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Triads or tetrads? Comparison of two methods for measuring the similarity in preferences under incomplete block design

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ABSTRACT

The measurement of preferences can be based on historical observations of consumer behaviour or on data describing consumer intentions. In the latter case, the measure-ment of preferences is performed using methods which express consumer attitudes at the time of research. However, most of these methods are very laborious, especially when a large number of objects is tested. In such cases incomplete analyses may prove useful. An incomplete analysis involves the division of objects into subgroups, so that each pair of objects appears at exactly the same frequency and all objects are in each subgroup.
The purpose of the work is to compare two incomplete methods for measuring the similarity of preferences, i.e. the triad method and the tetrad method. These methods can be used whenever similarities are measured on an ordinal scale. They have been com-pared in terms of their labour intensity and ability to map the known structure of ob-jects, even when all pairs of objects in subgroups cannot be presented equally frequent-ly.

Key words: measurement of preferences, triads, tetrads, multidimensional scaling.

1. Introduction

Preferences represent the basic concept in the theory of economics and, in particular, in the consumer choice theory. They reflect consumers' attitudes developed in the process of mutual interactions between consumers and their environment. They take the form of a binary relationship based on axiomatic properties of reflexivity, transitivity and consistency (e.g. Varian, 2005, pp. 63–64). Even though the relationship of preferences is very easy to determine experimentally (e.g. using a questionnaire survey), the measurement aimed at quantifying preferences is a problematic one. There are no precise and unambiguous definitions of many concepts, therefore it is difficult to measure both the intensity and the level of the conditions described by these concepts.

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An important tool in the study of the similarities of preferences is nonmetric multidimensional scaling, which is a technique for the analysis of similarity (or dissimilarity) data on a set of *n* objects (see, e.g. Borg and Groenen, 2005). Multidimensional scaling produces a multidimensional geometrical representation of objects in a low dimensional space (this is usually a two or three-dimensional space), where relationships between the objects correspond to geometric relationships of points representing objects on the perceptual map. In the nonmetric multidimensional scaling, dissimilarities are measured on the ordinal scale. In this case, given the dissimilarities δ_{ij} and δ_{kl} of two object pairs (O_i, O_j) and (O_k, O_l) from the set of *n* objects $O = (O_1, O_2, \ldots, O_n)$, the researcher is only interested which of the two dissimilarity δ_{ij} and δ_{kl} is greater (or smaller).

There are two ways of obtaining input dissimilarities in multidimensional scaling. When they are directly obtained from empirical subjective measurements of objects performed by subjects, they are called direct dissimilarities. By contrast, when they are not obtained from subjects, but calculated from a data matrix associated with these objects, they are labeled as derived dissimilarities. This article focuses only on direct dissimilarities.

When the number of objects is high, the number of direct assessments made by respondents becomes too large, and makes the dissimilarities task more difficult. In this article, two incomplete methods are proposed to solve this problem in order to make the similarity task easier, while keeping satisfactory scaling solutions. These methods are the method of triads and the method of tetrads. The idea of the presented methods is based on the theory of balanced incomplete block designs (see, e.g. Burton and Nerlove, 1976; Rink, 1987; Morris, 2010, pp. 109–111). The method of tetrads is an original proposal, the idea of which is based on the method of triads. These methods will be compared due to their labor intensity and the ability to map the known structure of preferences.

2. The methods of collecting preferences similarity data

The most important decision to be taken at the initial stage of preference scaling is the selection method for measuring similarities. So far, many more or less popular and widely used methods of direct similarities measurement have been developed (see, e.g. Bijmolt, 1996, pp. 30-31; Zaborski, 2001, pp. 40–43). There are three main approaches to collecting input similarities. The first approach is based on rankings and similarity ratings of the pairs of objects, the second uses grouping and sorting tasks in order to calculate similarities, and the third approach consists of pairwise comparisons of similarities. Some of them, suggested in the literature, are presented in Table 1.

