bar{z}_u})$,
$L_{u9i} = \bar{y}'_{ui}\ exp\ (\frac{\bar{z}_u - \bar{Z}}{\bar{z}_u + \bar{Z}})$,
$L_{u10i} = \bar{y}'_{ui} + \beta_{y'iz}(\bar{Z} - \bar{z}_u)$,
$L_{u11i} = \bar{y}'_{ui} + b_{y'iz}(u)(\bar{Z} - \bar{z}_u)$,
etc.,
where, k, θ_1 and θ_2 are constants chosen suitably, so that the mean squared errors of L_{u2i}, L_{u6i} and L_{u7i} may be optimized respectively.
Therefore, following Srivastava (1980) and Tracy *et al.*(1996) a class of estimator have been proposed which may contain the above discussed estimators as its members, under the considered randomized and scrambled response models based on unmatched sample as:

$$L_{ui} = U_i\ (\bar{y}'_{ui},\ \bar{z}_u) \tag{3}$$

where $i = 1$ and 2 denote the randomized and scrambled response model respectively given in section 2.2 and 2.3 and $U_i\ (\bar{y}'_{ui},\ \bar{z}_u)$ is a function of $\bar{y}'_{ui}$ and $\bar{z}_u$ such that
(i) The point $(\bar{y}'_{ui},\ \bar{z}_u)$ assumes the value in a closed convex subset R_2 of two dimensional real space containing the point $(\bar{Y}'_i, \bar{Z})$.
(ii) The function $U_i\ (\bar{y}'_{ui},\ \bar{z}_u)$ is continuous and bounded in R_2.
(iii) $U_i(\bar{Y}'_i,\ \bar{Z}) = \bar{Y}'_i$ and $U_{1i}(\bar{Y}'_i,\ \bar{Z}) = \frac{\partial U_i(\bar{y}'_{ui}, \bar{z}_u)}{\partial \bar{y}'_{ui}} = 1$, i.e., First order partial derivative of U_i with respect to $\bar{y}'_{ui}$ at $U_i\ (\bar{Y}'_i, \bar{Z}) = \bar{Y}'_i \Rightarrow U_{1i}\ (K_i) = \frac{\partial U_i(.)}{\partial \bar{y}'_{ui}}|_{K_i} = 1$, where $K_i = (\bar{Y}'_i, \bar{Z})$.
(iv) The first and second order partial derivatives of $U_i\ (\bar{y}'_{ui}, \bar{z}_u)$ exist and are continuous and bounded in R_2.

2.4.2 Estimators Based on the matched sample at current wave

For the matched sample of size m retained from previous wave, it is clear that there are two kind of auxiliary information available, one is non-sensitive additional auxiliary information (z) and other is information from previous wave based on sample of size n. Hence, motivated by Senapati and Sahoo(2006) let $f_{1i} = g_i\ (\bar{x}'_{mi},\ \bar{z}_m,\ \bar{z}_n)$ and $f_{2i} = h_i\ (\bar{x}'_{ni},\ \bar{z}_n)$ are two different classes of estimators of $\bar{X}'_i$ through samples of sizes m and n respectively such that $g_i\ (\bar{X}'_i,\ \bar{Z},\ \bar{Z}) = h_i(\bar{X}'_i,\ \bar{Z}) = \bar{X}'_i$. Let $(\bar{y}'_{mi},\ f_{1i},\ f_{2i})$ assumes values in a closed convex subspace R_3 of 3-dimensional real space containing the point $(\bar{Y}'_i,\ \bar{X}'_i,\ \bar{X}'_i)$. Also suppose $T_i\ (\bar{y}'_{mi},\ f_{1i},\ f_{2i})$ is a known function of $\bar{y}'_{mi},\ f_{1i},\ f_{2i}$ such that $T_i\ (\bar{Y}'_i,\ \bar{X}'_i,\ \bar{X}'_i) = \bar{Y}'_i$ and the three functions $g_i,\ h_i$, and T_i satisfies the regularity conditions stated by Srivastava (1980). Hence, a general class of estimators based on sample size m at current wave for estimating sensitive population mean under two models may be defined as

$$L_{mi} = T_i\ (\bar{y}'_{mi},\ f_{1i},\ f_{2i}) \tag{4}$$

where $i = 1$ and 2 denote the randomized and scrambled response models respectively quoted in section 2.2 and 2.3. Many well known estimators when modified for estimation of sensitive population mean can become a member of proposed class of estimators. Some of them are listed in Table 2.

Table 2. Estimators based on sample size m

Member	Estimator	Functional Form
L_{m1i}	$[\bar{y}'_{1mi} + k(\bar{x}'_{ni} - \bar{x}'_{mi})]$	when no additional non-sensitive auxiliary information is used then $f_{1i} = \bar{x}'_{mi}$ & $f_{2i} = \bar{x}'_{ni}$
L_{m2i}	$[\bar{y}'^{*}_{1mi} + \beta_{y'_i x'_i}(\bar{x}'^{*}_{1ni} - \bar{x}'^{*}_{1mi})]$, where, $\bar{y}'^{*}_{1mi} = [\bar{y}'_{mi} + \beta_{y'_i z}(\bar{Z} - \bar{z}_m)]$, $\bar{x}'^{*}_{1ni} = [\bar{x}'_{ni} + \beta_{x'_i z}(\bar{Z} - \bar{z}_n)]$ & $\bar{x}'^{*}_{1mi} = [\bar{x}'_{mi} + \beta_{x'_i z}(\bar{Z} - \bar{z}_m)$	$\bar{y}'^{*}_{1mi} + \beta_{y'_i x'_i}(f_{2i} - f_{1i})]$
L_{m3i}	$\left(\frac{\bar{y}'^{*}_{2mi}}{\bar{x}'^{*}_{2mi}}\right)\bar{x}'^{*}_{2ni}$, where, $\bar{y}'^{*}_{2mi} = \bar{y}'_{mi} + b_{y'_i z}(m)(\bar{Z} - \bar{z}_m)$, $\bar{x}'^{*}_{2ni} = \bar{x}'_{ni} + b_{x'_i z}(n)(\bar{Z} - \bar{z}_n)$ & $\bar{x}'^{*}_{2mi} = \bar{x}'_{mi} + b_{x'_i z}(m)(\bar{Z} - \bar{z}_m)$	$\frac{\bar{y}'^{*}_{2mi}}{f_{1i}} f_{2i}$
L_{m4i}	$\bar{y}'^{*}_{2mi} + b_{y'_i x'_i}(m)(\bar{x}'^{*}_{3ni} - \bar{x}'^{*}_{3mi})$, where, $\bar{x}'^{*}_{3ni} = \frac{\bar{x}'_{ni}}{\bar{z}_n}\bar{Z}$, $\bar{x}'^{*}_{3mi} = \frac{\bar{x}'_{mi}}{\bar{z}_m}\bar{Z}$	$\bar{y}'^{*}_{2mi} + b_{y'_i x'_i}(m)(f_{2i} - f_{1i})$
L_{m5i}	$\bar{y}'^{*}_{4mi} + b_{y'_i x'_i}(m)(\bar{x}'^{*}_{3ni} - \bar{x}'^{*}_{3mi})$, where, $\bar{y}'^{*}_{4mi} = \frac{\bar{y}'_{mi}}{\bar{z}_m}\bar{Z}$	$\bar{y}'^{*}_{4mi} + b_{y'_i x'_i}(m)(f_{2i} - f_{1i})$
L_{m6i}	$\frac{\bar{y}'^{*}_{2mi}}{\bar{x}'^{*}_{2mi}}\bar{x}'^{*}_{3ni}$	$\frac{\bar{y}'^{*}_{2mi}}{f_{1i}} f_{2i}$

2.4.3 Combined General Class of Estimators

Considering the convex linear combinations of the two classes of estimators L_{ui} and L_{mi} based on sample of size u and m respectively, the final estimator for population mean of coded response variable is given as

$$L_i = \Psi_i^* L_{ui} + (1-\Psi_i^*) L_{mi} \; ; \; i = 1 \; and \; 2 \tag{5}$$

where the class of estimators L_{ui} and L_{mi} are defined in equations 3 and 4 respectively and $\Psi_i^* \; \varepsilon \; [0,1]$ is a scalar quantity to be chosen suitably.
Many existing estimators for population mean at current wave by eminent researches in successive sampling can be the members of the proposed class when modified to work for estimation of sensitive population mean of coded response variable at current wave. Some of them are modified and given as:

