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Breaking Benford's law: a statistical analysis of COVID-19 data using the Euclidean distance statistic

Leonardo Campanelli[1]

ABSTRACT

Using the Euclidean distance statistical test of Benford's law, we analyse the COVID-19 weekly case counts by country. While 62% of the 100 countries and territories considered in the present study conforms to Benford's law at a significant level of $\alpha = 0.05$ and 17% at a significant level of $0.01 \leq \alpha < 0.05$, the remaining 21% shows a deviation from it (p values smaller than 0.01). In particular, 5% of the countries 'break' Benford's law with a p value smaller than 0.001.

Key words: Benford's law, COVID-19 data.

1. Introduction

At the end of the 19th century, Newcomb (1881) noticed that the first-digit distribution of logarithms were not uniform, as one would expect, but rather followed the rule

$$P_B(d) = \log\left(1 + \frac{1}{d}\right), \tag{1}$$

where $P_B(d)$ is the probability of the first significant digit d. About 60 years later, Benford (1938) rediscovered Newcomb's rule (hereafter Benford's law), extended the law to arbitrary logarithmic bases and to multiple digits, and successfully tested the law against 20 very different data sets, like physical constants, deaths rates, populations of cities, length of rivers, etc.

Although it is now known that some distributions satisfy Benford's law [see Morrow (2014) and references therein] and that particular principles lead to the emergence of the Benford phenomenon in data (Hill, 1995a, 1995b, and 1995c), no general criteria has been found that fully explain when and why Benford's law holds for a generic set of data. Although much work is still needed to understand the theoretical basis of the law, the number of its applications has grown in the last few decades [for theoretical insights and general applications of Benford's law, see Miller (2015)].

Probably, the most famous applications are to detecting tax (Nigrini, 1996), campaign finance (Cho and Gaines, 2007), and election (Roukema, 2013) frauds. Other interesting applications are in image processing (Pérez-González et al., 2007), where Benford's law can be used to look for hidden messages in pictures as well as to test whether or not the

[1] All Saints University School of Medicine, Toronto, Canada. E-mail: leonardo.s.campanelli@gmail.com. ORCID: https://orcid.org/0000-0002-7200-9990.

image has been compressed, and in natural sciences, where the law has been shown to hold for geophysical observables such as the length of time between geomagnetic reversals, depths of earthquakes, models of Earth's gravity, and geomagnetic and seismic structure (Sambridge et al., 2010).

In general, however, it is important to stress that the rejection/acceptance of tests on data whose underlying distribution is not known to follow Benford's law should not be used as a tool to uncover error or, more importantly, fraud. This is particularly true for COVID-19 data since there is no theoretical basis or sufficient empirical evidence that these data follow a Benford distribution.

The first application of Benford's law to the study of COVID-19 data, in particular to daily and cumulative case and death counts, is due to Sambridge and Jackson (2020), while the most recent work on the 'Benfordness' of COVID-19 data is by Farhadi (2021). Using different statistical tests, the authors of both studies conclude that, in general, COVID-19 data conform to a Benford's distribution and also indicate 'anomalies' in the data of some countries. The results of these and similar analyses, however, cannot be completely trusted for reasons discussed in Section 2. Here, we will describe the statistical approach used to test the compliance of COVID-19 data with Benford's law.

Our goal is, indeed, to show that, in general, the first-digit distribution of COVID-19 (weekly) case counts by country do conform to Benford's law. This opens the possibility of detecting, in a statistically robust way, anomalous deviations from the law by specific countries. Our results, presented in Section 3, will be discussed in Section 4. In Section 5, we draw our conclusions.

2. Data and Method

It is well known that the compliance of data sets with Benford's law improves as the range of the data increases. Daily confirmed cases and daily death cases are then not appropriate when checking for the compliance of COVID-19 first-digit distributions with Benford's law because they typically extend over very few orders of magnitude. Another possibility would be the use of cumulative data. The disadvantage of using this type of data is that as cumulative cases numbers begin to flatten (especially after a COVID-19 'wave' has passed), first digits tend to become all the same, thus distorting relative digit frequencies. In order to overcome the above problems for COVID-19 data, we will only analyse the data on weekly confirmed cases by country: they extend, at least for about 45.0% of the countries, over 4 order of magnitudes, and do not flatten.

Data are from the World Health Organization (WHO, 2021) and are updated to December 20, 2021 (two years from the start of the pandemic). Counts collected by WHO reflect laboratory-confirmed cases and include both domestic and repatriated cases. True cases are subject to a time-variable under/overestimation since case definition, case detection, testing strategies, and reporting practice differ among countries, territories, and areas. All counts are continuously verified by WHO and then may change based on retrospective updates necessary to reflect changes in case definition and/or reporting practices.

Of the 222 countries and territories affected by COVID 19, only 100 have COVID-19 weekly case counts with range spanning 4, or more, orders of magnitude. These countries and territories are shown in Table 1 and, following the WHO's convention (WHO, 2021), are grouped in six different regions: Africa, Americas, Eastern Mediterranean, Europe, South-Est Asia, and Western Pacific. Also shown in the table is the range of weekly cases, $[N_{\min}, N_{\max}]$, and the number of weeks, N.