Table 1. The methods of collecting similarities data

Method	Description
Sorting	The subject has to sort the objects into a number of groups, with relatively similar objects in each group
Paired comparisons	For all pairs of objects the subject has to indicate the most preferred object
Ratings	The subject has to rate each pair of objects on an ordinal scale, where the extreme values of the scale represent the maximum dissimilarity and maximum similarity of preferences
Ranking	The subject has to arrange the objects from the most to the least preferred
Ranking of pairs	The subject is requested to arrange all possible pairs of objects in order of decreasing similarity of preferences
Pick *k* out of *n*	The subject is asked to pick a number of objects which s/he considers most similar to a particular reference object. This process has to be done several times while rotating the reference object
Conditional ranking	One object is presented to the subject as a reference object, and the remaining objects have to be ordered on the basis of their preference similarity with the reference object. Each of the objects is in turn presented as the reference
Dyads	For each pair of pairs of objects (dyad) the subject has to select a more similar pair of the two
Triads	The subject has to indicate which objects of combinations of tree objects form the most similar pair, and which form the least similar pair

Source: Zaborski (2017).

The differences in the application of various measurement methods may result from the number of objects simultaneously presented to the respondents (e.g. in the method consisting in ranking, sorting or conditional ordering of similarities the respondents simultaneously assess all objects, while in the course of pairwise comparison or triad method, only two or three objects are presented in a sequence), the difficulty in assessing similarities (e.g. ordering for the entire set, especially with a large number of objects, is more problematic than selecting the preferred object from two or three items) and the total number of required ratings (in the case of ranking, it is just one assessment, and, e.g. for the triad method the number of assessments is a cubic function of the number of objects).

Table 2. Effects of the similarity data collection methods

Effects	Preference collection methods				
	ST	RT	CR	RN	TR
Subjective feelings:					
Fatigue	++	+	–	++	–
Boredom	+	+	–	+	–
Ease of expressing preferences	+	+	+	+/–	+
Command clarity	+	+	+	+	+
Preference judgements:					
Completion time	++	+	+/–	++	–
Missing values	+	++	+/–	+/–	–
Preference scaling results:					
Goodness of fit to the data	+	+	+	+	+
Recovery of known distances	–	+	+	+/–	+

Explanation: ST – sorting, RT – ratings, CR – conditional ranking, RN – ranking, TR – triads, ++ = very good, + = good, +/– = medium, – = poor
Source: own work based on Bijmolt, 1996, pp. 41-48; Zaborski, 2003.

The selection of a method affects subjective feelings of the respondents, i.e. fatigue, weariness resulting from making numerous assessments, or difficulties in expressing similarity assessments. As a result, the collected data may be incomplete or assessments which do not always fully reflect the respondents' attitudes may occur. Table 2 presents the impact of different preference collecting methods on subjective feelings of respondents, preference judgements and preference scaling results. It shows that by using methods that are not labor intensive, i.e. sorting or ranking, we are not able to fully reproduce the known structure of preferences. With ranking procedures, the respondent may become frustrated if asked to rank many more objects, and he/she may skip the question or select the most and least preferred, ignoring the rest. On the other hand, paired comparison methods require a large number of observations. When the number of objects becomes large, deriving all pairs can become tedious and time-consuming. The respondent may become tired answering the large number of paired comparisons that are necessary to collect similarity data. In such cases, incomplete tests may be helpful. The triad and tetrad methods presented in this paper under the incomplete block design allow for a significant reduction of the above-mentioned limitations, resulting from the use of other methods included in Table 1.

3. Presentation of methods

In the method of triads (see Roskam, 1970; Burton and Nerlove, 1976) the subject is asked to consider all possible groups of three objects (O_i, O_j, O_k) ($i, j, k = 1, 2, \ldots, n$, where $i \neq j \neq k \neq i$) at a time, taken from the full set of n objects $O = (O_1, O_2, \ldots, O_n)$. The subject has to indicate which two objects of each combination form the most similar pair, and which two objects form the least similar pair. On this basis the triad is formed, where the most similar objects are placed as the first and the second, and the least similar as the first and the third one. For example, if (O_i, O_j) is the most similar pair and (O_j, O_k) is the least similar pair, the triad is (O_j, O_i, O_k).