$L_{1i} = \Psi_{1i}^* L_{u1i} + (1-\Psi_{1i}^*) L_{m1i}$, (Modified Jessen (1942) estimator)
$L_{2i} = \Psi_{1i}^* L_{u10i} + (1-\Psi_{1i}^*) L_{m2i}$, (Modified Singh and Priyanka (2008))
$L_{3i} = \Psi_{2i}^* L_{u11i} + (1-\Psi_{2i}^*) L_{m3i}$, (Modified Singh and Karna (2009) estimator)

$$\left.\begin{aligned} L_{4i} &= \Psi_{4i}^* L_{u4i} + (1-\Psi_{4i}^*) L_{m4i}, \\ L_{5i} &= \Psi_{5i}^* L_{u4i} + (1-\Psi_{5i}^*) L_{m5i}, \\ L_{6i} &= \Psi_{6i}^* L_{u4i} + (1-\Psi_{6i}^*) L_{m6i}. \end{aligned}\right\} \text{(Modified Singh and Prasad (2010) estimator)}$$

etc.,

3. Features of proposed General Class of Estimators

3.1. Bias and Mean Squared Error

The bias and mean squared error of class of estimators L_{ui} and L_{mi} are derived up to first order approximations under large sample assumptions and using the following transformations.
$\bar{y}'_{ui} = \bar{Y}'_i (1+e_{1i})$, $\bar{y}'_{mi} = \bar{Y}'_i (1+e_{2i})$, $\bar{x}'_{mi} = \bar{X}'_i (1+e_{3i})$, $\bar{x}'_{ni} = \bar{X}'_i (1+e_{4i})$, $\bar{z}_m = \bar{Z} (1+e_5)$, $\bar{z}_u = \bar{Z} (1+e_6)$, $\bar{z}_n = \bar{Z} (1+e_7)$, $\bar{x}'_{ui} = \bar{X}'_i (1+e_{8i})$, $s^2_{x'_i}(m) = S^2_{x'_i} (1+e_{9i})$, $s_{y'_i z}(u) = S_{y'_i z} (1+e_{10i})$, $s_{y'_i z}(m) = S_{y'_i z} (1+e^*_{10i})$, $s^2_z(u) = S^2_z (1+e_{11})$, $s^2_z(m) = S^2_z (1+e^*_{11})$, $s^2_z(n) = S^2_z (1+e^{**}_{11})$, $s_{x'_i z}(n) = S_{x'_i z} (1+e_{12i})$, $s_{x'_i z}(m) = S_{x'_i z} (1+e^*_{12i})$, such that, $E(e_{si}) = 0$; $|e_{si}| < 1$; $E(e_k) = 0$; $|e_k| < 1$ where, $i = 1$ and 2; $s = 1, 2, 3, 4, 8, 9, 10$ and 12 and $k = 5, 6, 7 \; and \; 11$.

3.1.1 The Bias and Mean Squared Error of L_{ui}

The expressions of bias and mean squared error of the class of estimators L_{ui} are derived as

$$L_{ui} = U_i(\bar{y}'_{ui}, \; \bar{z}_u)$$

Expanding $U_i\ (\bar{y}'_{ui},\ \bar{z}_u)$ about the point $K_i\ =\ (\bar{Y}'_i, \bar{Z})$ in a first order Taylor series, we have

$$L_{ui} = [\ U_i(K_i) + \ (\bar{y}'_{ui} - \ \bar{Y}'_i)\ d_{i1} + (\bar{z}_u - \ \bar{Z})\ d_{i2} + \frac{1}{2}\ \left\{(\bar{y}'_{ui} - \ \bar{Y}'_i)^2\ d_{i11} + (\bar{z}_u - \ \bar{Z})^2\ d_{i22} + 2\ (\bar{y}'_{ui} - \ \bar{Y}'_i)(\bar{z}_u - \ \bar{Z})\ d_{i12}\right\} + \ldots] \quad (6)$$

where,
$d_{i1} = \ \frac{\partial U_i}{\partial \bar{y}'_{ui}}|_{K_i},\ \ d_{i2} = \ \frac{\partial U_i}{\partial \bar{z}_u}|_{K_i},\ \ d_{i11} = \ \frac{\partial^2 U_i}{\partial \bar{y}'^2_{ui}}|_{K_i},\ \ d_{i22} = \ \frac{\partial^2 U_i}{\partial \bar{z}^2_u}|_{K_i},$

$d_{i12} = \ \frac{\partial^2 U_i}{\partial \bar{y}'_{ui}\partial \bar{z}_u}|_{K_i}\ ;\ K_i = \ (\bar{Y}'_i,\ \bar{Z})$ and $i = \ 1$ and 2
Applying large sample approximations in equation 6, and retaining terms up to first order approximations we have,

$$\left(L_{ui}\ -\ \bar{Y}'_i\right) = \ \left[\bar{Y}'_i e_{1i} + \bar{Z} e_6 d_{i2} + \frac{1}{2}\left\{\bar{Y}'^2_i e^2_{1i} d_{i11} + \bar{Z}^2 e^2_6 d_{i22} + 2\bar{Y}'_i \bar{Z} e_{1i} e_6 d_{i12}\right\}\right] \quad (7)$$

Taking expectations on both sides in the above equation 7 and assuming the population size is sufficiently large, we get bias of L_{ui} up to first order approximation as

$$B(L_{ui}) = \frac{1}{u}\left[\frac{1}{2}(d_{i11}\bar{Y}'^2_i C^2_{y'_i} + \bar{Z}^2 C^2_z d_{i22}) + (\rho_{y'_i z} C_{y'_i} C_z \bar{Y}'_i \bar{Z} d_{i12})\right] \quad (8)$$

Now, squaring both sides of above equation 7 and retaining terms up to first order of approximations, we have

$$\left(L_{ui}\ -\ \bar{Y}'_i\right)^2\ =\ \left[\bar{Y}'^2_i e^2_{1i} + \bar{Z}^2 e^2_6 d^2_{i2} + 2\bar{Y}'_i \bar{Z} e_{1i} e_6 d_{i2}\right]$$

Taking expectations on both sides in the above equation and assuming the population is very large i.e.,$N \to \infty$, the mean squared error of L_{ui} is obtained as

$$M(L_{ui}) = \frac{1}{u}\left[\bar{Y}'^2_i C_{y'^2_i} + \bar{Z}^2 C^2_z d^2_{i2} + 2\rho_{y'_i z} C_{y'_i} C_z \bar{Y}'_i \bar{Z} d_{i2}\right]$$

which is optimized for $d_{i2} = -\rho_{y'_i z}$, Further, substituting optimized value of d_{i2} in the above equation we obtain the required optimum mean squared error of L_{ui} as

$$M(L_{ui})_{opt.} = \frac{1}{u}\left[\bar{Y}'^2_i C_{y'^2_i} + \bar{Z}^2 C^2_z \rho^2_{y'_i z} - 2\rho_{y'_i z} C_{y'_i} C_z \bar{Y}'_i \bar{Z} \rho_{y'_i z}\right]$$

Remark 5. *Since x' and y' are the same variables over two waves and z is the stable auxiliary variable so as pointed out by Murthy (1967), Cochran(1977), Reddy(1978), Feng and Zou (1996) and Singh and Ruiz-Espejo (2003) the coefficient of variation is stable in nature, so we assume that the coefficients of variation x', y' and z are almost equal (i.e., $C_{y'} \cong C_{x'} \cong C_z$).*

From the above remark 5 we state the following theorem.

Theorem 3.1. *To the first degree of approximations, the bias and mean squared error of L_{ui} under assumption given in remark 5 is*

$$B(L_{ui}) \;=\; \frac{1}{2u}\left[d_{i11}+d_{i22}+2d_{i12}\rho_{y_i^{'}z}\right]S^2_{y_i^{'}} \tag{9}$$

and

$$M(L_{ui})_{opt.} = \frac{1}{u}\left(1-\rho^2_{y_i^{'}z}\right)S^2_{y_i^{'}} \tag{10}$$

which is similar to the variance of linear regression estimator for population mean.

