CPI INFLATION IN AFRICA: FRACTIONAL PERSISTENCE, MEAN REVERSION AND NONLINEARITY

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ABSTRACT

Price stability has been one of the key mandates that apex monetary authorities strive to achieve globally. While most developed economies have achieved single digit inflation rates, most developing economies, especially African countries still experience alarming double-digit inflation rates. This paper therefore examined the dynamics of inflation in sixteen African countries. We employed the fractional persistence framework with linear trend and non-linear specifications based on Chebyshev’s polynomial in time. The results indicated nonlinear time trend in inflation for most of the countries. With the exception of Burkina Faso, which exhibited plausibility of naturally reverting to its mean level, the majority of the selected African countries would require stronger interventions to revert their observed inflationary levels to their mean levels.

Key words: Africa, Fractional Integration, Inflation Rate, Mean Reversion, Nonlinear Trend, Structural Break.

JEL: 22.

1. Introduction

The persistent increase in the general price level of goods and services in an economy over a period of time is a feat that cannot be ruled out during the process of policy formation with regards to economic activities. This being the result of the attendant consequence of its possible impacts and effects, either positively or negatively, on the purchasing power of the economy’s medium of exchange and unit of account within the economy (Paul et al., 1973). On the negative impact, the general price increase could lead to commodity scarcity, increased opportunity cost of holding money, investment drought as a result of

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uncertainty of future prices. Positively, it reduces the real burden of public and private debt, reduces unemployment due to nominal wage rigidity and provides monetary authorities with a tool to stabilize the economy, since interest rates are nominally kept above zero (Mankiw, 2001). The severity of this general price level could be low or moderate fluctuations in the real demand for goods and services, or changes in available supplies, especially, during scarcity. Consensually, a long sustained period of inflation is perceived to be the outcome of a faster growth rate of money supply in comparison with economic growth rate. Rather than a zero or negative inflation, a low inflation is preferred as it reduces the severity of economic recession by enabling the labour market to adjust more quickly in a downturn and, consequently, reducing the risk of liquidity trap, which may prevent monetary authorities from performing its stabilizing role for the economy (Svensson, 2003).

The literature is replete with various country specific studies that tried to investigate the dynamics of inflation. These include studies on Nigeria (Adenekan and Nwanna, 2004; Odusanya and Atanda, 2010; Imimole and Enoma, 2011; Bawa et al., 2016); Ethiopia (Wolde-Rufael, 2008); Ghana (Adu and Marbuah, 2011); South Africa (Nell, 2000, 2006; Hodge, 2002, 2006, 2009; Fedderke and Schaling, 2005; Burger and Du Plessis, 2006; Burger and Marinkov, 2005; Vermeulen, 2015); Egypt (Ali, 2011; Osama, 2014; Osama, 2014); Kenya (Kaushik, 2011; Kimani and Mutuku, 2013; Kirimi, 2014); Cameroun (Tabi and Ondoa, 2011). These studies basically focused on the impact of inflation on economic growth, local currency value, money supply and stock prices. In some other studies, inflation thresholds of 6% and 9%, respectively, were obtained for Nigeria (Fabayo and Ajilore, 2006; Ajide and Olukemi, 2010), while Phiri (2013) obtained inflation threshold of 22.5% for Zambia. Methodologically, Fielding et al., 2004, and Mikkelsen and Peiris, 2005, employed VAR in their study of inflation. Barnichon and Peiris, 2008, employed the heterogeneous panel cointegration methodology and established the significant role of the output gap and the real money gap on the evaluation of inflation, with the money gap playing a larger role. Caporale, Carcel and Gil-Alana (2015) investigated inflation persistence and nonlinearity using fractional integration approach in five African countries such as Angola, Botswana, Lesotho, Namibia and South Africa and found nonlinear persistence in the case of Angola and Lesotho, while linear persistence was found in the remaining three countries. Boateng et al. (2017) investigated inflation persistence in Ghana and South Africa by using CPI inflation. The authors applied the fractional autoregressive moving average model with heteroscedasticity innovations. The results obtained showed evidence of mean reverting persistence with asymmetric effects of shocks on the conditional mean of CPI inflation of the two countries. In further enhancing inflation forecast precision, the incorporation of the mixed data sampling methodology was suggested (see Salisu and Ogbonna, 2017). Although this is yet to be applied in the African context, it has proven to significantly improve an economy's inflation prediction, especially with regards to OECD member countries.