The most common tests in use for testing whether an observed sample of size N satisfies Benford's law are the Pearson's χ^2, Kolmogorov-Smirnov, and Kuiper tests. However, such tests are based on the null hypothesis of a continuous distribution, and are generally conservative for testing discrete distributions as the Benford's one (Noether, 1963). This problem can be overcome if one uses the results by Morrow (2014) who has recently found asymptotically valid test values for these statistics under the specific null hypothesis that Benford's law holds.

Other tests have been recently proposed, based on new statistics such as the 'max' statistic, m, introduced by Leemis et al. (2000), and the 'normalized Euclidean distance' statistic, d^*, introduced by Cho and Gaines (2007). At the moment of their introduction, however, the properties of the corresponding estimators were not well understood and no test values were reported. These problems were solved by Morrow (2014), who provided asymptotically test values for those statistics too.

Recently enough (Campanelli, 2021), we have found, by means of Monte Carlo simulations, the (empirical) cumulative distribution function (CDF) of the 'Euclidean distance' statistic, d_N^*, which is based on the statistic d^* and was introduced by Morrow (2014). It is defined as

$$d_N^* = \sqrt{N \sum_{d=1}^{9} [P(d) - P_B(d)]^2}, \tag{2}$$

where $P(d)$ is the observed first-digit frequency distribution.

In the following, we will use this statistic to study the first-digit distribution of COVID-19 weekly case counts by country since this is the only statistic, among the ones discussed before and analysed by Morrow, with known distribution. In particular, we will use its CDF to evaluate p values as $p = 1 - \text{CDF}(d_N^*)$.

3. Results

Our results are presented in Table 1 where we show, for each country, the Euclidean distance score, d_N^*, and the corresponding p value. Notice that the CDF of d_N^*, and then the p values, are reliable up to the second decimal place if $0.28 < d_N^* < 1.85$ and up to the third decimal place otherwise (Campanelli, 2021). In the first case, the uncertainty on p is ± 0.001, while in second case is ± 0.0001. In Table 1, the last digits in parentheses refer to these errors. For example, $p = 0.27(4)$ stands for $p = 0.274 \pm 0.001$, while $p = 0.000(2)$ stands for $p = 0.0002 \pm 0.0001$.

As shown in Figure 1, while the great majority of the countries (79%) conform to Benford's law ($p \leq 0.01$), 5% of them show a large deviation from it, having p values smaller than 0.001.

In Figure 2, we show the observed first-digit frequency distributions of weekly case

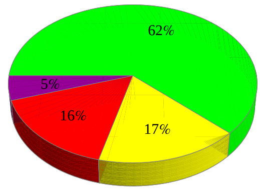

Figure 1: Percentages of the countries in a given range of p values of the Euclidean distance statistic for the first-digit distribution of COVID-19 weekly case counts by country: from top and clockwise, $p \geq 0.05$ (green), $0.01 \leq p < 0.05$ (yellow), $0.001 \leq p < 0.01$ (red), and $p < 0.001$ (purple).

counts for 15 selected countries superimposed to Benford's law. Represented countries are China (where the pandemic started), the United States of America (with the largest total number of cases), India (with the largest range of weekly case counts, $N_{\max}/N_{\min}$), Tanzania (with the smallest sample size N), Mauritius (with the smallest total number of cases), Algeria (with the smallest range of weekly case counts), Vietnam, Thailand, and Poland (the outliers in the first box plot of Figure 5 with the world largest p values), Honduras, Qatar, Belarus, Cuba, and Egypt (with the smallest p values), and Canada (with the smallest p value in the interval $0.001 \leq p < 0.01$). It is worth noticing that, although the first six countries in Figure 1 have very disparate statistical properties (such as sample size, total number of cases, and range of weekly cases), they all conform to Benford's law at a significant level of 0.01 (excluding Mauritius and Algeria, the other four countries conform to Benford's law at a significant level of 0.05). Moreover, the spatial distribution of p values is quite uniform, in the sense that countries with either large or small p values seem to be evenly scattered in the world, as shown in Figure 3. This suggests that there is no correlation between p value and geographical location of a country.

A better understanding of such a spatial variation of p values can be obtained by analysing the percentage of the countries in a given range of p values for each of the six regions of the world as defined by the WHO (2021). The result is shown in Figure 4. As it is clear from the pie charts, Africa conforms very well to Benford's law, all countries in this region having p values larger than 0.01. Also, South-Est Asian and Western Pacific countries conform well to Benford's law, the only countries with a p value less than 0.01 being Maldives (for South-Est Asia), and Philippines and Australia (for Western Pacific). Countries in Americas (Eastern Mediterranean), instead, show the largest deviation from Benford's law: only about 41% (53%) of them have p values bigger than 0.05, while about 12% (13%) have p values below 0.001.