3.1.2 The Bias and Mean Squared Error of L_{mi}

For deriving the bias and mean squared error of class of estimators L_{mi}, $f_{1i} = g_i\ (\bar{x}^{'}_{mi},\ \bar{z}_m,\ \bar{z}_n)$ and $f_{2i} = h_i\ (\bar{x}^{'}_{ni},\ \bar{z}_n)$ have been expanded around the points $(\bar{X}^{'}_i,\ \bar{Z},\ \bar{Z})$ and $(\bar{X}^{'}_i,\ \bar{Z})$ respectively by first order Taylor's series and neglecting the remainder terms we get,
$f_{1i} = \bar{X}^{'}_i + G_1(\bar{x}^{'}_{mi} - \bar{X}^{'}_i) + G_2[(\bar{z}_m - \bar{Z}) - (\bar{z}_n - \bar{Z})]$ and
$f_{2i} = \bar{X}^{'}_i + H_1(\bar{x}^{'}_{ni} - \bar{X}^{'}_i) + H_2(\bar{z}_n - \bar{Z})$.
Following Senapati and Sahoo(2006), we assume $H_1 = 1$ because $h_i(\bar{X}^{'}_i,\ \bar{Z}) = \bar{X}^{'}_i$ and $G_1 = 1,\ G_2 = -G_3$ because $g_i\ (\bar{x}^{'}_{mi},\ \bar{z}_m,\ \bar{z}_n)$ and $g_i\ (\bar{x}^{'}_{mi},\ \bar{z}_n,\ \bar{z}_m)$ assume the same value i.e., $\bar{X}^{'}_i$ at $(\bar{X}^{'}_i,\ \bar{Z},\ \bar{Z})$. Hence we have

$$f_{1i} = \bar{X}^{'}_i + (\bar{x}^{'}_{mi} - \bar{X}^{'}_i) + G_2[(\bar{z}_m - \bar{Z}) - (\bar{z}_n - \bar{Z})] \tag{11}$$

and

$$f_{2i} = \bar{X}^{'}_i + (\bar{x}^{'}_{ni} - \bar{X}^{'}_i) + H_2(\bar{z}_n - \bar{Z}) \tag{12}$$

Similarly, observing $F_1 = 1,\ F_2 = -F_3$ and expanding $T_i\ (\bar{y}^{'}_{mi},\ f_{1i},\ f_{2i})$ around the point $(\bar{Y}^{'}_i,\ \bar{X}^{'}_i,\ \bar{X}^{'}_i)$ by first order Taylor's series, we have,
$L_{mi} = \bar{Y}^{'}_i + F_1(\bar{y}^{'}_{mi} - \bar{Y}^{'}_i) + F_2[(f_{1i} - \bar{X}^{'}_i) - (f_{2i} - \bar{X}^{'}_i)]$, i.e.,

$$\begin{aligned} L_{mi} = \;& \bar{Y}^{'}_i + (\bar{y}^{'}_{mi} - \bar{Y}^{'}_i) + F_2[(f_{1i} - \bar{X}^{'}_i) - (f_{2i} - \bar{X}^{'}_i)] + \\ & \frac{1}{2}[(\bar{y}^{'}_{mi} - \bar{Y}^{'}_i)^2 F_{11} + (f_{1i} - \bar{X}^{'}_i)^2 F_{22} + (f_{2i} - \bar{X}^{'}_i)^2 F_{33} + \\ & 2(\bar{y}^{'}_{mi} - \bar{Y}^{'}_i)(f_{1i} - \bar{X}^{'}_i)F_{12} + 2(\bar{y}^{'}_{mi} - \bar{Y}^{'}_i)(f_{2i} - \bar{X}^{'}_i)F_{13} + \\ & 2(f_{1i} - \bar{X}^{'}_i)(f_{2i} - \bar{X}^{'}_i)F_{23}]. \end{aligned} \tag{13}$$

where,
$F_1 = \frac{\partial \bar{f}_i}{\partial \bar{y}^{'}_{mi}}|s^*_{1i} = 1,\ F_2 = \frac{\partial \bar{f}_i}{\partial \bar{x}^{'}_{mi}}|s^*_{1i},\ F_3 = \frac{\partial \bar{f}_i}{\partial \bar{x}^{'}_{ni}}|s^*_{1i},\ F_{11} = 0,\ F_{22} = \frac{\partial^2 \bar{f}_i}{\partial \bar{x}^{'2}_{mi}}|s^*_{1i},$
$F_{33} = \frac{\partial^2 \bar{f}_i}{\partial \bar{x}^{'2}_{ni}}|s^*_{1i},\ F_{12} = \frac{\partial^2 \bar{f}_i}{\partial \bar{y}^{'}_{mi}\partial \bar{x}^{'}_{mi}}|s^*_{1i},\ F_{13} = \frac{\partial^2 \bar{f}_i}{\partial \bar{y}^{'}_{mi}\partial \bar{x}^{'}_{ni}}|s^*_{1i},\ F_{23} = \frac{\partial^2 \bar{f}_i}{\partial \bar{x}^{'}_{mi}\partial \bar{x}^{'}_{ni}}|s^*_{1i},$

$G_1 = \frac{\partial \bar{g}_i}{\partial \bar{y}'_{mi}}|S^*_{2i} = 1,\ G_2 = \frac{\partial \bar{g}_i}{\partial \bar{x}'_{mi}}|S^*_{2i},\ G_3 = \frac{\partial \bar{g}_i}{\partial \bar{x}'_{ni}}|S^*_{2i},\ G_{11} = 0,$
$G_{22} = \frac{\partial^2 \bar{g}_i}{\partial \bar{x}'^2_{mi}}|S^*_{2i},\ G_{33} = \frac{\partial^2 \bar{g}_i}{\partial \bar{x}'^2_{ni}}|S^*_{2i},\ G_{12} = \frac{\partial^2 \bar{g}_i}{\partial \bar{y}'_{mi}\partial \bar{x}'_{mi}}|S^*_{2i},\ G_{13} = \frac{\partial^2 \bar{g}_i}{\partial \bar{y}'_{mi}\partial \bar{x}'_{ni}}|S^*_{2i},$
$G_{23} = \frac{\partial^2 \bar{g}_i}{\partial \bar{x}'_{mi}\partial \bar{x}'_{ni}}|S^*_{2i},\ H_1 = \frac{\partial \bar{h}_i}{\partial \bar{y}'_{mi}}|S^*_{3i} = 1,\ H_2 = \frac{\partial \bar{h}_i}{\partial \bar{x}'_{mi}}|S^*_{3i},\ H_3 = \frac{\partial \bar{h}_i}{\partial \bar{x}'_{ni}}|S^*_{3i},$
$H_{11} = 0,\ H_{22} = \frac{\partial^2 \bar{h}_i}{\partial \bar{x}'^2_{mi}}|S^*_{3i},\ H_{33} = \frac{\partial^2 \bar{h}_i}{\partial \bar{x}'^2_{ni}}|S^*_{3i},\ H_{12} = \frac{\partial^2 \bar{h}_i}{\partial \bar{y}'_{mi}\partial \bar{x}'_{mi}}|S^*_{3i},$
$H_{13} = \frac{\partial^2 \bar{h}_i}{\partial \bar{y}'_{mi}\partial \bar{x}'_{ni}}|S^*_{3i},\ H_{23} = \frac{\partial^2 \bar{h}_i}{\partial \bar{x}'_{mi}\partial \bar{x}'_{ni}}|S^*_{3i}$
where $S^*_{1i} = (\bar{Y}'_i,\ \bar{X}'_i,\ \bar{X}'_i),\ S^*_{2i} = (\bar{X}'_i,\ \bar{Z},\ \bar{Z})$ and $S^*_{3i} = (\bar{X}'_i,\ \bar{Z})$; $i =$ 1 and 2 correspond to randomized and scrambled response models considered.

