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Changepoint detection with the use of the RESPERM method – a Monte Carlo study

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Abstract

RESPERM (residuals permutation-based method) is a single changepoint detection method based on regression residuals permutation, which can be applied to many physiological situations where the regression slope can change suddenly at a given point. This article presents the results of a Monte Carlo study on the properties of the RESPERM method for single changepoint detection in a linear regression model. We compared our method with a well-known segmented method for detection breakpoint in linear models. The Monte Carlo study showed that when the input data are very noisy, the RESPERM method outperforms the segmented approach in terms of variance, and in the case of bias, the results of the two methods are comparable.

Key words: changepoint detection, RESPERM, permutation methods.

1. Introduction

The changepoint analysis plays an important role in many fields including time series analysis, quality control, economy, finance, genome research, signal processing, medical research, and many others. The changepoint problem is generally referred to as identifying the changes at unknown times and of estimating the location of changes in stochastic processes. This problem was initially discussed by Quandt (1958, 1960) and also Chow (1960). The changepoint problem can be formulated in several models and numerous methodological approaches have been implemented in examining these models. Maximum-likelihood estimation, Bayesian estimation, piecewise regression, quasi-likelihood and non-parametric regression as well as grid-searching are among the methods which have been applied to resolving challenges in changepoint problems (Julious, 2001). Changepoint detection methods can be online (detecting in real-time

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setting) or offline, that retrospectively detect changes when all samples are received. The problem of estimation the location of changepoints has been intensively studied in the literature including significance testing of estimate as well (a pioneering work of Chow (1960) should be mentioned here, but there are also many newer ones). A detailed review, as well as the classification and evaluation of different changepoint detection methods based on the selected criteria can be found, for example, in (Aminikhanghahi and Cook, 2017; Truong et al., 2020).

In this article we consider locating changepoint in a linear regression model with one changepoint. There are different nomenclatures to describe the so-called changepoint regression, such as „segmented" (Lerman, 1980), „broken-line" (Ulm, 1991), „structural change" (Bacon and Watts, 1971) and some others, in which the relationship between the response and the explanatory variable (or variables) is piecewise linear. There are two possibilities in changepoint regression. The first one is a continuous piecewise model, in which regression lines with different slopes are connected at unknown changepoints. In the second, discontinuous model, the regression lines jump at the changepoint (Figure 1).

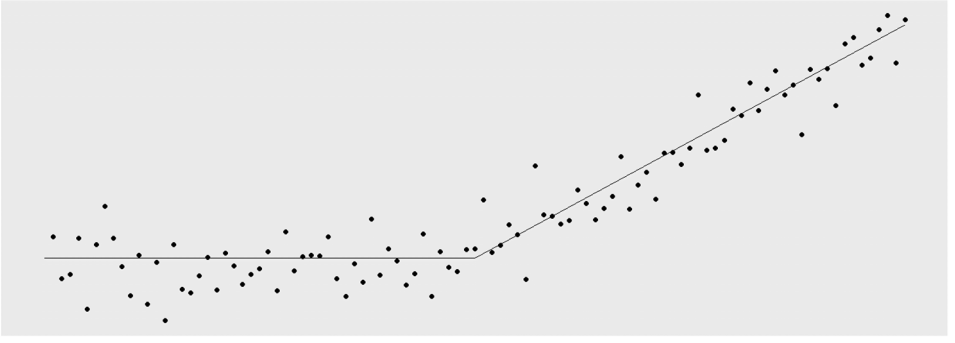

Figure 1. The illustrative example of one changepoint in linear regression model

The paper is presented as follows. After a brief introduction to the problem of changepoint detection in the literature, in Section 2 we present the general model for changepoint regression and describe, very shortly, two selected methods used in the comparison further. In Section 3 we present the concepts and the process of the Monte Carlo study. Sections 4 and 5 describe the Monte Carlo simulation study and the obtained results with discussion, respectively. The last section provides conclusion remarks.

2. The changepoint regression model and the selected methods

The general model of changepoint regression with J changepoints ($J+1$ regimes) can be written as:

$$y_i = \begin{cases} \beta_{0(1)} + \beta_{1(1)} x_i + \varepsilon_{i(1)} & x_i \le chp_1 \\ \beta_{0(2)} + \beta_{1(2)} x_i + \varepsilon_{i(2)} & chp_1 < x_i \le chp_2 \\ \vdots & \\ \beta_{0(j)} + \beta_{1(j)} x_i + \varepsilon_{i(j)} & chp_{j-1} < x_i \le chp_j \\ \vdots & \\ \beta_{0(J+1)} + \beta_{1(J+1)} x_i + \varepsilon_{i(J+1)} & chp_J < x_i \end{cases} \quad (1)$$

In this paper we will only consider the models where the changepoints are defined in terms of only one regressor denoted here as x. The model (1) considers only one regressor x along which the changepoints lie, however other regressors can also be included in the model. In the model (1) $i = 1, \ldots, n$ are the observation numbers, n is the total sample size, chp_j $j = 1, \ldots, J$ are the changepoint parameters for the regressor x which satisfy:

$$chp_1 < chp_2 < \ldots < chp_J$$

$\varepsilon_{i(j)}$ are independent, identically distributed random variables, having mean zero and possibly differing variances σ_j^2 $j = 1, \ldots, J$, respectively.