To gain further insight into the 'global distribution' of p values, we present the box-and-whisker plots for the p values of all countries (the world) and the countries in the six WHO regions in Figure 5. All distributions are positively skewed, with medians well below 0.5. This indicates that the first-digit distribution of COVID-19 weekly case counts by country deviates somehow from Benford's law on a 'global' scale. [2] Such a deviation is, however, to be expected for the reasons explained in Campanelli (2021). Indeed, Benford's law does not represent a true law of numbers: some distributions can be 'close' to but not exactly Benford's, and this regardless of data quality; also, Benford's law emerges in the limit of infinite range of the underlying distribution, condition which is never realized in practice.

4. Discussion

The results of our analysis show that the conformance to Benford's law cannot be rejected at a significant level of 0.01 for most of the countries (79%). This implies that (*i*) the first-digit distribution of COVID-19 weekly case counts by country follows Benford's law and, accordingly, (*ii*) it can be used to detect possible 'anomalies' in COVID-19 count data. Thus, in our case, data from Canada, Jordan, Puerto Rico, Greece, Philippines, Belgium, Tunisia, Latvia, Paraguay, Sweden, Guatemala, Pakistan, Kazakhstan, Maldives, Australia, and Russia show a possible anomalous behaviour ($0.001 \leq p < 0.01$), while anomalies are certainly present in the data of Honduras, Qatar, Belarus, Cuba, and Egypt ($p < 0.001$). [3]

Needless to say, the origin of such anomalies cannot be revealed by our statistical analysis, and further and specific investigations are needed to understand if the anomalous behaviour is the result of data manipulation or other factors.

One possibility in this direction is to look at a potential correlation between the Global Health Security Index (GHSI) and the level of Benfordness (Farhadi, 2021). The GHSI was introduced by the Johns Hopkins University (2021) and is an index of the capabilities of a country to respond to epidemics of potential international concern. Accordingly, one would expect that countries with a high GHSI score are prone to reporting reliable COVID-19 data, and then to conforming to Benford's law, while countries with a low GHSI are not.

However, our results shows that this is not the case. Indeed, countries with a very low GHSI, like the African countries, comply well with Benford' law, while advanced countries with very high GHSI scores, like Australia, Canada, and Sweden (ranked 4, 5, and 7, respectively) show a statistically significant deviation from it. Indeed, a global analysis of the GHSI scores versus the p values of the Euclidean distance statistic clearly indicates a lack of (positive) correlation between GHSI scores and p values. This, together with the fact the the great majority of the countries comply with Benford's law, suggests that non-

[2] Such a deviation can be quantified by a Kolmogorov-Smirnov (KS) statistical test for the distribution of p values, whose CDF is $\mathrm{CDF}(p) = p$. The values (degrees of freedom) of the KS statistic for all countries and the ones in the six regions are 0.4295 (100), 0.4487 (13), 0.6165 (17), 0.6567 (15), 0.3379 (38), 0.5208 (8), and 0.3639 (9), respectively. Accordingly, conformance to Benford's law is rejected at a significant level of 0.001 (Facchinetti, 2009) in the case of all countries, and the countries in Americas, Eastern Mediterranean, and Europe. It is rejected at a significant level of 0.01 for the African countries. It is not rejected at a significant level of 0.01 for the South-East Asian countries, and it is not rejected at a significant level larger than 0.20 for the case of the Western Pacific countries.

[3] According to Sambridge et al. (2010) there is evidence that, in general, infection diseases conform to Benford's law. Indeed, they found that the total numbers of cases of 18 infectious diseases reported to the WHO by 193 countries worldwide in 2007 follow a Benford distribution. However, their result was not supported by a goodness-of-fit analysis. Using the Euclidean statistical test, we find that the null hypothesis of conformance to Benford's law cannot be rejected at a significant level of 0.01 (Campanelli, 2022). [In particular, the dynamic range of the data is $N_{\max}/N_{\min} = 10^6$, the number of data points is $N = 987$, and the Euclidean distance score is $d_N^* = 1.419$, corresponding to a p value of $p = 0.02(7)$.]

compliance might indicate a possible data manipulation. In Table 1, we report the GHSI score and rank for each country [data are from Farhadi (2021)], while in Figure 6 we show the p values of the Euclidean distance statistic versus the corresponding GHSI scores (the number of countries with known GHSI score is $N = 91$). The Pearson's coefficient and the corresponding p value are $r = 0.06$ and $p = 0.0166$, respectively, indicating that the very week positive correlation between GHSI score and Benfordness is unlikely (at a significant level of 0.01).