After applying large sample approximations in equation 13 taking relevant expectations, simplifying and retaining terms up to first order of approximation we get the bias and mean squared error of L_{mi} for large N as:

$$\begin{aligned}B(L_{mi}) = \frac{1}{2}[\frac{1}{m}((&\bar{X}'^2_i C_{x'_i} G_{11} + \bar{Z}^2 C_{z^2} G_{22} + \bar{X}'_i \bar{Z} \rho_{x'_i z} C_{x'_i} C_z G_{12}) F_2 + F_{11} \bar{Y}'^2_i C_{y'^2_i} + \\ &(\bar{X}'^2_i C^2_{x'_i} + \bar{Z}^2 C^2_z G^2_2 + 2\bar{X}'_i \bar{Z} \rho_{x'_i z} C_{x'_i} C_z G_2) F_{22} + 2(\bar{X}'_i \bar{Y}'_i \rho_{y'_i x'_i} C_{x'_i} C_{y'_i} + \\ &\bar{Y}'_i \bar{Z} \rho_{y'_i z} C_{y'_i} C_z G_2) F_{12}) + \frac{1}{n}([(\bar{Z}^2 C^2_z G_{33} + \bar{X}'_i \bar{Z} \rho_{x'_i z} C_{x'_i} C_z G_{13} + \bar{Z}^2 C^2_z G_{23}) F_2 - \\ &(\bar{X}'_i C_{x'^2_i} H^2_1 + \bar{Z}^2 C^2_z H^2_2 + \bar{X}'_i \bar{Z} \rho_{x'_i z} C_{x'_i} C_z H_{12})] F_2 - (\bar{Z}^2 G^2_2 C^2_z + 2\bar{X}'_i \bar{Z} G_2 \rho_{x'_i z} C_{x'_i} C_z) F_{22} + \\ &(\bar{X}'_i C_{x'^2_i} + \bar{Z}^2 C^2_z H^2_2 + 2\rho_{x'_i z} C_{x'_i} C_z \bar{X}'_i \bar{Z} H_2) F_{33} - 2\rho_{y'_i z} C_{y'_i} C_z \bar{Y}'_i \bar{Z} G_2 F_{12} + \\ &2(\rho_{y'_i x'_i} C_{y'_i} C_{x'_i} \bar{X}'_i \bar{Y}'_i + \bar{Y}'_i \bar{Z} H_2 \rho_{y'_i z} C_{y'_i} C_z) F_{13} + 2(\bar{X}'^2_i C^2_{x'_i} + \rho_{x'_i z} \bar{X}'_i \bar{Z} C_{x'_i} C_z H_2) F_{23})]\end{aligned} \tag{14}$$

$$\begin{aligned}M(L_{mi}) = \ &\bar{Y}'_i \frac{1}{m} C_{y'^2_i} + \bar{X}'_i F^2_2 (\frac{1}{m} C_{x'^2_i} - \frac{1}{n} C_{x'^2_i}) + F^2_2 G^2_2 \bar{Z}^2 (\frac{1}{m} C_{z^2} - \frac{1}{n} C_{z^2}) + \\ &F^2_2 H^2_2 \bar{Z}^2 \frac{1}{n} C^2_z - 2\bar{Y}'_i \bar{X}'_i F_2 (\frac{1}{m} \rho_{x'_i y'_i} C_{y'_i} C_{x'_i} - \frac{1}{n} \rho_{x'_i y'_i} C_{y'_i} C_{x'_i}) + \\ &2\bar{Y}'_i \bar{Z} F_2 G_2 (\frac{1}{m} \rho_{y'_i z} C_{y'_i} C_z - \frac{1}{n} \rho_{y'_i z} C_{y'_i} C_z) + 2\bar{Y}'_i F_2 H_2 \bar{Z} \frac{1}{n} \rho_{y'_i z} C_{y'_i} C_z - \\ &2\bar{X}'_i \bar{Z} F^2_2 G_2 (\frac{1}{m} \rho_{x'_i z} C_{x'_i} C_z - \frac{1}{n} \rho_{x'_i z} C_{x'_i} C_z)\end{aligned} \tag{15}$$

which is further optimized for
$(F_2)_{opt.} = \frac{\rho_{y'_i x'_i} - \rho_{x'_i z} \rho_{y'_i z}}{\rho^2_{x'_i z} - 1}$ $(say\ F^*_2),\ (G_2)_{opt.} = \frac{\rho_{y'_i x'_i} \rho_{x'_i z} - \rho_{y'_i z}}{\rho_{x'_i z} \rho_{y'_i z} - \rho_{y'_i x'_i}}$ $(say\ G^*_2)$
and $(H_2)_{opt.} = \frac{\rho_{y'_i z} (\rho^2_{x'_i z} - 1)}{\rho_{y'_i x'_i} - \rho_{x'_i z} \rho_{y'_i z}}$ $(say\ H^*_2)$.

Further, substituting minimum value of $F_2,\ G_2$ and H_2 in the above equation we obtain the optimum mean squared error of L_{mi} as

$$
\begin{aligned}
M(L_{mi})_{opt.} =& \bar{Y}_i^{'}\frac{1}{m}C^2_{y_i^{'}} + \bar{X}_i^{'}F_2^{*2}(\frac{1}{m}C^2_{x_i^{'}} - \frac{1}{n}C^2_{x_i^{'}}) + F_2^2G_2^{*2}\bar{Z}^2(\frac{1}{m}C_z^2 - \frac{1}{n}C_z^2) + \\
& F_2^{2*}H_2^{*2}\bar{Z}^2\frac{1}{n}C_z^2 - 2\bar{Y}_i^{'}\bar{X}_i^{'}F_{*2}(\frac{1}{m}\rho_{x_i^{'}y_i^{'}}C_{y_i^{'}}C_{x_i^{'}} - \frac{1}{n}\rho_{x_i^{'}y_i^{'}}C_{y_i^{'}}C_{x_i^{'}}) + \\
& 2\bar{Y}_i^{'}\bar{Z}F_2^*G_2^*(\frac{1}{m}\rho_{y_i^{'}z}C_{y_i^{'}}C_z - \frac{1}{n}\rho_{y_i^{'}z}C_{y_i^{'}}C_z) + 2\bar{Y}_i^{'}F_2^*H_2^*\bar{Z}\frac{1}{n}\rho_{y_i^{'}z}C_{y_i^{'}}C_z - \\
& 2\bar{X}_i^{'}\bar{Z}F_2^{*2}G_2^*(\frac{1}{m}\rho_{x_i^{'}z}C_{x_i^{'}}C_z - \frac{1}{n}\rho_{x_i^{'}z}C_{x_i^{'}}C_z)
\end{aligned}
$$

From the remark 5 we state the following theorem.

Theorem 3.2. *To the first degree of approximations, the bias and mean squared error of L_{mi} under assumption given in remark 5, is given by*

$$
B(L_{mi}) = \left[\frac{1}{m}(a^*) + \frac{1}{n}(b^*)\right]\frac{S^2_{y_i^{'}}}{2} \tag{16}
$$

Where,

$$
\begin{aligned}
a^* =& \left(G_{11} + G_{22} + G_{12}\rho_{x_i^{'}z}\right)F_2 + F_{11} + \left(1 + G_2^2 + 2\rho_{x_i^{'}z}G_2\right)F_{22} + \\
& 2\left(\rho_{y_i^{'}x_i^{'}} + \rho_{y_i^{'}z}G_2\right)F_{12}, \\
b^* =& \left[\left(G_{33} + \rho_{x_i^{'}z}G_{13} + G_{23}\right) - \left(H_1^2 + H_2^2 + \rho_{x_i^{'}z}H_{12}\right)\right]F_2 - \left(G_2^2 + 2\rho_{x_i^{'}z}G_2\right)F_{22} \\
& + \left(1 + H_2^2 + 2\rho_{x_i^{'}z}H_2\right)F_{33} + 2\rho_{y_i^{'}z}F_{12} + 2\left(\rho_{y_i^{'}x_i^{'}} + \rho_{y_i^{'}z}H_2\right)F_{13} + \\
& 2\left(1 + H_2\rho_{x_i^{'}z}\right)F_{23}.
\end{aligned}
$$

and

$$
\begin{aligned}
M(L_{mi})_{opt.} =& [(\frac{1}{m} - \frac{1}{n})(F_2^{*2} + F_2^{*2}G_2^{*2} + 2\rho_{y_i^{'}x_i^{'}}F_2^* + 2\rho_{y_i^{'}z}G_2^*F_2^* + 2\rho_{x_i^{'}z}G_2^*F_2^{*2}) + \\
& \frac{1}{n}(F_2^{*2} + H_2^{*2} - 2\rho_{y_i^{'}z}F_2^*H_2^*) + \frac{1}{m}]S^2_{y_i^{'}}
\end{aligned} \tag{17}
$$

Theorem 3.3. *Bias of the general class of estimators L_i to the first order of approximations are obtained as*

$$
B(L_i) = \Psi_i^* B(L_{ui}) + (1 - \Psi_i^*)B(L_{mi}) \tag{18}
$$

Substituting the values of $B(L_{ui})$ and $B(L_{mi})$ from the equations 9 and 16 in the above equation, we have the expression for the bias of the general class of estimators L_i in equation 18.

