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New generators for minimal circular generalised neighbour designs in blocks of two different sizes

Muhammad Nadeem[1], Khadija Noreen[2], H. M. Kashif Rasheed[3], Rashid Ahmed[4], Mahmood Ul Hassan[5]

Abstract

Minimal neighbour designs (NDs) are used when a response of a treatment (direct effect) is affected by the treatment(s) applied in the neighbouring units. Minimal generalised NDs are preferred when minimal NDs cannot be constructed. Through the method of cyclic shifts (Rule I), the conditions for the existence of minimal circular generalised NDs are discussed, in which v/2 unordered pairs do not appear as neighbours. Certain generators are also developed to obtain minimal circular generalised NDs in blocks of two different sizes, where k2 = 3, 4 and 5. All these designs are constructed using i sets of shifts for k1 and two for k2.

Key words: direct effects, neighbour effects, method of cyclic shifts, generalised NDs, GN2-designs.

Mathematics Subject Classification (2010): 05B05; 62K10; 62K05.

1. Introduction

There are several situations where response of a treatment (direct effect) is affected by the treatment(s) applied in neighbouring units. Such effects are known as neighbour effects which become the major source of bias. Such bias is minimized with the use of neighbour designs (NDs):

- If each pair of treatments appears once as neighbour then it is minimal ND.

[1] Department of Statistics, The Islamia University of Bahawalpur, Pakistan.
ORCID: https://orcid.org/0000-0002-4034-3564.
[2] Department of Statistics, The Islamia University of Bahawalpur, Pakistan.
ORCID: https://orcid.org/0000-0002-0377-0030.
[3] Department of Statistics, The Islamia University of Bahawalpur, Pakistan.
ORCID: https://orcid.org/0000-0002-5965-7803.
[4] Department of Statistics, The Islamia University of Bahawalpur, Pakistan.
E-mail: mailto:rashid701@hotmail.com. ORCID: https://orcid.org/0000-0001-9703-7296.
[5] Department of Statistics, Stockholm University, Stockholm, Sweden.
ORCID: https://orcid.org/0000-0003-2889-0263.

- A block formed in a cycle in such a way that its first and last units are considered as adjacent neighbours is called a circular block. In circular blocks, each unit has one left-neighbour and one right-neighbour.
- A block in which each treatment appears either once or not at all is called a binary block.
- The circular generalized neighbour designs (CGNDs) in which $\lambda'_1 = 1$ and $\lambda'_2 = 0$ are called minimal CGNDs (MCGNDs) and are a better alternative to the minimal NDs.

Rees (1967) used neighbour designs in virus research. Partially NDs and generalized NDs (GNDs) should be used in the situations where NDs require a large number of experimental units. Misra *et al.* (1991) relaxed the condition of the constancy of $\lambda'$and constructed GNDs. Chaure and Misra (1996), Nutan (2007), Kedia and Misra (2008) constructed GN_2- designs and GN_3-designs. Ahmed *et al.* (2009), Zafaryab *et al.* (2010) and Shehzad *et al.* (2011) presented procedures to generate MCGNDs for limited cases. Iqbal *et al.* (2012) presented CGNDs for k = 3. Ahmed and Akhtar (2012) presented partially balanced NDs in circular blocks for some specific cases. In the literature, some minimal CGNDs might be constructed through *i* sets of shifts for k_1 and one set for k_2, where $k_2 = 3$, 4 and 5. These designs can also be constructed for other combinations of v, k_1 and k_2 using *i* sets of shifts for k_1 and two for k_2 which have not been constructed. In this article, generators are developed to obtain MCGNDs in two different block sizes for (i) $k_2 = 3$, (ii) $k_2 = 4$, (iii) $k_2 = 5$. All these designs are constructed through the method of cyclic shifts (Rule I) using *i* sets of shifts for k_1 and two sets for k_2. In the proposed designs $v/2$ unordered pairs do not appear as neighbours while all others appear once.

In Section 2, the method of cyclic shifts (Rule I) is described for MCGNDs. In Section 3, conditions are discussed for the existence of MCGNDs in which $v/2$ unordered pairs do not appear as neighbours. In Section 4, generators are developed for MCGNDs in two different block sizes when $k_2 = 4$, 5 and 6.

2. Method of Construction

The method of cyclic shifts was introduced by Iqbal (1991) for the construction of BIBDs, NDs, RMDs, Polygonal designs, etc. Its Rule I is explained here for MCGNDs.