Related works. – Our results about the Benfordness of COVID-19 data are in disagreement with those obtained by Sambridge and Jackson (2020) for some specific countries. This is not surprising since Sambridge and Jackson analysed cumulative counts which are expected to deviates from Benford's law after a COVID-19 wave has passed (see discussion above). Moreover, their analysis was not statistically robust being only based on the evaluation of the Pearson correlation coefficient r between observed and expected counts. In particular, Sambridge and Jackson found that out of the 53 analysed countries none had a very weak correlation with Benford's law ($|r| \leq 0.20$), and only China had weak correlation ($0.20 < |r| \leq 0.40$). In contrast, our analysis clearly shows that China conforms well to Benford's law, the p values for the Euclidean distance statistic being 0.81(4). Also, Qatar and Greece were found to have moderate ($0.40 < |r| \leq 0.60$) and strong correlation ($0.60 < |r| \leq 0.80$), respectively, while our results show that these two countries do not conform to Benford's law to a high significant level [their p values are 0.000(0) and 0.00(2), respectively]. Moreover, Egypt, Australia, Canada, Russia, Belgium, and Sweden presented very strong correlation ($0.80 < |r| \leq 1.00$) with Benford's law, while our result is that conformance to Benford's law for these countries can be rejected at a significant level less than 0.01.

Wei and Vellwock (2020) analysed cumulative and daily cases, and cumulative and daily deaths, from 20 countries. Their goodness-of-fit test was based on the normalized Euclidean distance estimator, d^*, defined as

$$d^* = \frac{1}{D} \sqrt{\sum_{d=1}^{9} [P(d) - P_B(d)]^2}, \tag{3}$$

where $D = \sqrt{\sum_{d=1}^{8} P_B^2(d) + [P(9) - 1]^2} \simeq 1.03631$ is a normalization factor that assures that the normalized Euclidean distance is bounded by 0 and 1. The measure of fit to check concordance with Benford's law was taken to be the one proposed by Goodman (2016), according to which compliance with Benford's law occurs when $d^* \leq 0.25$. However, such a rule of thumb has been shown to be statistically unfounded in Campanelli (2021) and, generally, gives untrustworthy results for a number of data points either much less or much bigger than 40 (in particular the rule has a very low statistical power for a number of data points $N \gg 40$.) In fact, in Wei and Vellwock (2020), cumulative cases for Italy, Spain, and U.K. gave high d^* scores (0.50, 0.45, and 0.32, respectively), while our result is that conformance to Benford's law for these countries cannot be rejected at a significant level of 0.05. Moreover, while our analysis reject the null at a significant level less than 0.01 for Russia, Philippines, and South Africa, Wei and Vellwock found full conformance to Benford's law, the d^* values for these countries being well below 0.25 (0.20, 0.11, and 0.10, respectively). Finally, daily cases for Philippines, Pakistan, South Africa, and Belgium, were found to have low d^* scores (0.12, 0.07, 0.06, and 0.05, respectively), while our analysis reject conformance to Benford's law for these countries at a significant level less than 0.01. Finally, it is worth noticing that, in our case, the use of d^* statistic together with Goodman's rule-of-thumb would give a highly questionable compliance with Benford's law for all countries excepted Honduras ($d^* = 0.260$) and Tanzania ($d^* = 0.251$).

Recently enough, Farhadi (2021) has performed a detailed analysis of COVID-19 data from 153 countries by using two standard statistical tests – the Kolmogorov-Smirnov and χ^2 tests (both based on a 0.05 significance level) – and the Goodman's rule of thumb for the normalized Euclidean statistic. In his analysis, he combined daily case counts, daily deaths, and daily 'new tests', the latter variable indicating the number of individuals identified for being contaminated with COVID 19. Farhadi found that 27% of the countries showed 'full conformance' to Benford's (the null hypothesis of compliance with Benford' law was not rejected by the three statistical tests), 69% a 'partial conformance' (the null was rejected by only one of the three statistical tests), while 4% of the countries did not conform to Benford's law (the null was rejected by all three statistical tests). [4] Qualitatively speaking, then, Farhadi's findings agree with our conclusions that the first-digit distribution of COVID-19 counts follows Benford's law and can be used to flag anomalies. However, the method and data used by Farhadi (based on a combination of daily cases, deaths, and new tests) differ substantially from ours and a direct quantitative comparison with his results is not statistically feasible.

5. Conclusions

We have analysed the COVID-19 weekly case counts by country, as provided by the World Health Organization, updated to December 20, 2021. We worked under the null hypothesis that the first-digit distribution of those counts follows a Benford's distribution. The choice of weekly confirmed cases instead of daily ones came from the requirement of having counts that extended over many orders of magnitudes so to improve the compliance of the data sets with Benford's law. For the same reason we did not consider daily and weekly death counts. Also, cumulative cases were not considered as their numbers flatten (especially at the end of a 'wave'), thus distorting relative digit frequencies. Out of the 222 countries affected by COVID 19, we considered only the ones with weekly counts spanning at least 4 orders of magnitude. This choice reduced the study to the analysis of the data from 100 countries and territories. In order to test the null hypothesis, we used the Euclidean distance test introduced in Morrow (2014) and developed in Campanelli (2021), which avoids the specific problems introduced by other statistical tests.

Our analysis shows that the majority of the countries (62%) conforms to Benford's law at a significant level of 0.05. However, 5% of the countries (Honduras, Qatar, Belarus, Cuba, and Egypt) 'break' Benford's law with p values smaller than 0.001.

[4] As expected at the light of our discussion about the Goodman's rule of thumb, Farhadi found that, while the Kolmogorov-Smirnov and χ^2 tests produce similar results, the Goodman's rule of thumb was too conservative to signal anomalies in the distributions of the first digit.