Theorem 3.4. *Mean squared error of the general class of estimators* L_i *to first order of approximations are obtained as*

$$M(L_i) = \Psi_i^{*2} M(L_{ui})_{opt.} + (1-\Psi_i^*)^2 M(L_{mi})_{opt.} \tag{19}$$

The optimized values of $M(L_{ui})$ and $M(L_{mi})$ are computed in equation 10 and equation 17 respectively and as the two classes of estimators L_{ui} and L_{mi} are based on two non-overlaping samples of sizes u and m respectively.
So, $cov(L_{ui},\, L_{mi}) = 0$. Hence, using these values in above equation 19 we get the mean squared error of L_i.

3.2. Optimum Mean Squared Error of the Proposed class of Estimator

The mean squared error of class of estimators L_i is a function of unknown constant Ψ_i^* therefore, it is minimized with respect to Ψ_i^* and hence the optimum value of Ψ_i^* is obtained as

$$\Psi_{iopt.}^* = \frac{M[L_{mi}]_{opt.}}{M[L_{ui}]_{opt.} + M[L_{mi}]_{opt.}} \tag{20}$$

Substituting the value of $\Psi_{iopt.}^*$ from equation 20 in equation 19, we get the optimum mean squared error of the class of estimator L_i as

$$M[L_i]_{opt.} = \frac{M[L_{ui}]_{opt.} \times M[L_{mi}]_{opt.}}{M[L_{ui}]_{opt.} + M[L_{mi}]_{opt.}} \tag{21}$$

Further, substituting the values $M[L_{ui}]_{opt.}$ and $M[L_{mi}]_{opt.}$ from equations 10 and equation 17 in equation 21, the simplified values of $M[L_i]_{opt.}$ is derived as

$$M[L_i]_{opt.} = \frac{B_{1i}^* \mu_i + B_{2i}^*}{\mu_i^2 A_{3i}^* - \mu_i B_{3i}^* + A_{1i}^*} \left(\frac{S^2_{y_i'}}{n} \right) \tag{22}$$

where,
$A_{1i}^* = 1 - \rho^2_{y_i' z},\ A_{2i}^* = d^* + 1,\ A_{3i}^* = d^* - H_2^{*2} F_2^{*2} + 2\rho_{y_i' z} H_2^* F_2^*,$
$d^* = F_2^{*2} + G_2^{*2} F_2^{*2} + 2\rho_{y_i' x_i'} F_2^* + 2\rho_{y_i' z} F_2^* G_2^* + 2\rho_{x_i' z} F_2^{*2} G_2^*,$
$B_{1i}^* = A_{1i}^* A_{3i}^*,\ B_{2i}^* = A_{1i}^* A_{2i}^* - A_{1i}^* A_{3i}^*,\ B_{3i}^* = A_{1i}^* - A_{2i}^* + A_{3i}^*.$

3.3. Optimum Rotation Rate

Rotation rate is an important aspect in successive sampling as it is directly related to total cost of survey. More the sample rotated/ matched from previous wave, lesser number of units will be required to be drawn at current wave. Hence, mean squared error of the estimator L_i $(i = 1$ and $2)$ derived in equation 22 which is a function of μ_i, have been optimized with respect to μ_i $(i = 1$ and $2)$. The optimum value of

μ_i say $\hat{\mu}^*_{fi}$ have been obtained which satisfies the condition given as:

$$0 \ < \ min\left\{ \frac{-C^*_{2i} + \sqrt{C^{*\,2}_{2i} + C^*_{1i}C^*_{3i}}}{C^*_{1i}},\ \frac{-C^*_{2i} - \sqrt{C^{*\,2}_{2i} + C^*_{1i}C^*_{3i}}}{C^*_{1i}} \right\} < \ 1 \qquad (23)$$

where, $C^*_{1i} = B^*_{1i}A^*_{3i}$, $C^*_{2i} = A^*_{3i}B^*_{2i}$ and $C^*_{3i} = A^*_{1i}B^*_{1i} + B^*_{3i}B^*_{2i}$.
Substituting the applicable value of $\hat{\mu}^*_{fi}$ in equation 23, we have the optimum value of the mean squared error of the general class of estimators L_i as,

$$M(L_i)_{opt.^*} = \frac{B^*_{1i}\mu^*_{fi} + B^*_{2i}}{\mu^{2*}_{fi}A^*_{3i} - \mu^*_{fi}B^*_{3i} + A^*_{1i}} \left(\frac{S^2_{y'_i}}{n} \right) \ ; \ i \ = \ 1 \ and \ 2. \qquad (24)$$

4. Performance of Proposed Composite class of estimator

The proposed general class of estimator have been compared with the member of its class listed in section 2.4.3. Therefore their optimum fraction of sample to be drawn afresh at current wave and the optimum mean squared error have been computed and are presented below in Table 3 and Table 4 respectively.

Table 3. Optimum rotation rate for proposed estimators

Estimator	Optimum Rotation Rate
L_{1i}	$\hat{\mu}_{Ji}$ satisfies $0 < min\left\{ \frac{1+\sqrt{1-\rho^2_{y'_i x'_i}}}{\rho^2_{y'_i x'_i}},\ \frac{1-\sqrt{1-\rho^2_{y'_i x'_i}}}{\rho^2_{y'_i x'_i}} \right\} < 1$
L_{2i}	$\hat{\mu}_{spi}$ satisfies $0 \ < min\left\{ \frac{-A^*_{1i}+\sqrt{A^*_{1i}(A^*_{1i}+D^*_{1i})}}{D^*_{1i}},\ \frac{-A^*_{1i}-\sqrt{A^*_{1i}(A^*_{1i}+D^*_{1i})}}{D^*_{1i}} \right\} \ < \ 1$
L_{3i}	$\hat{\mu}_{ski}$ satisfies $0 \ < min\left\{ \frac{I_{2i}+\sqrt{I^2_{2i}-I_{1i}I_{3i}}}{I_{1i}},\ \frac{I_{2i}-\sqrt{I^2_{2i}-I_{1i}I_{3i}}}{I_{1i}} \right\} < \ 1$
L_{4i}	$\hat{\mu}_{sp1i}$ satisfies $0 \ < min\left\{ \frac{I_{12i}+\sqrt{I^2_{12i}-I_{11i}I_{13i}}}{I_{11i}},\ \frac{I_{12i}-\sqrt{I^2_{12i}-I_{11i}I_{13i}}}{I_{11i}} \right\} < \ 1$
L_{5i}	$\hat{\mu}_{sp2i}$ satisfies $0 \ < min\left\{ \frac{I_{22i}+\sqrt{I^2_{22i}-I_{21i}I_{23i}}}{I_{21i}},\ \frac{I_{22i}-\sqrt{I^2_{22i}-I_{21i}I_{23i}}}{I_{21i}} \right\} < \ 1$
L_{6i}	$\hat{\mu}_{sp3i}$ satisfies $0 \ < min\left\{ \frac{I_{32i}+\sqrt{I^2_{32i}-I_{31i}I_{33i}}}{I_{31i}},\ \frac{I_{32i}-\sqrt{I^2_{32i}-I_{31i}I_{33i}}}{I_{31i}} \right\} < \ 1$