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be l sets of shifts, with $j = 1, 2, \ldots, l$ and $1 \le q_{ji} \le v-1$. If each of 1, 2, …, v-1 appears once in S*, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}, (q_{j1}+ q_{j2}+\ldots+ q_{j(k-1)}) \bmod v, v-q_{j1}, v-q_{j2}, \ldots, v-q_{j(k-1)}, v-(q_{j1}+q_{j2}+\ldots+q_{j(k-1)}) \bmod v]$ then it will provide minimal CND. If (i) $\lambda_1' = 1$ and $\lambda'_2 = 0$, or (ii) $\lambda_1' = 1$ and $\lambda'_2 = 2$ then the design is called MCGND.

Example 2.1. The following sets produce MCGND for v = 24, k_1 = 5 and k_2 = 3.

$$S_1 = [3,4,5,10], S_2 = [7,11], \quad S_3 = [8,15]$$

To generate the design from these sets of shifts, take v blocks for S_1 = [3,4,5,10]. Assign 0, 1, …, v-1 as the first unit element for each block respectively. Add 3 (mod v) to the each first unit element to obtain the second unit elements. Similarly, add 4 (mod v) to the each second unit element to obtain the third unit elements. Then add 5 and 10 in the similar way and get Table 1.

Table 1: Blocks generated from S_1 = [3,4,5,10]

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}
0	1	2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15	16	17	18
12	13	14	15	16	17	18	19	20	21	22	23
22	23	0	1	2	3	4	5	6	7	8	9
B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}	B_{21}	B_{22}	B_{23}	B_{24}
12	13	14	15	16	17	18	19	20	21	22	23
15	16	17	18	19	20	21	22	23	0	1	2
19	20	21	22	23	0	1	2	3	4	5	6
0	1	2	3	4	5	6	7	8	9	10	11
10	11	12	13	14	15	16	17	18	19	20	21

Take v more subjects for S_2 = [7,11] and generate Table 2 in the similar way as obtained through S_1.

Table 2: Blocks generated from S_2 = [7,11]

B_{25}	B_{26}	B_{27}	B_{28}	B_{29}	B_{30}	B_{31}	B_{32}	B_{33}	B_{34}	B_{35}	B_{36}
0	1	2	3	4	5	6	7	8	9	10	11
7	8	9	10	11	12	13	14	15	16	17	18
18	19	20	21	22	23	0	1	2	3	4	5
B_{37}	B_{38}	B_{39}	B_{40}	B_{41}	B_{42}	B_{43}	B_{44}	B_{45}	B_{46}	B_{47}	B_{48}
12	13	14	15	16	17	18	19	20	21	22	23
19	20	21	22	23	0	1	2	3	4	5	6
6	7	8	9	10	11	12	13	14	15	16	17

Take v more subjects for S_3 = [8,5] and generate Table 3 in the similar way as obtained through S_1.

Table 3: Blocks generated from S_3 = [8,5]

B_{49}	B_{50}	B_{51}	B_{52}	B_{53}	B_{54}	B_{55}	B_{56}	B_{57}	B_{58}	B_{59}	B_{60}
0	1	2	3	4	5	6	7	8	9	10	11
8	9	10	11	12	13	14	15	16	17	18	19
23	0	1	2	3	4	5	6	7	8	9	10
B_{61}	B_{62}	B_{63}	B_{64}	B_{65}	B_{66}	B_{67}	B_{68}	B_{69}	B_{70}	B_{71}	B_{72}
12	13	14	15	16	17	18	19	20	21	22	23
20	21	22	23	0	1	2	3	4	5	6	7
11	12	13	14	15	16	17	18	19	20	21	22

Table 1, 2 and 3 jointly present the MCGND for v = 24, k_1 = 5 and k_2 = 3, using 72 blocks. In this design 12 unordered pairs (0,12), (1,13), …, (11,23) among the total 276 do not appear as neighbours while all remaining 264 unordered pairs appear once.

3. Conditions for the Existence of MCGNDs in Which v/2 Pairs Do Not Appear as Neighbours

Condition 3.1: Let m = (v-2)/2 and A = [1, 2, …, m]. If m (mod 4) ≡ 0 then A will provide MCGNDs for $v = 2ik_1+4k_2+2$ in which (m+1) unordered pairs do not appear as neighbours while all other appear exactly once.