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Appendices

Table 1: The Euclidean distance d_N^* in Eq. (1) and its corresponding p value for the first-digit distribution of Covid-19 weekly case counts for 100 countries. Also indicated are the range of cases, $[N_{\min}, N_{\max}]$, the number of weeks, N, and the GHSI score and rank. Counts are from WHO (2021) and are updated to December 20, 2021. (Digits in parentheses at the third and fourth decimal places indicate a statistical error on those digits of ± 1).

Country	Range	N	GHSI score	GHSI rank	d_N^*	p
Africa						
Algeria	$[5, 10524]$	96	23.6	173	1.4079	0.02(9)
Botswana	$[1, 15884]$	87	–	–	1.0374	0.24(3)
Ethiopia	$[3, 19940]$	94	40.6	84	0.8937	0.43(8)
Kenya	$[3, 19023]$	94	47.1	55	1.2771	0.06(7)
Mauritius	$[1, 10258]$	80	–	–	1.4535	0.02(1)
Mozambique	$[2, 13268]$	92	28.1	153	1.1051	0.17(5)
Namibia	$[1, 12944]$	89	35.6	104	1.1731	0.12(2)
Nigeria	$[1, 12531]$	95	37.8	96	0.6236	0.84(9)
South Africa	$[7, 162987]$	95	54.8	34	1.5317	0.01(2)
Tanzania	$[4, 24307]$	23	–	–	1.2457	0.08(0)
Uganda	$[1, 22511]$	90	–	–	0.7271	0.70(8)
Zambia	$[1, 19058]$	93	28.7	152	1.3057	0.05(7)
Zimbabwe	$[1, 26671]$	93	38.2	92	0.9221	0.39(5)
Americas						
Argentina	$[16, 219910]$	95	58.6	25	1.5532	0.01(1)
Brazil	$[6, 533024]$	96	59.7	22	1.2301	0.08(9)
Bolivia	$[7, 19834]$	94	35.8	102	1.0026	0.28(4)
Canada	$[2, 60784]$	100	75.3	5	1.8364	0.00(1)
Colombia	$[5, 204556]$	95	44.2	65	1.3949	0.03(2)
Costa Rica	$[9, 17469]$	95	45.1	62	1.3167	0.05(3)
Cuba	$[8, 64196]$	94	35.2	110	2.0674	0.000(1)
Dominican Republic	$[4, 11168]$	95	38.3	91	1.3509	0.04(3)
Ecuador	$[5, 14597]$	95	50.1	45	1.3332	0.04(8)
Guatemala	$[5, 26678]$	94	37.2	125	1.6470	0.00(5)
Honduras	$[6, 10595]$	94	27.6	156	2.6172	0.000(0)
Mexico	$[5, 128779]$	96	57.6	28	1.1353	0.15(0)
Paraguay	$[5, 20955]$	95	37.5	103	1.6844	0.00(4)
Peru	$[9, 60739]$	95	49.2	49	1.0690	0.20(9)
Puerto Rico	$[7, 32162]$	93	–	–	1.7721	0.00(2)
Uruguay	$[6, 26378]$	94	41.3	81	0.8801	0.46(0)
U.S.A.	$[12, 1745361]$	101	83.5	1	0.7242	0.71(2)

Table 1: continued

Country	Range	N	GHSI score	GHSI rank	d_N^*	p
Eastern Mediterranean						
Afghanistan	$[3,12314]$	96	32.3	120	1.2214	0.09(3)
Egypt	$[5,10778]$	96	39.9	87	1.9710	0.000(4)
Iran	$[47,269975]$	97	37.7	97	1.3850	0.03(4)
Iraq	$[2,83098]$	96	25.8	167	1.2867	0.06(4)
Jordan	$[5,57666]$	95	42.1	80	1.7989	0.00(1)
Lebanon	$[5,33605]$	97	43.1	73	1.1526	0.13(7)
Libya	$[1,19510]$	92	25.7	168	1.4154	0.02(8)
Morocco	$[6,64784]$	95	43.7	68	1.1256	0.15(8)
Oman	$[6,17783]$	96	43.1	73	1.0093	0.27(6)
Pakistan	$[2,40287]$	95	35.5	105	1.6157	0.00(6)
Palestinian Territories	$[8,17509]$	96	–	–	1.0512	0.22(8)
Qatar	$[7,13049]$	96	41.2	82	2.4137	0.000(0)
Saudi Arabia	$[5,30925]$	95	49.3	47	1.2266	0.09(1)
Tunisia	$[5,52076]$	95	33.7	122	1.7322	0.00(2)
U.A.E	$[2,26285]$	100	46.7	56	0.9135	0.40(8)
Europe						
Armenia	$[1,14417]$	95	50.2	44	0.7368	0.69(3)
Austria	$[8,96094]$	96	58.5	26	0.8381	0.52(8)
Azerbaijan	$[2,29155]$	96	34.2	117	0.6744	0.78(5)
Belarus	$[1,14213]$	96	35.3	108	2.2927	0.000(0)
Belgium	$[1,125246]$	96	61.0	19	1.7387	0.00(2)
Bosnia and Herzegovina	$[2,11122]$	95	42.8	79	0.6642	0.79(9)
Bulgaria	$[2,32962]$	95	45.6	61	1.4023	0.03(0)
Croatia	$[1,37433]$	96	53.3	38	0.6771	0.78(1)
Czechia	$[27,127489]$	95	52.0	42	0.9291	0.38(5)
Denmark	$[3,78981]$	96	70.4	8	1.4868	0.01(7)
Estonia	$[1,11930]$	96	57.0	29	1.4258	0.02(6)
Finland	$[1,16510]$	98	68.7	10	0.7175	0.72(3)
France	$[1,504469]$	100	68.2	11	0.8111	0.57(2)
Georgia	$[3,33665]$	96	52.0	42	0.9460	0.36(0)
Germany	$[2,406754]$	99	66.0	14	0.8982	0.43(1)
Greece	$[7,47411]$	96	53.8	37	1.7715	0.00(2)
Hungary	$[7,70400]$	95	54.0	35	1.1028	0.17(7)
Ireland	$[1,53846]$	96	59.0	23	1.0170	0.26(7)
Israel	$[1,65917]$	97	47.3	54	0.7419	0.68(5)
Italy	$[3,257579]$	98	56.2	31	1.3064	0.05(6)
Kazakhstan	$[6,56120]$	94	40.7	83	1.5923	0.00(8)