Careful study on the dynamics of inflationary process in Africa will therefore help in the choice of policy models and estimation methods. There is still an ongoing debate on whether the inflation rate in Africa is a stationary I(0) or nonstationary I(1) process. As our contribution in this paper, we carried out
extensive time series analysis using fractional integration framework to investigate African inflationary persistence and nonlinearity.

The methodological approach in this paper is hardly applied in investigating inflation dynamics and other economic time series. Following Granger and Hyung (2004) and Ohanissian et al. (2008), a process follows long range dependency if over a long time span, far apart observations are still strongly correlated with the current observations. The time processes in mean reverting series are not integrated of order 1 (non-stationary) but are integrated of fractional order less than 1 and the test of fractional order confirms that the I(1) hypothesis should be rejected. In economic theory, mean reversion means that the series can still revert itself to its mean level after the initial shock on the economy, as propelled by a high inflation rate. Nonstationarity in inflation rates means that shocks to inflation have a permanent effect and strong policies would be required by monetary/economic agencies to revert the inflation rate back to its mean level. Stationary or mean reverting inflation means that inflation incurs a lower cost for the monetary/economic agency in the pursuit of monetary policies (Cecchetti and Debelle, 2006). Stationarity/nonstationarity of the inflation rate is controversial, while many authors believe that the series follow I(0) stationary process based on the fact that the generating time series is log-price I(1). Other authors are of the opinion that the series is nonstationary I(1), and it should be included in the system of cointegrating variables (Gil-Alana, Shittu and Yaya, 2012). Using fractional persistence, inflation is neither I(0) nor I(1) but I(d), where d is a value between 0 and 1. Noting that long memory models overestimate the degree of persistence of the series in the presence of structural breaks (Ben Nasr et al., 2014; Gil-Alana, Cunado and Gupta, 2015), and also, with the availability of long time series for many countries, these are very likely. Thus, we supplement our long memory model to accommodate for nonlinear deterministic trends as in Cuestas and Gil-Alana (2016)\(^5\). The approach employed Chebyshev polynomials in cosine function of time up to third orders in fractional persistence framework to determine nonlinearity in time series, in a smooth fashion, rather than an abrupt fashion as in Gil-Alana (2008).

Gil-Alana, Shittu and Yaya (2012) analyzed Nigerian inflation rates using long range dependence in fractional integration incorporating structural breaks. In their results, they observed long memory behaviour in inflation rates with different periods of breaks. Gil-Alana, Yaya and Solademi (2016) examined unit roots, structural breaks and nonlinearity in inflation rates in G7 countries. Based on classical unit root decisions, the authors first observed inclusive results in the stationarity/nonstationarity of inflation rates in these countries. A test based on fractional unit root analysis showed nonstationarity I(1) process for inflation rates in the case of UK, Canada, France, Japan and the US, while in the case of Italy, evidence of I(d > 1) was observed and in the case of Germany, mean reversion was observed.

Specifically, in this paper, we investigate long range dependency, mean reversion and nonlinearity in inflation dynamics of African countries using fractional persistence approach. This paper is the first, among many, investigating

\(^5\) It is a known fact that fractional persistence, nonlinearities and structural breaks are closely related properties in time series (see Diebold and Inoue, 2001; Kapetanios, Shin and Snell, 2003; Granger and Hyung, 2004; Gil-Alana, Cunado and Gupta, 2015).
unit root in African CPI inflation and inflation rates. The findings clearly expose readers to nonlinear dynamics of CPI, which has effect on the construct of cointegrating econometric models involving CPI inflation. Importance of time dynamics of inflation to monetary policy agents in Africa also gingered this write up.