Table 4. Mean Squared Error

Estimator	Optimum Mean Squared Error
L_{1i}	$\left(\frac{1-\hat{\mu}_{Ji}\rho^2_{y'_i x'_i}}{1-\hat{\mu}^2_{Ji}\rho^2_{y'_i x'_i}}\right)\frac{S^2_{y'_i}}{n}$
L_{2i}	$\left(\frac{[A^*_{1i}(A^*_{1i}+\hat{\mu}_{spi}D^*_{1i})]}{A^*_{1i}+\hat{\mu}^2_{spi}D^*_{1i}}\right)\left(\frac{S^2_{y'_i}}{n}\right)$ where $D^*_{1i}=2\rho^2_{y'_i z}\rho_{y'_i x'_i}-\rho^2_{y'_i x'_i}(1+\rho^2_{y'_i z})$
L_{3i}	$\left(\frac{\hat{\mu}_{ski}g_{1i}-g_{2i}}{\hat{\mu}^2_{ski}K_{3i}-\hat{\mu}_{ski}g_{3i}-K_{1i}}\right)\left(\frac{S^2_{y'_i}}{n}\right)$ where $k_{1i}=1-\rho^2_{y'_i z}$, $k_{2i}=2-\rho^2_{y'_i z}-2\rho_{y'_i x'_i}-\rho^2_{x'_i z}+2\rho_{y'_i z}\rho_{y'_i z}$, $k_{3i}=2\rho_{y'_i x'_i}+\rho^2_{x'_i z}-2\rho_{x'_i z}\rho_{y'_i z}-1$, $g_{1i}=k_{1i}k_{3i}, g_{2i}=k_{1i}k_{2i}+k_{1i}k_{3i}$, $g_{3i}=k_{2i}-k_{1i}+k_{3i}$, $I_{1i}=k_{3i}g_{1i}$, $I_{2i}=k_{3i}g_{2i}$ and $I_{3i}=k_{1i}g_{1i}+g_{2i}g_{3i}$
L_{4i}	$\left(\frac{\hat{\mu}_{sp1i}g_{11i}-g_{12i}}{\hat{\mu}^2_{sp1i}K_{13i}-\hat{\mu}_{sp1i}B_{13i}-K_{11i}}\right)\left(\frac{S^2_{y'_i}}{n}\right)$ where $k_{11i}=2-2\rho_{y'_i z}$, $k_{12i}=1-\rho^2_{y'_i z}-2\rho^2_{y'_i x'_i}\rho_{x'_i z}+2\rho_{y'_i z}\rho_{y'_i x'_i}\rho_{x'_i z}$, $k_{13i}=2\rho^2_{y'_i x'_i}\rho_{x'_i z}-2\rho_{y'_i z}\rho_{y'_i x'_i}\rho_{x'_i z}$, $g_{11i}=k_{11i}k_{13i}, g_{12i}=k_{11i}k_{12i}+k_{11i}k_{13i}$, $g_{13i}=k_{12i}-k_{11i}+k_{13i}$, $I_{11i}=k_{13i}g_{11i}$, $I_{12i}=k_{13i}g_{12i}$ and $I_{13i}=k_{11i}g_{11i}+g_{12i}g_{13i}$
L_{5i}	$\left(\frac{\hat{\mu}_{sp2i}g_{21i}-g_{22i}}{\hat{\mu}^2_{sp2i}K_{23i}-\hat{\mu}_{sp2i}g_{23i}-K_{11i}}\right)\left(\frac{S^2_{y'_i}}{n}\right)$ where $k_{22i}=2-2\rho^2_{y'_i x'_i}\rho_{x'_i z}-2\rho_{y'_i z}-2\rho_{y'_i x'_i}+2\rho_{y'_i x'_i}\rho_{x'_i z}+2\rho_{y'_i x'_i}\rho_{y'_i z}$, $k_{23i}=2\rho^2_{y'_i x'_i}\rho_{x'_i z}+2\rho_{y'_i x'_i}-2\rho_{y'_i x'_i}\rho_{x'_i z}-2\rho_{y'_i x'_i}\rho_{y'_i z}$, $g_{21i}=k_{11i}k_{23i}$, $g_{22i}=k_{11i}k_{22i}+k_{11i}k_{23i}$, $g_{23i}=k_{22i}-k_{11i}+k_{23i}, I_{21i}=k_{23i}g_{21i}$, $I_{22i}=k_{23i}g_{22i}$ and $I_{23i}=k_{11i}g_{21i}+g_{22i}g_{23i}$
L_{6i}	$\left(\frac{\hat{\mu}_{sp3i}B_{31i}-g_{32i}}{\hat{\mu}^2_{sp3i}K_{33i}-\hat{\mu}_{sp3i}B_{33i}-K_{11i}}\right)\left(\frac{S^2_{y'_i}}{n}\right)$ where $k_{32i}=2-\rho^2_{x'_i z}-\rho^2_{y'_i z}-2\rho_{y'_i x'_i}+2\rho_{y'_i z}\rho_{x'_i z}$, $k_{33i}=2\rho^2_{x'_i z}-2\rho_{x'_i z}+2\rho_{y'_i x'_i}-\rho_{y'_i z}\rho_{x'_i z}$, $g_{31i}=k_{11i}k_{33i}, g_{32i}=k_{11i}k_{32i}+k_{11i}k_{33i}$, $g_{33i}=k_{32i}-k_{11i}+k_{33i}, I_{31i}=k_{33i}g_{31i}, I_{32i}=k_{33i}g_{32i}$ and $I_{33i}=k_{1i}g_{31i}+g_{32i}g_{33i}$

5. Estimators for sensitive population mean at current wave

Replacing the population mean of coded response $\bar{Y}'_i$ $(i=1,\ 2)$ in equation 1 and equation 2 by its estimators L_i and L_{ij} $(i=1,\ 2\ ;\ j=1,\ 2,\ 3,\ 4,\ 5,\ 6)$, the corresponding estimators for sensitive population mean at current wave $\hat{\bar{Y}}_i$ and $\hat{\bar{Y}}_{ij}$ respectively is obtained and are given in Table 5.
Since, the estimators $\hat{\bar{Y}}_i$ and $\hat{\bar{Y}}_{ij}$ are biased, the mean squared errors of sensitive population mean estimators $\bar{Y}_{ji}$; $j=1,\ 2,\ 3,\ 4,\ 5,\ 6$ has also been computed under two considered models and are presented in Table 5.

Table 5. Sensitive population mean estimators $\hat{\hat{Y}}_i,\ \hat{\hat{Y}}_{ij}$ and Mean squared error of the estimators $\hat{\hat{Y}}_i,\ \hat{\hat{Y}}_{ij}$ under the models M_{AR} and M_{DP}

i	Model	Sensitive population mean estimator	Mean squared error of sensitive population mean
1	M_{AR}	$\hat{\hat{Y}}_1 = \frac{L_1 - p_2\bar{W}_2 - (1-p_1-p_2)\bar{W}_3}{p_1 + p_2\bar{W}_1}$	$M[\hat{\hat{Y}}_1] = \frac{M[L_1]_{opt.*}}{[p_1 + p_2\bar{W}_1]^2}$
		$\hat{\hat{Y}}_{1j} = \frac{L_{1j} - p_2\bar{W}_2 - (1-p_1-p_2)\bar{W}_3}{p_1 + p_2\bar{W}_1}$	$M[\hat{\hat{Y}}_{1j}] = \frac{M[L_{1j}]_{opt.*}}{[p_1 + p_2\bar{W}_1]^2}$
2	M_{DP}	$\hat{\hat{Y}}_2 = \frac{L_2 - \varphi_y^*\bar{W}_1}{\varphi_y^* + (1-\varphi_y^*)\bar{W}_2}$	$M[\hat{\hat{Y}}_2] = \frac{M[L_2]_{opt.*}}{[\varphi_y^* + (1-\varphi_y^*)\bar{W}_2]^2}$
		$\hat{\hat{Y}}_{2j} = \frac{L_{2j} - \varphi_y^*\bar{W}_1}{\varphi_y^* + (1-\varphi_y^*)\bar{W}_2}$	$M[\hat{\hat{Y}}_{2j}] = \frac{M[L_{2j}]_{opt.*}}{[\varphi_y^* + (1-\varphi_y^*)\bar{W}_2]^2}$

6. Comparison

The percent relative efficiency of proposed general class of estimator for sensitive population mean $\hat{\hat{Y}}_i$ with respect to the estimator $\hat{\hat{Y}}_{ij}$ have been computed as

$$E_{ji} = \frac{M(Y_{ij})}{M(Y_i)} \times 100\ ;\ \forall\ \ i\ =\ 1\ and\ 2\ and\ j=\ 1,\ 2,\ 3,\ 4,\ 5\ and\ 6. \qquad (25)$$

Remark 6. *In the present paper we have considered additive, multiplicative and upshot of additive and multiplicative type scrambled response models on two wave successive sampling. The three scrambling variable $W_1,\ W_2$ and W_3 used to perturb the true response through randomized or scrambled response models may follow any distribution. Hence, following Pollock and Bek (1976), Eichhorn and Hayre(1983) and Arcos et al.(2015), we consider scrambling variable W_1 to follow normal distribution with mean 1 and variance 1. However, the scrambling variable W_2 has been assumed to follow normal distribution with mean 0 and variance 1 and W_3 has been assumed to follow normal distribution with mean 1 and variance 2.*