In the method of tetrads the respondent also has the task to indicate the most similar pair and the least similar pair, but for all possible groups of four objects (O_i, O_j, O_k, O_l) $i, j, k, l = 1, 2, \ldots, n$, where $i \neq j \neq k \neq l \neq i \neq k$ and $j \neq l$. On this basis the tetrad is formed, where the most similar objects are placed as the first and the second, and the least similar as the first and the fourth one. For example, if (O_i, O_j) is the most similar pair and (O_j, O_l) is the least similar pair, the tetrad is (O_j, O_i, O_k, O_l). If the object from the most similar pair (O_i, O_j) is not present in a pair of the least similar objects then the most similar objects are placed as the second and the third. In this situation one should also ask the respondent to indicate the second most similar pair of objects. For example, if (O_i, O_j) is the most similar pair, (O_i, O_k) is the second similar pair and (O_k, O_l) is the least similar pair, the tetrad is $(O_k, O_i, O_j, O_l)^*$.

Although the advantage of the methods presented above is a relative simplicity of the judgments required of the subjects, so they can be useful techniques for preference data collection, the number of triads and tetrads increases very rapidly with the number of objects. The number of ratings which a respondent must make for n objects in the method of triads is equal to the number of three element combinations of n-element set and it amounts to:

$$C_n^3 = \frac{n(n-1)(n-2)}{6}. \tag{1}$$

For tetrads it is a four element combinations of n-element set:

$$C_n^4 = \frac{n(n-1)(n-2)(n-3)}{24}, \tag{2}$$

so beyond about n=7, the presentation of the full sets becomes totally unfeasible and very laborious for the subject.

When the number of triads or tetrads is considered too large to be practical, according to the theory of balanced incomplete block designs, it can be reduced in such a way that all pairs of objects are presented equally frequently, but less than their potential maximum number. If λ denotes the number of three or four-elements combinations (blocks) in which each pair of objects occurs, than the reduced number of blocks L_λ must satisfy both of these defining relations (see, e.g. Rink, 1987):

$$\begin{cases} nr = kL_\lambda \\ (n-1)\lambda = (k-1)r \end{cases}, \tag{3}$$

where:

k is the number of objects in one block (k=3 for triads and k=4 for tetrads),

r is the number of replication of each object in the reduced blocks,

$\lambda=1,\ldots, n-2$ for triads,

$\lambda=1,\ldots, (n-1)(n-2)/2$ for tetrads.

According to the equations (3), the number of incomplete blocks in the method of triads is equal:

$$L_\lambda = C_n^3 \frac{\lambda}{n-2} = \frac{\lambda n(n-1)}{6}, \tag{4}$$

and in the method of tetrads:

$$L_\lambda = C_n^4 \frac{2\lambda}{(n-2)(n-3)} = \frac{\lambda n(n-1)}{12}. \tag{5}$$

The number of triads and tetrads for different values of λ and n is shown in Table 3. Because it is not possible to define a reduced number of blocks for all combinations of λ and n, not all the cells in Table 3 are filled.

Table 3. The number of triads and tetras for different values of λ and n

n	**Triads**						**Full set of triads**	**Tetrads**						**Full set of tetrads**
	λ							**λ**						
	1	**2**	**3**	**4**	**5**	**6**		**1**	**2**	**3**	**4**	**5**	**6**	
6	–	10	–	20	×	×	20	–	–	–	–	–	15	15
7	7	14	21	28	35	×	35	–	7	–	14	–	21	35
8	–	–	–	–	–	56	56	–	–	14	–	–	28	70
9	12	24	36	48	60	72	84	–	–	18	–	–	36	126
10	–	30	–	60	–	90	120	–	15	–	30	–	45	210
11	–	–	55	–	–	110	165	–	–	–	–	–	55	330
12	–	44	–	88	–	132	220	–	–	33	–	–	66	495
13	26	52	78	104	130	156	286	13	26	39	52	65	78	715
14	–	–	–	–	–	182	364	–	–	–	–	–	91	1001
15	35	70	105	140	175	210	455	–	–	–	–	–	105	1365
16	–	80	–	160	–	240	560	20	40	60	80	100	120	1820
17	–	–	136	–	–	272	680	–	–	68	–	–	136	2380

Source: own work.