Following the introductory section, the rest of the paper is presented as follows: Section 2 presents fractional persistence framework for nonlinear deterministic trend. Section 3 presents the results and discussion while Section 4 renders concluding remarks and policy implications.

2. Methodology

A time series process \( \{ y_t, t = 0, \pm 1, \pm 2, \ldots \} \) is integrated of order zero, \( I(0) \), if it is a covariance stationary process with a spectral density function that is positive and finite at zero frequency. Thus, a process is integrated of order \( d \) if it can be represented as,

\[
(1 - B)^d y_t = u_t, \quad t = 0, \pm 1, \pm 2, \ldots
\]  

(1)

with \( y_t = 0 \) for \( t \leq 0 \), and \( d > 0 \) where \( B \) is the backward shift operator such that \( By_t = y_{t-1} \) and \( u_t \) is \( I(0) \) process. The parameter \( d \) therefore determines the size of differences needed to render a series stationary \( I(0) \). Recall, in the case of classical unit integration, \( d \) is restricted as integer, while in fractional persistence, a much richer degree of flexibility in the values of \( d \) is allowed.

A very appealing case of fractional persistence is \( I(0 < d < 0.5) \) time series process, known as long memory. In the sense that the spectral density function of the process is unbounded at the lowest frequency. By a way of time domain definition, let \( \gamma(h) = (y_t, y_{t+h}) \) be the autocovariances at lag \( h \) of the stationary process \( \{ y_t; t \in \mathbb{Z} \} \), then the autocovariance of such long memory process is unbounded and infinite, that is,

\[
\sum_{h=-\infty}^{\infty} \left| \gamma(h) \right| = \infty
\]

(2)

Thus, in terms of hyperbolic decay of autocovariances,

\[
\gamma(h) \sim h^{2d-1} \ell_1(h)
\]

(3)
As $h \to \infty$ and $\ell_1(h)$ is a slowly varying function.$^6$

The values of fractional $d$ have many implications from both economic and statistical viewpoints. For example, if $d = 0$ in equation (1), $y_t = u_t$, the process $y_t$ is then said to be $I(0)$, stationary with autocovariances decaying exponentially. For $0 < d < 0.5$, as in long memory process, the autocovariances as well as autocorrelations decay at much slower and hyperbolic rates compared to when $d = 0$. For $0.5 < d < 1$, $y_t$ becomes nonstationary as the variance of the partial sums increases in magnitude. In economic terms, $d < 1$ implies that the series is “mean reverting”, in the sense that shocks to the series disappears in the long run, and the series reverts back to its mean level. For $d \geq 1$, this is a nonstationary stance, where the effect of any shocks to the series persists forever.

Actually, the mean reversion case is relevant in the context of the inflation rate, since shocks imparts differently in the short and long run, depending on the value of the fractional differencing parameter $d$.

Robinson (1994) incorporates equation (1) into the conventional regression model of the form,

$$x_t = \alpha + \beta t + y_t, \quad (1 - B)^d y_t = u_t, \quad t = 0, \pm 1, \pm 2, \ldots \quad (4)$$

with equation (1), where $x_t$ is now the observed time series, $\alpha$ and $\beta$ are the coefficients corresponding to the intercept and a linear time trend. Since $u_t$ is $I(0)$, this allows the usage of a Whittle function in the frequency domain to compute the estimates of $\alpha$, $\beta$ and the fractional differencing parameter $d$ as well as the confidence intervals of the estimates. The approach tests the null hypothesis,

$$H_o : d = d_0 \quad (5)$$

in equations (1) and (4) for a grid of real values $d_0$. Thus, the null model tested is,

$$x_t = \alpha + \beta t + y_t, \quad (1 - B)^{d_0} y_t = u_t, \quad t = \pm 1, \pm 2, \ldots \quad (6)$$

with $I(0)$ disturbances.