7. Numerical Presentation

Population Source:[Priyanka and Mittal (2016)]
The population comprise of $N = 315$ units. Let x and y denote the average monthly expenditure on drug usage by undergraduate students in 2015 and 2016 respectively. However z denote the average monthly pocket money of undergraduate students from all sources. The parameters of considered population are computed as:
$N\ =\ 315,\ S_x^2\ =\ 1.2463\times10^6,\ \ S_y^2\ =\ 2.1926\times10^6,\ \ S_z^2\ =\ 1.4670\times10^7,$
$\bar{X} = 370.5238,\ \bar{Y} = 504.8095,\ \bar{Z} = 4.0233\times10^3,\ \ \rho_{yx} = 0.8937,$
$\rho_{xz} = 0.6491,\ \rho_{yz} = 0.7012.$
The artificial data for $W_1,\ W_2$ and W_3 have also been generated as per assumption in remark 6. It is observed that $\bar{W}_1 = 1.0871,\ S_{W_1}^2 = 0.5832,\ \bar{W}_2 = -0.0248,\ S_{W_2}^2 = 1.1695.$ and $\bar{W}_3 = 0.9731,\ S_{W_3}^2 = 4.4527$
The optimum values of $\hat{\mu}_i's$ for L_i and L_{ji} and percent relative efficiencies E_{ji} have been computed for the above data under two considered models and are presented in Figure 1 to Figure 13 and Table 6.

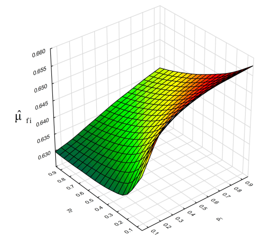

Figure 1: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_1$

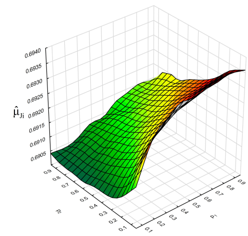

Figure 2: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{11}$

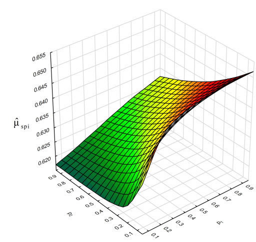

Figure 3: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{12}$

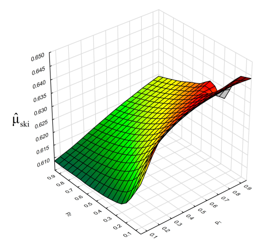

Figure 4: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{13}$

Table 6. Optimum fraction of sample drawn afresh and percent relative efficiencies under scrambled response model. where $\alpha = \varphi_y^* = \varphi_x^*$

i	M_{DP}	$\hat{\mu}_{f2}$	$\hat{\mu}_{J2}$	$\hat{\mu}_{sp2}$	$\hat{\mu}_{sk2}$	$\hat{\mu}_{sp12}$	$\hat{\mu}_{sp22}$	$\hat{\mu}_{sp32}$	E_{12}	E_{22}	E_{32}	E_{42}	E_{52}	E_{62}
	α													
2	0.1	0.6562	0.6935	0.6501	0.6430	0.3782	0.6798	0.6880	142.97	100.93	102.05	105.51	122.06	138.91
	0.3	0.6489	0.6926	0.6414	0.6339	0.3996	0.6694	0.6793	151.88	101.18	102.37	104.13	119.76	137.04
	0.5	0.6413	0.6916	0.6320	0.6242	0.4149	0.6584	0.6706	162.02	101.47	102.74	103.25	117.78	135.40
	0.7	0.6345	0.6908	0.6234	0.6153	0.4240	0.6485	0.6630	171.89	101.78	103.11	102.83	116.30	134.17
	0.9	0.6302	0.6904	0.6179	0.6096	0.4278	0.6423	0.6584	178.46	101.99	103.37	102.70	115.51	133.51

8. Simulation Study

The simulation study have been carried out by considering $10,000$ different samples using Monte Carlo simulation for the data mentioned in section 7. The simulated percent relative efficiency E_{sji} of $\hat{\hat{Y}}_i$ with respect to $\hat{\hat{Y}}_{ij}$; $j = 1, 2, \ldots, 6$ and $i = 1$ and 2 respectively have been computed for many combinations of constants and the results are presented in Table 7.

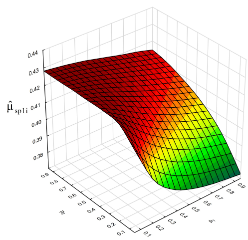

Figure 5: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{14}$

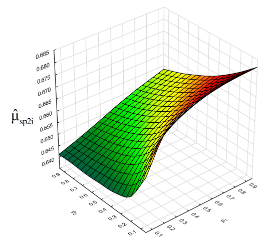

Figure 6: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{15}$

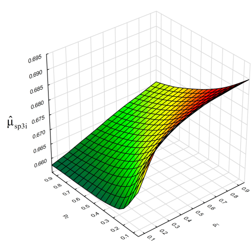

Figure 7: Optimum value of fraction of sample drawn afresh for estimator $\hat{\hat{Y}}_{16}$

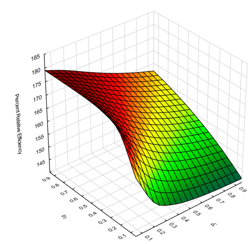

Figure 8: Percent Relative Efficiency E_{11}

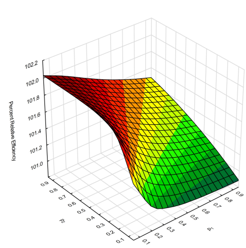

Figure 9: Percent Relative Efficiency E_{21}

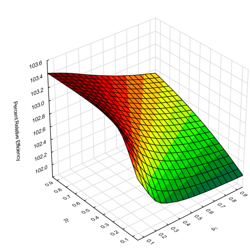

Figure 10: Percent Relative Efficiency E_{31}

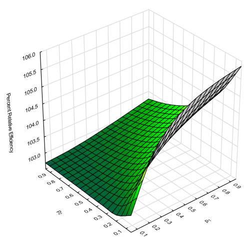

Figure 11: Percent Relative Efficiency E_{41}

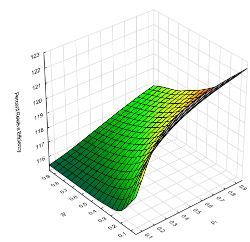

Figure 12: Percent Relative Efficiency E_{51}

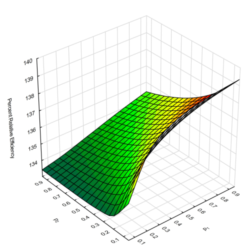

Figure 13: Percent Relative Efficiency E_{61}

Table 7. Simulation results for E_{sji} ; $j = 1, 2, \ldots, 6$; $i = 1$ and 2, $\alpha = \varphi_y^* = \varphi_x^*$

i	Model		E_{s1i}	E_{s2i}	E_{s3i}	E_{s4i}	E_{s5i}	E_{s6i}
1	M_{AR}							
	(p_2)	(p_1)						
		0.1	169.69	100.65	224.69	120.45	131.53	137.86
	0.1	**0.5**	186.63	100.91	196.39	119.78	128.99	135.80
		0.9	188.52	100.93	199.80	120.15	129.17	136.02
		0.1	155.46	100.47	274.59	121.17	134.37	140.23
	0.3	**0.5**	176.95	100.76	210.07	120.13	130.34	136.88
		0.9	183.39	100.85	205.21	120.23	129.75	136.46
		0.1	151.21	100.42	298.82	121.37	135.35	141.05
	0.5	**0.5**	169.71	100.65	227.63	120.60	131.67	137.99
		0.9	177.92	100.78	206.66	119.97	130.08	136.66
		0.1	149.21	100.39	313.36	121.47	135.84	141.47
	0.7	**0.5**	164.75	100.59	230.77	120.36	132.12	138.31
		0.9	173.36	100.71	214.69	120.16	130.79	137.24
		0.1	148.02	100.38	303.94	121.13	135.77	141.37
	0.9	**0.5**	161.38	100.54	258.97	121.24	133.46	139.50
		0.9	169.70	100.65	225.38	120.48	131.56	137.89
2	M_{DP}	φ_y^*						
		0.1	148.01	100.38	295.23	120.92	135.58	141.19
		0.5	169.68	100.66	221.400	120.27	131.36	137.70
		0.9	188.53	100.93	2010.6	120.25	129.27	136.11

9. Scrambling implementation Versus pseudonymous/incognito Questionnaires

As it is familiar that randomized and scrambled response estimators are less efficient than estimators obtained using direct questioning method. Here we discussed that the data is collected by scrambled response which is to be compared with data collected with pseudonymous/incognito questionnaire. For ascertaining the privacy protection additional cost has to be incurred.