Condition 3.2: Let m = (v-2)/2 and m (mod 4) ≡ 3. Then, A = [1, 2,…, (3m-1)/4, (3m+7)/4, (3m+11)/4,…, m, 5(m+1)/4] will provide MCGNDs for $v = 2ik_1+4k_2+2$ in which (m+1) unordered pairs do not appear while all others appear once.

4. MCGNDs in Blocks of Two Different Sizes

Here, some generators are developed through Rule I to obtain MCGNDs in blocks of two different sizes using i sets for k_1 and two for k_2. In these designs v/2 pairs do not appear as neighbours while all other appear once. (i+2) sets are obtained as follows:

1. Divide values of 'A' selected from Section 3, in i classes of size k_1 and two of size k_2 such that the sum of values in each class is divisible by v.
2. Deleting any one value from each class will result in (i+2) sets of shifts to produce MCGND.

4.1. MCGNDs when k_2 = 3

Generator 4.1.1. Construct MCGNDs for k_2 for $v = 2ik_1+14$, $k_1 = 4l+2$, k_2 = 3, i odd, m (mod 4) ≡ 0 and l integer. Here:

- Consider A = [1, 2,…, m].

Example 4.1.1. The following sets produce MCGND for v = 26, k_1 = 6 and k_2 = 3.

S_1 = [2,3,4,5,11], S_2 = [9,10], S_3 = [8,12]

Designs constructed through this method for v ≤ 100, k_1 = 6, 10, 14 and 18 are presented in Table 4 in Appendix.

Generator 4.1.2. Construct MCGNDs for k_2 for $v = 2ik_1+14$, k_1 (mod 4) ≡ 1, k_2 = 3, I (mod 4) ≡ 1, m (mod 4) ≡ 3. Here:

- Consider A = [1, 2,…, (3m-1)/4, (3m+7)/4, (3m+11)/4,…, m, 5(m+1)/4].

Example 4.1.2. The following sets produce MCGND for v = 24, k_1 = 5 and k_2 = 3.

S_1 = [3,4,5,11], S_2 = [8,10], S_3 = [7,15]

Designs constructed through this method for v ≤ 100, k_1 = 5, 9, 13 and 17 are presented in Table 5 in Appendix.

4.2. MCGNDs when $k_2 = 4$

Generator 4.2.1. Construct MCGNDs for k_2 for $v = 2ik_1+18$, $k_1 = 4l+2$, $k_2 = 4$, i even, $m \pmod 4 \equiv 0$. Here:

- Consider A = [1, 2,…, m].

Example 4.2.1. The following sets produce MCGND for $v = 42$, $k_1 = 6$ and $k_2 = 4$.

S_1 = [3,4,5,8,20], S_2 = [11,13,16,17,18], S_3 = [7,14,15], S_4 = [10,12,19]

Designs constructed through this method for $v \le 100$, k_1 = 6, 10, 14 and 18 are presented in Table 6 in Appendix.

Generator 4.2.2. Construct MCGNDs for k_2 for $v = 2ik_1+18$, $k_1 \pmod 4 \equiv 1$, $k_2 = 4$, $i \pmod 4 \equiv 3$, $m \pmod 4 \equiv 3$. Here:

- Consider A = [1, 2,…, $(3m\text{-}1)/4$, $(3m+7)/4$, $(3m+11)/4$,…, m, $5(m+1)/4$].

Example 4.2.2. The following sets produce MCGND for $v = 48$, $k_1 = 5$ and $k_2 = 4$.

S_1 = [4,5,17,19], S_2 = [8,9,14,16], S_3 = [10,11,12,13], S_4 = [22,23,30], S_5 = [7,15,20]

Designs constructed through this method for $v \le 100$, k_1 = 5, 9 and 13 are presented in Table 7 in Appendix.

4.3. MCGNDs when $k_2 = 5$

Generator 4.3.1. Construct MCGNDs for $v = 2ik_1+22$, $k_1 = 4l+2$, $k_2 = 5$, i odd, $m \pmod 4 \equiv 0$ and l integer. Here:

- A = [1, 2,…, m].

Example 4.3.1. The following sets produce MCGND for $v = 34$, $k_1 = 6$ and $k_2 = 5$.

S_1 = [3,4,5,7,13], S_2 = [6,8,9,10], S_3 = [12,14,15,16]

Designs constructed through this method for $v \le 100$, k_1 = 6, 10, 14 and 18 are presented in Table 8 in Appendix.