For both methods it is possible to enter the judgement on paired comparisons into a similarity matrix. The creation of the triangular similarity matrix is possible by giving the pair of objects the number of points, which is equal to the number of pairs in a block, for which it can be assumed that the similarity is smaller than the similarity of a given pair. The number of points assigned to pairs from the set of hypothetical blocks (triads and tetrads) marked with the consecutive letters of the alphabet is presented in Table 4 and Table 5.

Table 4. Number of points for pairs in example triad

Objects	Most similar pair	Least similar pair	Triad	Number of points for pairs in triads		
ABC	AB	AC	ABC	AB=2	AC=0	BC=1

Source: own work.

Table 5. Number of points for pairs in example tetrads

Objects	Most similar pair	Least similar pair	Tetrad	Number of points for pairs in tetrads					
ABCD	AB	AD	ABCD	AB=5	AC=1	AD=0	BC=3	BD=1	CD=3
ABCD	AB	CD	CABD*1)	AB=5	AC=4	AD=1	BC=1	BD=3	CD=0
			CBAD*2)	AB=5	AC=1	AD=3	BC=4	BD=1	CD=0
			DABC*3)	AB=5	AC=1	AD=4	BC=3	BD=1	CD=0
			DBAC*4)	AB=5	AC=3	AD=1	BC=1	BD=4	CD=0

Explanation: the second most similar pair of objects is: 1) AC; 2) BC; 3) AD; 4) BD

Source: own work.

The value of an element p_{ij} in the i-th row and the j-th column of the similarity matrix is equal to the sum of points awarded to a pair consisting of the i-th and the j-th objects in all blocks.

To discover the similarity structure of preferences by using nonmetric multidimensional scaling, the similarity matrix should be transformed into a matrix of dissimilarities, especially if all pairs of objects in blocks cannot be presented equally frequently. The dissimilarities δ_{ij} are determined in accordance with the formula:

$$\delta_{ij} = 1 - \frac{p_{ij}}{\max r \cdot m_{ij}}, \tag{6}$$

where m_{ij} is the number of pairs (i, j) in blocks, $\max r$ is the maximum number of points that can be obtained by a pair of objects in a block (for triads $\max r = 2$ and for tetrads $\max r = 5$). The denominator in the second component of the equation (6) indicates the maximum possible number of points for the pair (i, j), i.e. when in all blocks it was considered to be the pair of the most similar objects.

4. The comparison of methods

In order to make the study results independent on respondents' subjective effects (fatigue, boredom, task insight), the comparison of the presented above methods was made on the basis of the given distance matrix (see Table 6). The matrix shows the dissimilarities in the preferences of the University of the Third Age members in relation to the selected forms of activities (see Zaborski, 2014). As a result of multidimensional scaling based on the dissimilarity matrix, a configuration of points representing activities was obtained (Figure 1).

Table 6. The preferences dissimilarity matrix

Activities	1	2	3	4	5	6	7	8	9
1. English	0.000								
2. German	0.694	0.000							
3. Computer skills	1.372	1.128	0.000						
4. Gymnastics	0.908	1.111	0.766	0.000					
5. Yoga	0.596	1.007	1.062	0.370	0.000				
6. Swimming	1.117	1.276	0.712	0.209	0.568	0.000			
7. Weight training	1.395	1.413	0.530	0.522	0.892	0.342	0.000		
8. Nordic walking	0.754	1.291	1.333	0.578	0.318	0.723	1.065	0.000	
9. Painting and handcraft	1.196	0.663	0.637	1.071	1.190	1.138	1.104	1.507	0.000

Source: own work.