The changepoints locations given by chp_j are the unknown parameters to be estimated, but the number of changepoints in the observed sample is assumed to be known. The model above also assumes that regressor x can be ordered (that means partitioned by the changepoints chp_j), and sufficient number of observations can be placed in each intervals, for which the data came from different regimes (models data generation) for reliable estimation and inference. It is up to the user to determine what is „sufficient", but the rule of thumb may be to ensure at least 10 observations in each regime (Sheykhfard et al., 2020; Applied Regression Analysis, 2023).

The above model describes both continuous and discontinuous scenario, but to enforce the connected regression lines, the regression parameters must be constrained so that:

$$\beta_{0(j)} + \beta_{1(j)} \cdot chp_j = \beta_{0(j+1)} + \beta_{1(j+1)} \cdot chp_{j+1}. \quad (2)$$

There are many methods in the literature to detect the location of the unknown changepoints and estimate the regression model (1). We will consider two methods: the SEGMENTED (Muggeo, 2008), and RESPERM (Sommer et al., 2022) methods which will be shortly described below.

2.1. Segmented method

Muggeo (2003) proposed a method called *segmented regression*, which allows for multiple unknown changepoints but is restricted to continuous regression lines.

We briefly present this method for the single changepoint model ($J = 1$) with location in chp. The model (1) with constraints (2) for the segmented regression can be estimated iteratively via the following linear function of predictors:

$$\beta_0 + \beta_{1(1)}x_{i1} + (\beta_{1(2)} - \beta_{1(1)})(x_{i1} - chp_0)I(x_{i1} > chp_0) - \gamma \cdot I(x_{i1} > chp_0), \tag{3}$$

where $I(A)$ is an indicator function for an event A, chp_0 is an initial estimate for the changepoint, and γ is a re-parametrization of chp_0 that appears as a linear and continuously valued parameter which facilitates the estimation procedure. Muggeo (2003) recommends maximum likelihood (ML) under Gaussian errors with constant variance across regimes (homoscedasticity). The model enables for simultaneous ML inference on all model parameters, including the changepoint location. The procedure for segmented regression can be sketched as follows:

(1) choose an initial changepoint estimate chp_0,
(2) given the current estimated changepoint chp_0 estimate model (3) by Gaussian ML and update the changepoint via $\widehat{chp} = chp_0 + \hat{\gamma}/(\hat{\beta}_{1(2)} - \hat{\beta}_{1(1)})$,
(3) if $\hat{\gamma}$ is sufficiently closed to zero then stop, else set $chp_0 = \widehat{chp}$ and go to step (2),
(4) iterate steps (2) and (3) until termination.

In the above procedure $\hat{\gamma}$ measures the distance between the two fitted regression lines at the current estimate $\widehat{chp}$. It is not clear that this method can be extended to cover the discontinuous case. Muggeo also proposed the segmented package in R (Muggeo, 2008), which enables to estimate the parameters in GLM with segmented, continuous relationships via ML.

2.2. The residuals permutation-based method (RESPERM)

The RESPERM method was designed for detecting a changepoint in the EEG signal waveform in an experiment with showing a new brain-learned face. By definition, the EEG method is just for studying waveforms over time - hence this method uses only 1 regressor.

The RESPERM method considers a discrete set (i.e. a grid) of possible changepoint locations; for each possible changepoint an optimal set of parameter estimates for each of the two regimes is determined. The finally selected changepoint optimizes the chosen estimation criterion - Cohen's effect size (Cohen 1988), estimated based on the permutation method. Moreover, this method allows for different variances in each regime.

Let us consider n experimental data observations, which could be denoted by (x_i, y_i) for $i = 1, 2, \ldots, n$. We will consider two simple linear regression models with changepoint *chp*:

$$y = \beta_{01} + \beta_{11}x + \varepsilon_1 \quad \text{for } x \leq chp,$$

$$y = \beta_{02} + \beta_{12}x + \varepsilon_2 \quad \text{for } x \geq chp,$$

where y is the dependent variable, x is the regressorβ_{01}, β_{11}, β_{02} and β_{12} are parameters of linear models and ε_1, ε_2 are error terms.