By considering the effects of structural breaks on the time series under investigation, we considered the smooth change rather than the abrupt change

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$^6$ A positive measurable function defined on some neighbourhood $[a, \infty)$ of infinity is said to be slowly varying in Karamata’s sense if and only if for any $c > 0$, $\ell_1(cx)/\ell_1(x)$ converges to 1 as $x$ tends to infinity (Palma, 2007).
implied by structural breaks. The Chebyshev polynomials in time were first used in the context of unit root in Bierens (1997) since the function is bounded and orthogonal, in cosine function of time. In the context of fractional persistence, Cuestas and Gil-Alana (2016) made the proposition. The testing regression framework is of the form,

\[ x_t = \sum_{i=0}^{m} \theta_i P_{IN}(t) + y_t; \quad t = \pm1, \pm2, \ldots \]  

(7)

where \( m \) is the order of the Chebyshev polynomials and \( P_{IN}(t) \) is the Chebyshev polynomial, given as,

\[ P_{IN}(t) = \sqrt{2} \cos \left[ i\pi \left( t - 0.5 \right) / N \right], \quad t = 1, 2, \ldots, N; \quad i = 1, 2, \ldots \]  

(8)

with \( P_{0N}(t) = 1 \) (see Gil-Alana, Cunado and Gupta, 2015; Yaya, Gil-Alana and Carcel, 2015; Gil-Alana, Yaya and Solademi, 2016 and Caporale, Carcel and Gil-Alana, 2017 for some applications). Now, incorporating equation (8) in equation (7) with equation (1), we obtain simultaneously the fractional persistence estimate \( d \) along with nonlinear parameters \( \theta_0, \theta_1, \theta_2, \ldots, \theta_m \). For \( m = 0 \), the entire model system contains only an intercept and a linear trend, while \( m \geq 1 \) indicates a nonlinear model.

By restricting ourselves to a case where \( m = 3 \), we have \( \theta_0, \theta_1, \theta_2 \) and \( \theta_3 \). Nonlinearity is observed when at least one of \( \theta_1, \theta_2, \) and \( \theta_3 \) is significant. The estimates of the regression here are tested using a Lagrange Multiplier (LM) test of the same form as in Robinson (1994).

**Results and discussion**

The data considered in this paper are the monthly consumer price index (CPI) of some selected African countries, obtained from the International Monetary Fund (IMF) website. The countries include: Burkina Faso (BKF), Cameroon (CAM), Cote D’Ivoire (COTE’D), Egypt (EGY), Ethiopia (ETH), Gambia (GMB), Ghana (GHN), Kenya (KNY), Madagascar (MADA), Mauritania (MAU), Morocco (MOR), Niger (NIGR), Nigeria (NGR), Senegal (SEN), South Africa (SA) and Swaziland (SWA). The series under consideration spans a 48-year period between January 1969 and December 2016, amounting to a sample size of 576.

Plots of CPI inflation are given in Figure 1. Clearly, we observe an increasing trend in all the 16 plots with price stability between 1970 and 1975. There is a general upward movement of CPI in almost all the countries, with some noticeable upward shift around 1995 for some countries (see Burkina Faso (BKF), Cameroun (CAM), Cote D’Ivoire (COTE’D), Kenya (KNY), Niger (NIGR), Nigeria (NGR) and Senegal (SEN)). Between 1993 and 1994, CAM, COTE’D, NIGER, SEN and SA experienced a sharp increase in CPI, while late 2010 marked the

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7 Ouliaris et al. (1989) proposed regular polynomials to approximate GDP data generating process.
index base year for all the countries, since this allows for easier comparison
among the selected countries in Africa