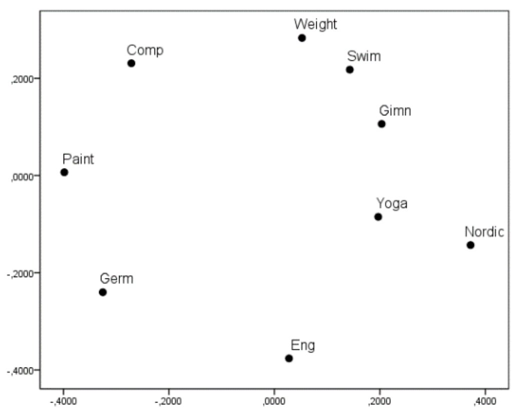

Figure 1. Preference map received based on the dissimilarity matrix

Source: own work.

In order to check how the incomplete study affects the preferences scaling results, five sets of triads (for λ=1, 2, …,5) and three sets for tetrads (for λ=3, λ=6 and λ=9) were generated. All sets are presented in Table 7.

Table 7. Sets of triads and tetrads

λ	Triads											
λ=1	1 2 3	6 4 5	8 7 9	7 4 1	9 2 5	3 6 8	1 6 9	8 4 2	3 5 7	8 5 1	7 6 2	3 9 4
λ=2	1 5 9	3 2 8	4 6 7	2 9 6	3 4 1	8 5 7	7 3 9	4 5 2	6 8 1	8 4 9	5 6 3	2 1 7
	1 2 3	6 4 5	8 7 9	7 4 1	9 2 5	3 6 8	1 6 9	8 4 2	3 7 5	8 5 1	7 6 2	3 9 4
λ=3	2 1 4	2 5 3	4 6 3	5 4 7	8 5 6	6 7 9	1 8 7	9 2 8	3 9 1	3 4 1	4 5 2	5 6 3
	4 6 7	8 5 7	8 6 9	7 9 1	2 1 8	3 9 2	2 1 6	7 3 2	8 4 3	5 4 9	6 5 1	7 6 2
	3 7 8	8 4 9	1 5 9	1 6 3	7 4 2	8 5 3	6 4 9	1 5 7	8 6 2	7 3 9	4 8 1	9 2 5
λ=4	the complement of triads set for λ=3											
λ=5	the complement of triads set for λ=2											
	Tetrads											
λ=3	1243	6512	2187	7351	6481	3961	8419	1597	9328	5429	7692	7432
	8539	6479	5463*	8562	3678*	8547						
λ=6	1243	6512	2187	7351	6481	3961	7681	8419	1597	8429	5429	7692
	7432	6562	8539	6479	5463*	8673*	8547	7491	8469*	4539	3748	6451
	2467*	4512	8423	2957	9218	2936	7391	8569	2587*	1263	5673*	8513
λ=9	7351	6512	2187	7439	7612	7382	7681	8419	1597	8429	5429	7692
	6592	6562	8539	3921	6432	1469*	8547	7491	8469*	4539	3748	6451
	1243	4512	8423	7351	6481	3961	7391	8569	2587*	1263	5673*	8513
	7432	2957	7645*	6479	5463*	8673*	7681	8479*	8563	8519	8542	3451
	2467*	5368	8412	2957	9218	2936						

Explanation: * – the most similar objects are placed second and third

Source: own work.

For each set similarity matrices were calculated, then they were transformed into dissimilarity matrix according to the formula (6) and the multidimensional scaling with the use of MINISSA program was performed. In the case of the method of triads the program TRISOSCAL, which uses MINISSA algorithm for multidimensional scaling, was used. MINISSA and TRISOSCAL are available in the multidimensional scaling package NewMDSX (Coxon and Davies, 1982). MINISSA performs the basic model of nonmetric MDS by taking data in the form of the full square symmetric matrix (or its lower triangle) of dissimilarities, whose elements are to be transformed to give the distances of the solution. This transformation will preserve the rank order of the input data.