The main goal of this method is to detect a change in the slope in the linear regression model. If $\beta_{11} \neq \beta_{12}$ then *chp* is a breakpoint in the considered linear model. To detect the breakpoint *chp* we use the Cohen effect size. Cohen (1988) defines an effect size *d* as follows:

$$d = \frac{m_A - m_B}{\sigma}, \tag{4}$$

where m_A and m_B are populations means under considerations expressed in raw (original) measurement units and σ is the standard deviation of either population of measurements. Instead of m_A, m_B and σ we use β_{11}, β_{12} and standard deviations of beta's σ_β.If the errors are very non-normal then the standard methods may not be reliable and a resampling method may offer some improvement (*Davidson* and Hinkley, 1997). The permutation of the residuals method is used to estimate the standard deviation σ_β. So, (4) could be rewritten in the form

$$d = \frac{\beta_{12} - \beta_{11}}{\sqrt{\frac{(k-1)S^2_{\beta 11} + (n-k-1)S^2_{\beta 12}}{n-2}}} \tag{5}$$

where *k* is the number of observations in the first group.

For the estimated two linear models the residuals are obtained separately for each one. Then, the residuals are permuted N_{perm}=1000 times and for each case, the coefficients β_{11}, β_{01} (the slope and the intercept of the first line) and β_{12} , β_{02} (the slope and the intercept of the second line) were estimated. Based on these estimates, the standard deviations $S_{\beta 11}$ and $S_{\beta 12}$ of the β_{11} and β_{12} coefficients are assessed.

Let us consider two sets $\mathbf{S_1} = \{(x_i, y_i): i = 1, 2, \ldots, k\}$ and $\mathbf{S_2} = \{(x_i, y_i): i = k + 1, k + 2, \ldots, n\}$ where $k = s, s+1, \ldots, n-s$ and s is the parameter of the method. Based on these two sets we get two regression lines with slopes $\hat{\beta}_{11}$and $\hat{\beta}_{12}$ and intercepts $\hat{\beta}_{01}$and $\hat{\beta}_{02}$. Let *d* now measure Cohen's effect (5) of the slope of the linear regression. We find k^* which maximizes Cohen's effect size *d* using the formula:

$$k^* = \min_{k \in K} \left\{ k: d(k) = \max_{k \in K} d(k) \right\} \tag{6}$$

where K = {s, s+1, …, *n*-s} and *s* is the parameter of the permutation-based method. So, the changepoint can be expressed as

$$chp = x_{k^*}.$$

3. Monte Carlo study

In the Monte Carlo study, the series of n = 100 observations with one changepoint were generated. The observations were generated according to the following model:

$$y = \begin{cases} 2 + p\varepsilon_j & \text{for } x \leq 50, \\ 2 + (x - 50) + pq\varepsilon_j & \text{for } x > 50, \end{cases} \tag{7}$$

where the covariate x = 1, 2, …, 100, coefficient p describes the level of the noise (p=1 for minor noise, p=3 for major noise, p = 5 for dominant noise), coefficient q = *1* for equal variances and q=2/3 for unequal variances, changepoint is established to chp = 50, ε_j (j = 1, 2, 3, 4) is the error term:

$\varepsilon_1 = \frac{1}{3}\varepsilon_N$, where ε_N has a standard normal distribution,

$\varepsilon_2 = \varepsilon_U - 0.5$, where ε_U has the uniform distribution on the [0, 1] interval,

$\varepsilon_3 = \varepsilon_{\beta 22} - 0.5$, where $\varepsilon_{\beta 22}$ has the beta distribution with shape parameters $s_1 = 2$, $s_2 = 2$ (symmetric distribution),

$\varepsilon_4 = \varepsilon_{\beta 26} - 0.25$, where $\varepsilon_{\beta 26}$ has the beta distribution with shape parameters $s_1 = 2$, $s_2 = 6$ (asymmetric distribution).

The main characteristics of these distributions are presented in Table 1.

Table 1. Expectations and variance of the distributions of errors in the present simulations

Error ε	E(ε)	D²(ε)	Error ε	E(ε)	D²(ε) Noise		
					Minor	Major	Dominant
ε_N	0	1	ε_1	0	$\frac{1}{9}$	1	$\frac{25}{9}$
ε_U	0	$\frac{1}{12}$	ε_2	0	$\frac{1}{12}$	$\frac{9}{12}$	$\frac{25}{12}$
$\varepsilon_{\beta 22}$	0.50	$\frac{1}{20}$	ε_3	0	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{5}{4}$
$\varepsilon_{\beta 26}$	0.25	$\frac{1}{48}$	ε_4	0	$\frac{1}{48}$	$\frac{9}{48}$	$\frac{25}{48}$

Source: own calculations.