We consider the case of fractional persistence with linear trend, as proposed
in Robinson (1994), using white noise, $AR(1)$ and seasonal $AR(1)$
disturbances. The results are given in Table 1. In the three cases, the estimates
of persistence $d$ are significant throughout, except in the cases of MADA, MAU
and SA, where there were no convergence under $AR(1)$ disturbances. In the
case of the white noise disturbance assumption, only BKF indicated tendency of
mean reversion (i.e. $d < 1$), while explosive behaviours of $d > 1$ were observed
in the cases of CAM, COTE’D, EGY, ETH, GMB, GHN, KNY, MADA, MAU, NIGR,
NGR, SEN and SA. Evidence of unit root was observed for MOR and SWA. By
considering $AR(1)$ disturbances, mean reversion is found in BKF, COTE’D,
MOR and SEN, and evidence of $d > 1$ is found in CAM, EGY, ETH, GMB, GHN,
KNY, NIGR, NGR and SWA. With seasonal $AR(1)$ disturbance, both BKF and
MOR indicated mean reversion, while NIGR and SWA exhibited unit roots. The
remaining countries experienced explosive behaviour of $d > 1$. Judging by the
fractional persistence with linear trend result alone, one would conclude that
inflation rates in the selected African countries might continue to drift farther from
their mean level, without any possibility of naturally reverting to their mean level.
By considering nonlinear deterministic trend in the fractional persistence framework, thereby testing fractional persistence simultaneously with nonlinearity in CPI inflation, we found only BKF to experience mean reversion (see Tables 2 and 3 for \( m = 3 \) and \( m = 2 \), respectively), while unit roots were found in MOR and SWA, and the behaviours of the remaining 13 countries were found to be explosive. These results are summarized in Table 4. In terms of nonlinearity for \( m = 3 \), we found significant evidence of nonlinearities in BKF, CAM, COTE’D, EGY, GMBIA, MAU, MOR, SEN, SA and SWA, while evidence of linearity was found in ETH, GHN, KNY, MADA, NIGR and NGR. Using \( m = 2 \), we observed
reduction in nonlinearity detection, as we observed BKF, CAM, COTE’D, MAU, MOR, SEN and SWA to be nonlinear. These results are summarized in Table 5.

Table 2. Nonlinear Fractional persistence based on Chebyshev Inequality for \( m = 3 \)

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>( \hat{d} )</th>
<th>( \hat{\theta}_0 )</th>
<th>( \hat{\theta}_1 )</th>
<th>( \hat{\theta}_2 )</th>
<th>( \hat{\theta}_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKF</td>
<td>0.9082***a [0.0358]</td>
<td>-0.0118 [11.43]</td>
<td>-29.8581*** [5.104]</td>
<td>0.8418 [2.826]</td>
<td>-3.6135' [1.962]</td>
</tr>
<tr>
<td>CAM</td>
<td>1.1124***b [0.0340]</td>
<td>0.0830 [15.36]</td>
<td>-33.8404*** [8.870]</td>
<td>0.6447 [3.900]</td>
<td>-1.9529 [2.484]</td>
</tr>
</tbody>
</table>

Note: Each cell contains the estimated coefficient with corresponding standard errors given in squared brackets. 'a' indicates evidence of I(d) with \( d < 1 \), while 'b' indicates evidence of I(d) with \( d > 1 \). ***., ** and * denote statistical significance at 1%, 5% and 10% levels, respectively.
Table 3. Nonlinear Fractional persistence based on Chebyshev Inequality for \( m = 2 \)