The quality of matching the resulting points' configuration to the configuration determined based on the distance matrix (Table 6) was tested by the Procrustes statistic (see Cox and Cox, 2001; Borg and Groenen, 2005):

$$R^2 = \frac{\left\{tr\left(\mathbf{X}^{*T}\mathbf{Y}\mathbf{Y}^T\mathbf{X}^*\right)^{\frac{1}{2}}\right\}^2}{tr(\mathbf{X}^{*T}\mathbf{X}^*)tr(\mathbf{Y}^T\mathbf{Y})}, \tag{7}$$

where $\mathbf{X}^* = \mathbf{X}(\mathbf{X}^T\mathbf{Y}\mathbf{Y}^T\mathbf{X})^{\frac{1}{2}}(\mathbf{Y}^T\mathbf{X})^{-1}$ – optimally rotated configuration $\mathbf{X}$ ($\mathbf{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n]^T$ – the configuration of points determined on the basis of the incomplete blocks), $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \boldsymbol{y}_n]^T$ – the configuration of points determined on the basis of the distance matrix. $R^2 \in (0; 1\rangle$, where 1 means a perfect matching. Because all configurations have the centroids at the origin and the average distance of points from the origin is equal to 1, the Procrustes analysis is limited only to the stage of optimal rotation. The quality of matching of the resulting configurations of points to the configuration on Figure 1 tested by the Procrustes statistic is presented in Table 8.

Table 8. Procrustes statistics for different sets of triads and tetrads

	Triads					Tetrads		
λ	λ=1	λ=2	λ=3	λ=4	λ=5	λ=3	λ=6	λ=9
L_λ	12	24	36	48	60	18	36	54
R^2	0.6071	0.9401	0.9438	0.9484	0.9749	0.9647	0.9483	0.9715

Source: own work.

With the exception of the set of twelve triads, the quality of matching the other sets does not differ significantly (they are in the range from 0.94 to 0.97) and should be considered as very good. Therefore, due to the practical application of both methods, the further study was limited to the triad sets for λ=2, λ=3 and λ=4, and the tetrad sets for λ=3 and λ=6. To verify how the choice of blocks affects the preference scaling results, nine sets of triads were generated (three for each value of λ), and six sets of tetrads (three for λ=3, and three for λ=6). As it was previously mentioned, it is not possible to determine reduced sets for all combinations of λ and n, and in consequence, all pairs of objects cannot be presented equally frequently. So each set was modified by subtracting randomly selected two, four and six triads/tetrads. Finally 36 sets of triads and 24 sets of tetrads were obtained. Based on the dissimilarity matrices for all sets, multidimensional scaling with the use of MINISSA program was performed. The quality of matching of the resulting configuration to the initial configuration (Figure 1) was tested by the Procrustes statistic. In addition, for λ=2 (in the case of triads) and for λ=3 (in the case of tetrads) each set was successively reduced by two triads/tetrads, until the value of the Procrustes statistics started to fall drastically. The reduction in the number of blocks was done in such a way that in each block (as much as possible) each pair was present at least once. The results of the study are presented in Table 9 and Table 10.

It can be seen that for all generated sets of tetrads results should be regarded as almost perfect. Even if the number of tetrads in sets was reduced by 10, the results indicate a very good matching in relation to the scaling carried out for the data set in Table 6. There is only a small difference in the obtained results between reduced (maximum to 8) sets of tetrads. The difference between the best and the worst solution for all sets in this group is less than 0.08 (excluding the results for Te_p^{-12} and Te_p^{-14}).