In order to evaluate the data scrambling benefits, the estimator under randomized and scrambled response model have been compared with direct questioning method. If no scrambling mechanism have been used at any wave then the similar estimator under direct method is proposed as

$$L_D = \chi L_{uD} + (1-\chi)L_{mD} \; ; \; \chi \; \varepsilon \; [0,\ 1] \tag{26}$$

where

$$L_{uD} = V^* \left(\bar{y}_u,\ \bar{z}_u\right), \tag{27}$$

$$L_{mD} = T^* \left(\bar{y}_m,\ f_1^*,\ f_2^*\right) \tag{28}$$

where, $V^* \left(\bar{y}_u,\ \bar{z}_u\right)$ is a function of $\left(\bar{y}_u,\ \bar{z}_u\right)$ such that

$V^* \left(\bar{Y},\ \bar{Z}\right) = \bar{Y} \Rightarrow V_1^*(K^*) = \frac{\partial V^*(.)}{\partial \bar{y}_u}|_{K^*} = 1$ with $K^* = \left(\bar{Y},\ \bar{Z}\right)$ and $V^*\left(\bar{y}_u,\ \bar{z}_u\right)$ satisfies the following conditions:

1. The function $V^* \left(\bar{y}_u,\ \bar{z}_u\right)$ is continuous and bounded in R.
2. The first, second and third partial derivatives of $V \left(\bar{y}_u,\ \bar{z}_u\right)$ exist and are continu-

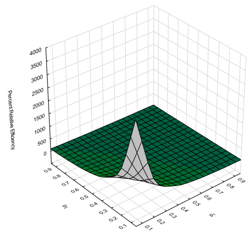

Figure 14: Percent Relative Efficiency E_{1D}

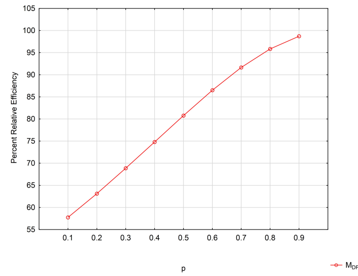

Figure 15: Percent Relative Efficiency E_{2D}

ous and bounded in R.
$T^*\ (\bar{y}_m,\ f_1^*,\ f_2^*)$ is a function of $(\bar{y}_m,\ f_1^*,\ f_2^*)$ such that $f_1^* = g^*\ (\bar{x}_m,\ \bar{z}_m,\ \bar{z}_n),\ f_2^* = h^*\ (\bar{x}_n,\ \bar{z}_n)$ and $T^*\ (\bar{y}_m,\ f_1^*,\ f_2^*)$ is a function of $(\bar{y}_m,\ f_1^*,\ f_2^*)$ such that $T^*\ (\bar{Y},\ \bar{X},\ \bar{X}) = \bar{Y},\ g^*\ (\bar{X},\ \bar{Z},\ \bar{Z}) = h^*(\bar{X},\ \bar{Z}) = \ \bar{X}$ and three functions $T^*,\ g^*,\ h^*$ satisfy the regularity conditions as considered for L_{ui} given in equation 10.
The minimum mean squared error of the class of estimator L_D to the first order approximations is given as

$$M[L_D]_{opt.^*} = \frac{B_{d1}^*\hat{\mu}_d + B_{d2}^*}{\hat{\mu}_d^2 A_{d3}^* - \hat{\mu}_d B_{d3}^* + A_{d1}^*}\left(\frac{S_y^2}{n}\right) \tag{29}$$

where,
$A_{d1}^* = 1-\rho_{xz}^2,\ A_{d2}^* = d_d+1,\ A_{d3}^* = d_d-(H_{d2}^*)^2(F_{d2}^*)^2+2H_{d2}^*F_{d2}^*\rho_{yz},$
$d_d = (F_{d2}^*)^2+(G_{d2}^*)^2(F_{d2}^*)^2+2F_{d2}^*\rho_{yx}+2F_{d2}^*G_{d2}^*\rho_{yz}+2(F_{d2}^*)^2G_{d2}^*\rho_{xz},\ B_{d1}^* = A_{d1}^*A_{d3}^*,$
$B_{d2}^* = A_{d1}^*A_{d2}^*-A_{d1}^*A_{d3}^*,\ B_{d3}^* = A_{d1}^*-A_{d2}^*+A_{d3}^*$ and

$\hat{\mu}_d$ satisfies

$$0\ <\ min\left\{\frac{-C_{d2}^*+\sqrt{C_{d2}^{*2}+C_{d1}^*C_{d3}^*}}{C_{d1}^*},\ \frac{-C_{d2}^*-\sqrt{C_{d2}^{*2}+C_{d1}^*C_{d3}^*}}{C_{d1}^*}\right\} < \ 1 \tag{30}$$

where $C_{d1}^* = B_{d1}^*A_{d3}^*,\ C_{d2}^* = A_{d3}^*B_{d2}^*$ and $C_{d3}^* = A_{d1}^*B_{d1}^*+B_{d3}^*B_{d2}^*$.

$$E_{iD}\ = \frac{M(L_D)_{opt.^*}}{M(\hat{\hat{Y}}_i)_{opt.^*}} \times 100 \tag{31}$$

The percent relative efficiencies have been computed for the data represented in section 7 for different choices of $\{p_1,\ p_2,\ \varphi_y^*\}\,\varepsilon\,\{0.1,\ 0.2,\ 0.3,\ 0.4,\ \ldots,\ 0.9\}$ and are presented in graphical form in Figure 14 to Figure 15 for the two considered models respectively.

10. Demonstration of Results

1. From Figure.1 - Figure.13, following can be concluded.
(i) It can be seen that optimum value of fraction of sample to be drawn afresh exists for all considered estimators under both the randomized and scrambled response model.
(ii) The proposed general class of estimators performs appreciably good in terms of percent relative efficiency under the considered models when compared with other modified estimators L_{ij} ; $j = 1, 2, \ldots, 6$ which are also the members of proposed class of estimators L_i ; $i = 1$ and 2.
(iii) For M_{AR}, it can be seen that for fixed value of p_1 if p_2 increases E_{1j} decreases. However for fixed value of p_2 if p_1 increases E_{1j} also increases.
(iv) Both the models, M_{AR} and M_{DP} are performing almost similar in terms of percent relative efficiency.
(v) The Scrambled response model M_{DP} performs appreciably good in terms of optimum fraction of sample to be drawn afresh than the model M_{AR}.
(vi) The Randomized response model M_{AR} is more general as it provide wider scope to the respondents and moderate optimum fraction of sample to be drawn afresh as well as percent relative efficiency.
(vii) Out of scrambled and randomized response models, M_{DP} is showing stable behaviour as per assumptions of successive sampling.

2. From the simulation result in Table 7 it can be focused that the proposed general class of estimator is efficient than others considered under both randomized and scrambled response model.

3. From Figure 14 and Figure 15 it is indicated that when the proposed estimator is compared with direct method, for some combinations percent relative loss has been observed which is in accordance with the theory as scrambling or randomization procedures in general yields loss in efficiency.

11. Epilogue

The propounded general class of estimator for estimating sensitive population mean at current wave under considered scrambled and randomized response models accomplishes good percent relative efficiency when proposed general class of estimator L_i is compared with modified estimators L_{ij} ; $j = 1, 2, \ldots, 6$ and $i = 1$ and 2. Out of the two considered techniques, the model under scrambled response technique proves more stable in context of successive sampling with proposed estimator on two successive waves. However, depending on the sensitive nature of the character under study the two available techniques can be explored with the proposed general class of estimator. Therefore, depending on the given situation the scrambled or randomized response models may be selected to be applied with proposed general class of estimators on successive waves.

Acknowledgements

Authors are thankful to honourable reviewers for deeply reading the paper which lead to improvement over the earlier version of the paper. Authors are also thankful to SERB, New Delhi, India for providing the financial assistance to carry out the present work.

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