Generator 4.3.2. Construct MCGNDs for $v = 2ik_1+22$, $k_1 \pmod 4 \equiv 1$, $k_2 = 5$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 3$. Here:

- A = [1, 2,…, $(3m\text{-}1)/4$, $(3m+7)/4$, $(3m+11)/4$,…, m, $5(m+1)/4$].

Example 4.3.2. The following sets produce MCGND for $v = 32$, $k_1 = 5$ and $k_2 = 5$.

S_1 = [2,5,9,15], S_2 = [10,13,14,20], S_3 = [4,6,8,11]

Designs constructed through this method for $v \le 100$, k_1 = 5, 9, 13 and 17 are presented in Table 9 in Appendix.

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Appendix

Table 4: MCGNDs for $v = 2ik_1+14$, $k_1 = 4l+2$, $k_2 = 3$, *i* odd, *l* integer and $v \leq 100$

v	k_1	k_2	Sets of Shifts
26	6	3	[1,2,3,4,5,11]+[7,9,10]+[6,8,12]
50	6	3	[1,3,4,6,12,24]+[2,7,8,9,11,13]+ [14,15,16,17,18,20]+[10,19,21]+[5,22,23]
74	6	3	[2,4,5,6,23,34]+[7,8,9,10,11,29]+ [1,12,13,15,16,17]+ [18,19,22,26,31,32]+ [14,24,25,27,28,30]+ [20,21,33]+[3,35,36]
98	6	3	[1,2,3,6,38,48]+[9,10,11,12,26,30]+ [13,14,15,16,18,22]+ [7,8,19,20,21,23]+ [25,27,28,29,43,44]+ [24,31,34,35,33,39]+ [4,32,37,40,41,42]+ [17,36,45]+[5,46,47]
34	10	3	[1,2,3,4,6,7,8,12,9,16]+[10,11,13]+[5,14,15]
74	10	3	[2,4,5,6,7,8,21,29,32,34]+[9,17,12,13,14,15,18,19,20,11]+ [1,16,22,23,24,25,26,27,28,30]+ [10,31,33]+[3,35,36]
42	14	3	[1,2,3,4,6,7,8,9,10,12,13,14,17,20]+[11,15,16]+[5,18,19]
98	14	3	[1,2,3,4,7,8,9,10,12,13,14,20,45,48]+ [6,15,16,17,18,19,21,22,23,24,27,35,26,25]+ [28,29,30,31,32,33,36,37,34,38,39,40,41,42]+[11,43,44]+[5,46,47]
50	18	3	[1,2,3,4,6,7,8,9,24,12,13,20,15,14,17,16,11,18]+[10,19,21]+[5,22,23]

Table 5: MCGNDs for $v = 2ik_1+14$, $k_1 \pmod 4 \equiv 1$, $k_2 = 3$, $i \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
24	5	3	[1,3,4,5,11]+[6,8,10]+[2,7,15]
64	5	3	[1,2,6,27,28]+[4,5,17,18,20]+ [10,12,13,14,15]+ [7,8,11,16,22]+ [19,21,23,25,40]+ [9,26,29]+[3,30,31]
32	9	3	[1,2,4,5,6,8,11,14,13]+[7,10,15]+[3,9,20]
40	13	3	[1,2,3,6,7,8,9,12,10,13,14,16,19]+[4,11,25]+[5,17,18]
48	17	3	[1,2,3,4,6,7,8,10,11,15,13,14,16,12,17,23,30]+[9,19,20]+[5,21,22]

Table 6: MCGNDs for $v = 2ik_1+18$, $k_1 = 4l+2$, $k_2 = 4$, *i* even and *l* integer and $v \leq 100$