Table 9. Procrustes statistics for different sets of triads

	λ=2			λ=3			λ=4		
	Tr_1	Tr_2	Tr_3	Tr_4	Tr_5	Tr_6	Tr_7	Tr_8	Tr_9
Tr_p^0	0.9401	0.9009	0.9444	0.9438	0.9606	0.9402	0.9484	0.9572	0.9392
Tr_p^{-2}	0.9705	0.9009	0.9566	0.9535	0.9633	0.9398	0.9487	0.9535	0.9373
Tr_p^{-4}	0.8723	0.9181	0.9403	0.9434	0.9420	0.9355	0.9473	0.9508	0.9316
Tr_p^{-6}	0.8772	0.9010	0.8949	0.9313	0.9427	0.9364	0.9448	0.9581	0.9204
$\overline{Tr}_p$	0.9181			0.9444			0.9448		
CV(%)	3.74			1.04			1.16		
Tr_p^{-8}	0.8480	0.9033	0.9097						
Tr_p^{-10}	0.8610	0.8862	0.8126						

Explanation: Tr_p^{-k} – set Tr_p (p=1,2,…,9) reduced by k triads; CV – the coefficient of variation

Source: own work.

Table 10. Procrustes statistics for different sets of tetrads

	λ=3			λ=6		
	Te_1	Te_2	Te_3	Te_4	Te_5	Te_6
Te_p^0	0.9647	0.9727	0.9323	0.9483	0.9814	0.9518
Te_p^{-2}	0.9507	0.9674	0.9062	0.9412	0.9723	0.9556
Te_p^{-4}	0.9452	0.9226	0.9034	0.9431	0.9828	0.9622
Te_p^{-6}	0.9450	0.9356	0.9437	0.9541	0.9688	0.9447
$\overline{Te}_p$	0.9409			0.9589		
CV (%)	2.37			1.51		
Te_p^{-8}	0.9586	0.9658	0.9563			
Te_p^{-10}	0.9575	0.9425	0. 9184			
Te_p^{-12}	0.8213	0.9077	0.8125			
Te_p^{-14}	0.8235	0.4878	0.2434			

Explanation: Te_p^{-k} – set Te_p (p=1,2,…,6) reduced by k tetrads; CV – the coefficient of variation

Source: own work.

The coefficient of variation of the Procrustes statistic value for sets containing from 8 to 18 tetrads is about 0.024, and from 30 to 36 tetrads about 0.015. It attests the fact that the choice of a set of tetrads (as in the case of the method of triads) has no significant effect on the results of preference scaling, even when all pairs of objects cannot be presented equally frequently. The analysis showed that the results clearly deteriorated only when the number of tetrads in sets was less than 8, but in these cases, not all pairs appear in sets. In the case of the triads method, similar results were obtained when the number of triads in the set is over 20, which means that recovery of a known structure of preferences requires respondents to make about three times more assessments than in the method of tetrads.

5. Conclusions

The results of many studies (see, e.g. Humphreys, 1982; Bijmolt, 1996; Zaborski, 2003) indicate that preference scaling based on various direct methods of measuring dissimilarities gives similar solutions. However, the selection method affects subjective feelings of respondents, which may result in different quality of input data. Therefore, the choice of the method of measurement should be guided primarily by two criteria: the method should not be labour-intensive, and expressing opinions on similarities should not cause problems to respondents. The methods which are proposed in the article do not satisfy the first of the above conditions. In the case of the triads method the number of ratings which a respondent must make for *n* objects is equal to the number of three element combinations of an *n*-element set. In the method of tetrads it is the number of four element combinations of an *n*-element set. The article indicates the possibility of reducing the number of sets presented to respondents in such a way that each pair of objects appears equally frequently, but less than their potential maximum number. In the example for 9 objects it was shown that scaling based on 8 tetrads gave a good solution. Using the method of triads, where a respondent is asked to pick out the most similar and the least similar pair from the three element sets, obtaining comparable results requires over three times more assessments. It was also demonstrated that the choice of the incomplete sets has no significant effect on the results of nonmetric multidimensional preference scaling, even when all pairs of objects cannot be presented equally frequently. This conclusion is particularly relevant for the creation of reduced sets when the number of objects does not allow to fulfil the condition of an equal number of pairs. The analysis indicated that the tetrad method can be used if each pair of objects appears in sets at least once, while for the method of triads each pair should appear at least twice.

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