Table 1: continued

Country	Range	N	GHSI score	GHSI rank	d_N^*	p
Latvia	[3, 16957]	95	62.9	17	1.6877	0.00(4)
Lithuania	[1, 20730]	96	55.0	33	0.7973	0.59(5)
Moldova	[1, 11680]	95	42.9	78	1.4101	0.02(9)
Netherlands	[2, 156007]	96	75.6	3	1.0607	0.21(8)
Norway	[1, 33281]	97	64.6	16	1.0084	0.27(7)
Poland	[6, 192441]	95	55.4	32	0.4811	0.96(3)
Portugal	[2, 86549]	95	60.3	20	1.4460	0.02(3)
Romania	[3, 104668]	96	45.8	60	1.1152	0.16(6)
Russia	[5, 281305]	95	44.3	63	1.5750	0.00(9)
Serbia	[1, 49995]	95	–	–	0.9512	0.35(3)
Slovakia	[1, 61514]	95	47.9	52	1.2145	0.09(7)
Slovenia	[2, 22657]	95	67.2	12	1.0019	0.28(5)
Spain	[1, 245818]	99	65.9	15	0.6382	0.83(2)
Sweden	[1, 46511]	97	72.1	7	1.6545	0.00(5)
Turkey	[6, 414312]	94	52.4	40	1.4798	0.01(8)
U.K.	[1, 683874]	100	77.9	2	1.0711	0.20(7)
Ukraine	[1, 153131]	95	38.0	94	1.4855	0.01(7)
South-East Asia						
Bangladesh	[7, 99693]	94	35.0	113	1.3715	0.03(7)
India	[1, 2738957]	97	46.5	57	1.2935	0.06(1)
Indonesia	[10, 350273]	95	56.6	30	1.2031	0.10(4)
Maldives	[1, 11401]	94	33.8	121	1.5900	0.00(8)
Myanmar	[4, 40004]	92	–	–	0.8451	0.51(6)
Nepal	[4, 61814]	91	35.1	111	0.9980	0.29(0)
Sri Lanka	[5, 41519]	95	33.9	120	1.2816	0.06(6)
Thailand	[1, 150652]	102	73.2	6	0.4247	0.98(2)
Western Pacific						
Australia	[3, 45560]	100	75.5	4	1.5845	0.00(8)
China	[1, 31333]	104	48.2	51	0.6523	0.81(4)
Japan	[1, 156931]	101	59.8	21	1.1777	0.11(9)
Malaysia	[3, 150933]	100	62.2	18	0.8115	0.57(2)
Mongolia	[1, 36698]	91	–	–	1.0876	0.19(1)
Philippines	[1, 144991	97	47.6	53	1.7711	0.00(2)
Singapore	[4, 25950]	101	58.7	24	0.8253	0.54(9)
South Korea	[3, 47825]	101	70.2	9	1.2551	0.07(7)
Vietnam	[1, 125955	97	49.1	50	0.4202	0.98(4)