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>( \hat{d} )</th>
<th>( \hat{\theta}_0 )</th>
<th>( \hat{\theta}_1 )</th>
<th>( \hat{\theta}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKF</td>
<td>0.9243***a [0.0344]</td>
<td>-3.0395 [13.37]</td>
<td>-30.1741*** [5.620]</td>
<td>0.8907 [3.058]</td>
</tr>
<tr>
<td>GHN</td>
<td>1.3496***b [0.0301]</td>
<td>-78.8250 [56.70]</td>
<td>-33.0979*** [9.046]</td>
<td>0.6723 [3.965]</td>
</tr>
<tr>
<td>MADA</td>
<td>1.6008***b [0.0506]</td>
<td>-79.8863 [120.1]</td>
<td>27.8198 [83.53]</td>
<td>3.6559 [23.97]</td>
</tr>
<tr>
<td>MOR</td>
<td>0.9897***a [0.0444]</td>
<td>-8.5827 [52.72]</td>
<td>-31.9241*** [3.632]</td>
<td>-2.9077 [1.834]</td>
</tr>
<tr>
<td>NGR</td>
<td>1.1170***b [0.0408]</td>
<td>-22.4837 [32.83]</td>
<td>-25.1671 [19.98]</td>
<td>0.3935 [8.769]</td>
</tr>
<tr>
<td>NIGR</td>
<td>1.3788***b [0.0304]</td>
<td>-71.5437 [54.34]</td>
<td>15.5853 [37.25]</td>
<td>10.2329 [12.66]</td>
</tr>
<tr>
<td>SEN</td>
<td>1.1569***b [0.0415]</td>
<td>-6.9309 [27.40]</td>
<td>-29.1434* [17.45]</td>
<td>-2.4043 [7.320]</td>
</tr>
</tbody>
</table>

Note: Each cell contains the estimated coefficient with corresponding standard errors given in squared brackets. ‘a’ indicates evidence of I(d) with \( d < 1 \), while ‘b’ indicates evidence of I(d) with \( d > 1 \).

***, ** and * denote statistical significance at 1%, 5% and 10% levels, respectively.

Table 4. Summary of the Results in terms of value of \( d \)

<table>
<thead>
<tr>
<th>Mean reversion ((d &lt; 1))</th>
<th>Unit roots ((d = 1))</th>
<th>Explosive behaviour ((d &gt; 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise disturbances</td>
<td>BKF</td>
<td>MOR, SWA</td>
</tr>
<tr>
<td>AR (1) Disturbances</td>
<td>BKF, COTE’D, MOR, SEN</td>
<td>MADA, MAU, SA</td>
</tr>
<tr>
<td>Seasonal AR (1) Disturbances</td>
<td>BKF, MOR</td>
<td>NIGR, SWA</td>
</tr>
<tr>
<td>Nonlinear trend with ( m = 3 )</td>
<td>BKF</td>
<td>MOR, SWA</td>
</tr>
<tr>
<td>Nonlinear trend with ( m = 2 )</td>
<td>BKF</td>
<td>MOR, SWA</td>
</tr>
</tbody>
</table>
Table 5. Summary of the Results in terms of Nonlinearities

<table>
<thead>
<tr>
<th>m = 3</th>
<th>Evidence of Nonlinearities</th>
<th>No Evidence of Nonlinearities</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKF, CAM, COTE’D, EGY, GMB, MAU, MOR, SEN, SA, SWA</td>
<td>ETH, GHN, KNY, MADA, NIGR, NGR</td>
<td></td>
</tr>
<tr>
<td>m = 2</td>
<td>BKF, CAM, COTE’D, MAU, MOR, SEN, SWA</td>
<td>EGY, ETH, GMB, GHN, KNY, MADA, NIGR, NGR, SA</td>
</tr>
</tbody>
</table>

4. Concluding Remarks and Policy

In this paper, we have examined time behaviour of African inflation using CPI as an inflation proxy variable. We considered 16 African countries, with data in monthly frequency, spanning between January 1969 and December 2016. We considered memory property, fractional persistence and nonlinearity using a newly developed approach of Cuestas and Gil-Alana (2016). The main results indicated that African inflationary dynamics are mostly explosive and nonlinear, and strong policy intervention is required to bring inflation back to original trend levels in each of these countries. Thus, mean reversion is likely to occur in CPI inflation of Burkina Faso. In the choice of methodology for analyzing inflation in Africa, this work recommends careful selection of the estimation approach, particularly, in countries where nonlinearities are detected.

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