v	k_1	k_2	Sets of Shifts
42	6	4	[2,3,4,5,8,20]+[9,11,13,16,17,18]+[6,7,14,15]+[1,10,12,19]
66	6	4	[1,3,4,6,20,32]+[7,8,9,11,12,19]+[16,17,18,26,27,28]+ [10,21,22,25,23,31]+ [13,14,15,24]+[2,5,29,30]
90	6	4	[5,18,34,40,41,42]+[7,8,9,14,15,37]+[6,11,12,16,17,28]+ [1,4,19,21,22,23]+ [25,26,27,29,30,43]+[13,31,32,33,35,36]+ [3,10,38,39]+[2,20,24,44]
58	10	4	[3,4,5,6,8,9,13,23,17,28]+[10,11,12,15,16,18,19,20,26,27]+ [1,14,21,22]+[2,7,24,25]
98	10	4	[1,2,3,4,6,7,8,9,10,48]+[13,14,15,16,17,18,19,20,23,41]+ [12,21,22,24,28,30,33,39,40,45]+ [32,34,35,36,37,38,43,44,46,47]+ [11,27,29,31]+[5,25,26,42]
74	14	4	[3,4,5,6,7,9,10,12,19,20,29,33,31,34]+ [2,14,15,16,17,21,24,27,22,23,25,28,26,36]+[1,11,30,32]+[8,13,18,35]
90	18	4	[1,2,3,4,5,7,8,11,12,13,14,15,16,18,19,39,40,43]+ [20,22,23,24,25,26,27,28,29,30,34,31,33,35,32,36,41,44]+[6,9,37,38]+ [10,17,21,42]

Table 7: MCGNDs for $v = 2ik_1+18$, $k_1 \pmod 4 \equiv 1$, $k_2 = 4$, $i \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
48	5	4	[3,4,5,17,19]+[1,8,9,14,16]+[2,10,11,12,13]+[21,22,23,30]+[6,7,15,20]
88	5	4	[2,4,5,34,43]+[7,8,9,22,42]+[3,12,13,28,32]+[14,17,18,19,20]+ [6,10,23,24,25]+[29,31,37,38,41]+[16,30,36,39,55]+[11,15,27,35]+ [1,21,26,40]
72	9	4	[4,6,7,8,10,20,26,28,35]+[9,12,13,14,15,16,17,18,30]+ [2,3,5,19,23,24,22,25,21]+[1,11,29,31]+[32,33,34,45]
96	13	4	[1,2,3,4,5,6,7,8,9,10,11,12,18]+[15,16,17,19,20,21,22,23,24,25,26,27,33]+[29,30,3 1,32,34,35,37,38,39,40,42,46,47]+[13,14,28,41]+[43,44,45,60]

Table 8: MCGNDs for $v = 2ik_1+22$, $k_1 = 4l+2$, $k_2 = 5$, i odd, l integer, and $v \leq 100$

v	k_1	k_2	Sets of Shifts
34	6	5	[2,3,4,5,7,13]+[1,6,8,9,10]+[11,12,14,15,16]
58	6	5	[1,2,3,4,22,26]+[7,8,9,10,11,13]+[16,17,19,20,21,23]+ [5,6,14,15,18]+[12,24,25,27,28]
82	6	5	[2,4,5,6,29,36]+[9,10,11,12,18,22]+ [7,13,14,15,16,17]+[1,3,8,21,23,26]+ [19,24,25,28,31,37]+ [30,32,33,34,35]+[20,27,38,39,40]
42	10	5	[2,3,4,6,7,8,9,10,15,20]+[1,5,11,12,13]+[14,16,17,18,19]
82	10	5	[1,2,3,4,5,6,7,8,9,37]+[11,13,14,15,16,17,18,19,20,21]+ [10,22,23,24,25,26,27,28,29,32]+ [30,31,33,34,36]+[12,35,38,39,40]
50	14	5	[2,3,4,6,7,8,10,11,12,13,15,16,24,19]+[1,5,9,17,18]+[14,20,21,22,23]
58	18	5	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,21,17,16]+[19,20,23,26,28]+ [18,22,24,25,27]

Table 9: MCGNDs for $v = 2ik_1+22$, $k_1 \pmod 4 \equiv 1$, $k_2 = 5$, $i \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
32	5	5	[1,2,5,9,15]+[7,10,13,14,20]+[3,4,6,8,11]
72	5	5	[3,5,9,10,45]+[4,8,17,21,22]+[2,13,15,19,23]+[6,12,16,18,20]+ [1,7,11,24,29]+[25,26,28,30,35]+[14,31,32,33,34]
40	9	5	[1,5,6,7,8,9,12,13,19]+[2,3,10,11,14]+[4,16,17,18,25]
48	13	5	[1,2,3,4,5,6,7,8,9,10,11,13,17]+[12,15,16,23,30]+[14,19,20,21,22]
56	17	5	[1,2,3,4,5,6,7,8,9,16,12,13,23,19,15,11,14]+[17,18,20,22,35]+ [10,24,25,26,27]