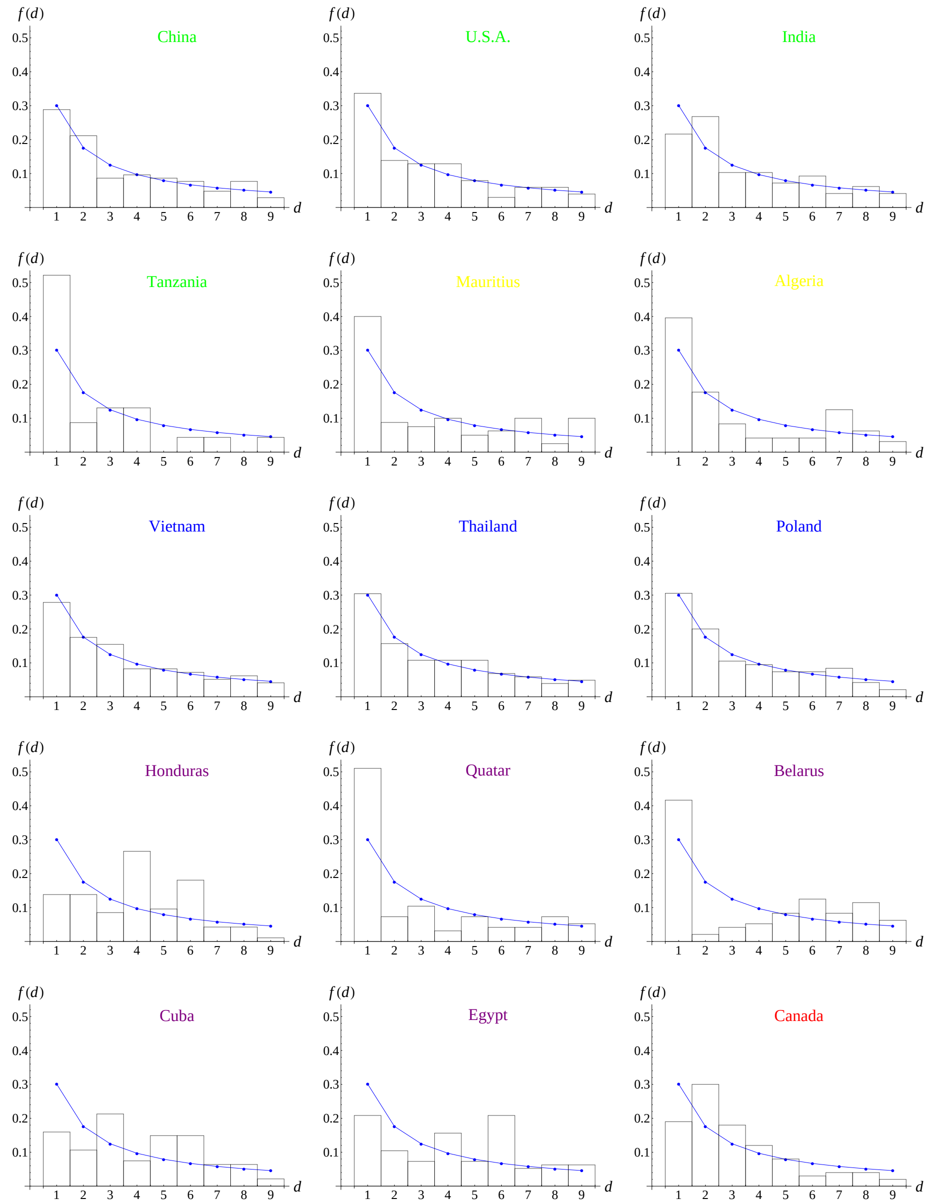

Figure 2: Observed first-digit frequencies of the Covid-19 weekly case counts for 15 selected countries: China (with the largest sample size N), USA (with the largest total number of cases), India (with the largest range of weekly case counts), Tanzania (with the smallest sample size N), Mauritius (with the smallest total number of cases), Algeria (with the smallest range of weekly case counts), Vietnam, Thailand, and Poland (the outliers in the first box plot of Fig. 5 with the world largest p values), Honduras, Qatar, Belarus, Cuba, and Egypt (with the smallest p values), and Canada (with the smallest p value in the interval $0.001 \leq p < 0.01$). The (blue) continuous lines represent Benford's law.

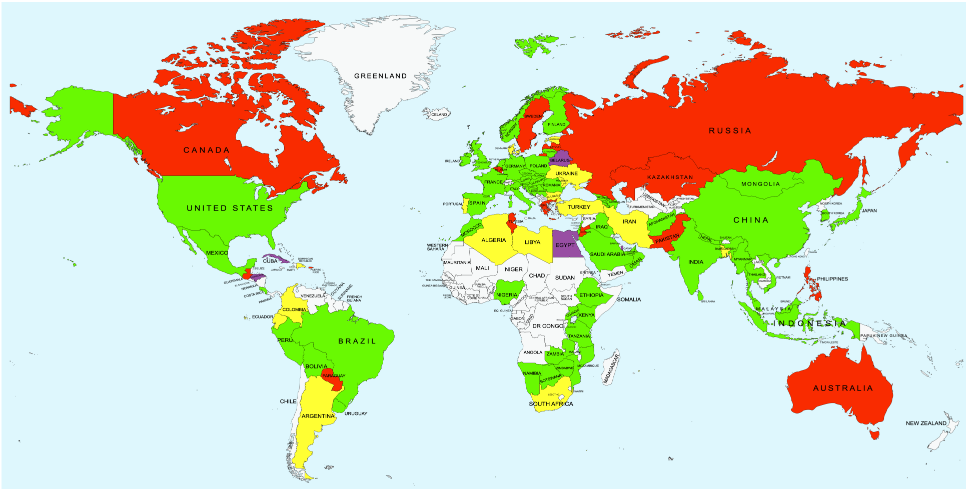

Figure 3: Spatial distribution of the p values of the Euclidean distance statistic for the first-digit distribution of Covid-19 weekly case counts by country. Ranges of p values are as follows: $p \geq 0.05$ (green), $0.01 \leq p < 0.05$ (yellow), $0.001 \leq p < 0.01$ (red), and $p < 0.001$ (purple). Light grey regions correspond to countries where Covid-19 weekly case counts have ranges below 4 orders of magnitude and then are excluded by our statistical analysis.

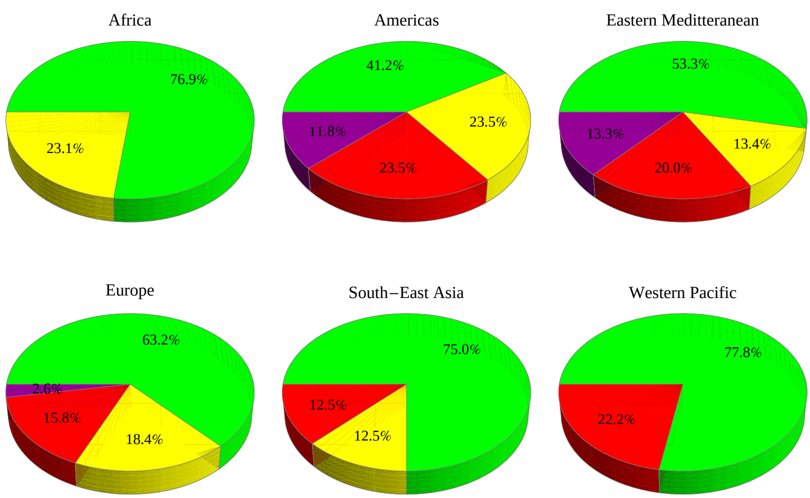

Figure 4: Percentages of countries in the six regions of the world (as defined by the World Health Organization) in a given range of p values of the Euclidean distance statistic for the first-digit distribution of Covid-19 weekly case counts by country. Ranges of p values in each pie chart are as follows: from top and clockwise, $p \geq 0.05$ (green), $0.01 \leq p < 0.05$ (yellow), $0.001 \leq p < 0.01$ (red), and $p < 0.001$ (purple).

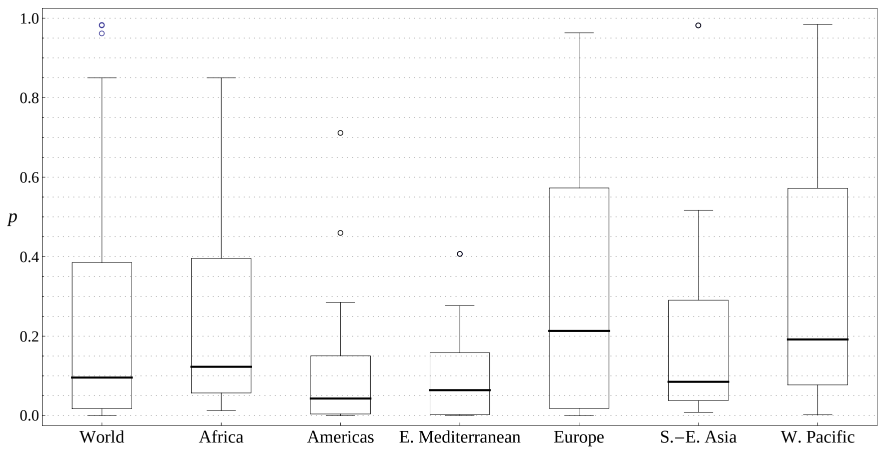

Figure 5: Box-and-whisker plots for the p values of the Euclidean distance statistic for the first-digit distribution of Covid-19 weekly case counts of all countries (the world) and countries in the six regions of the world, as defined by the World Health Organization.

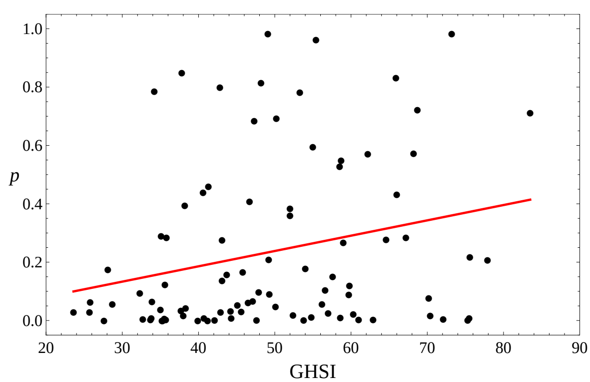

Figure 6: The p value of the Euclidean distance statistic for the first-digit distribution of Covid-19 weekly case counts by country as a function of the Global Health Security Index (GHSI) score. The (red) line is the regression line.