The expected values of errors in all models are 0 but differ in variance from 1/48 (for minor noise) up to 25/9 (dominant noise). The first part of Table 1 shows the four variants of the considered distributions (normal, uniform, beta symmetric and beta asymmetric) and their parameters. The second part of Table 1 shows the distributions and parameters used in the Monte Carlo study. Figure 2 shows the empirical density functions of errors for each model.

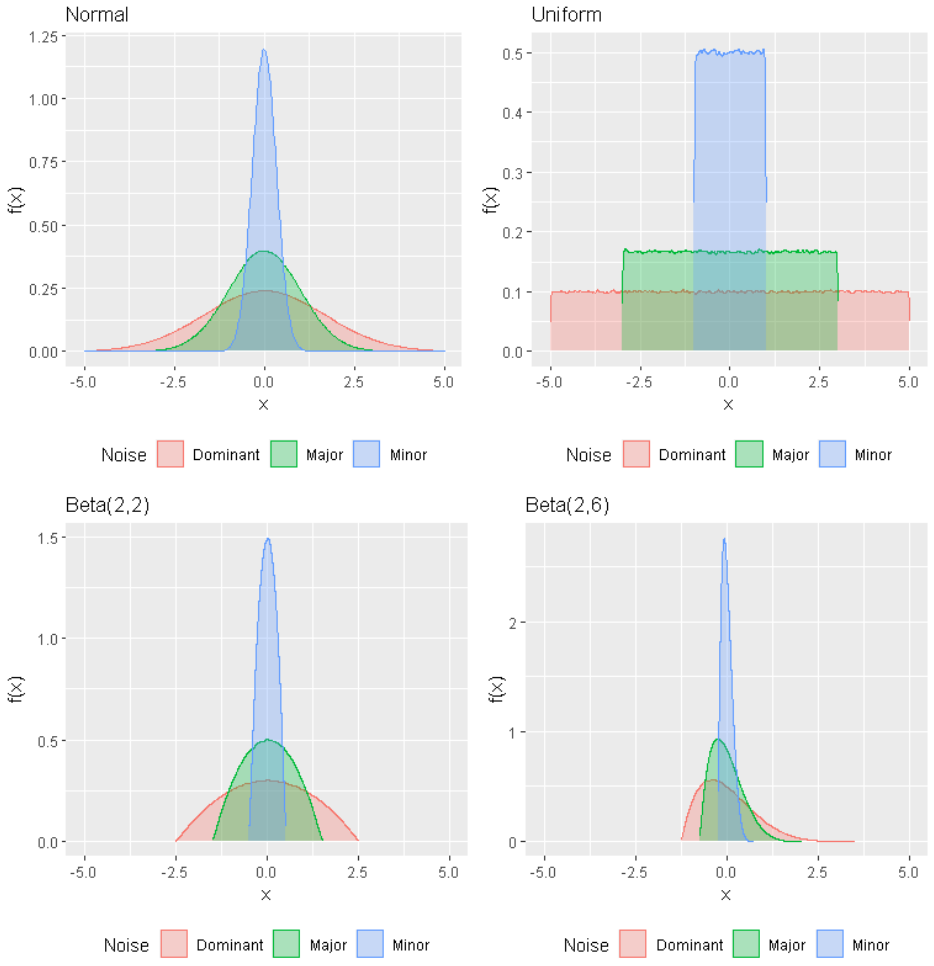

Figure 2. The empirical distributions of considered errors in computer simulations

Source: own calculations.

Six variants of error models are considered in the Monte Carlo study. The first one is the model with equal variances for the two parts considered and the second one for unequal variances. In the second case, the density function of errors from the first part were multiplied by the constant 2/3. In both of these variants, the noise is at a low level (minor noise). The next two variants of the Monte Carlo study are similar to the first two, but the density functions of errors were multiplied by 3 (major noise). The last two variants of the Monte Carlo study are similar to the first two, but the density functions of errors were multiplied by 5 (dominant noise). Typical random series of observations for equal variances and unequal variances are shown in Figure 2. The first part in model

(7) (from 0 to 50) is a linear model with a slope β_{11} equal to 0. The second part (from 50 to 100) has the slope β_{12} equal to 0.5.

The two cases presented at the top of Figure 3 relate to noise at the minor level. The next two cases in the middle row relate to noise at the major level. The last two cases at the bottom relate to noise at the dominant level. In the Monte Carlo study, the changepoint was set to *chp* = 50.

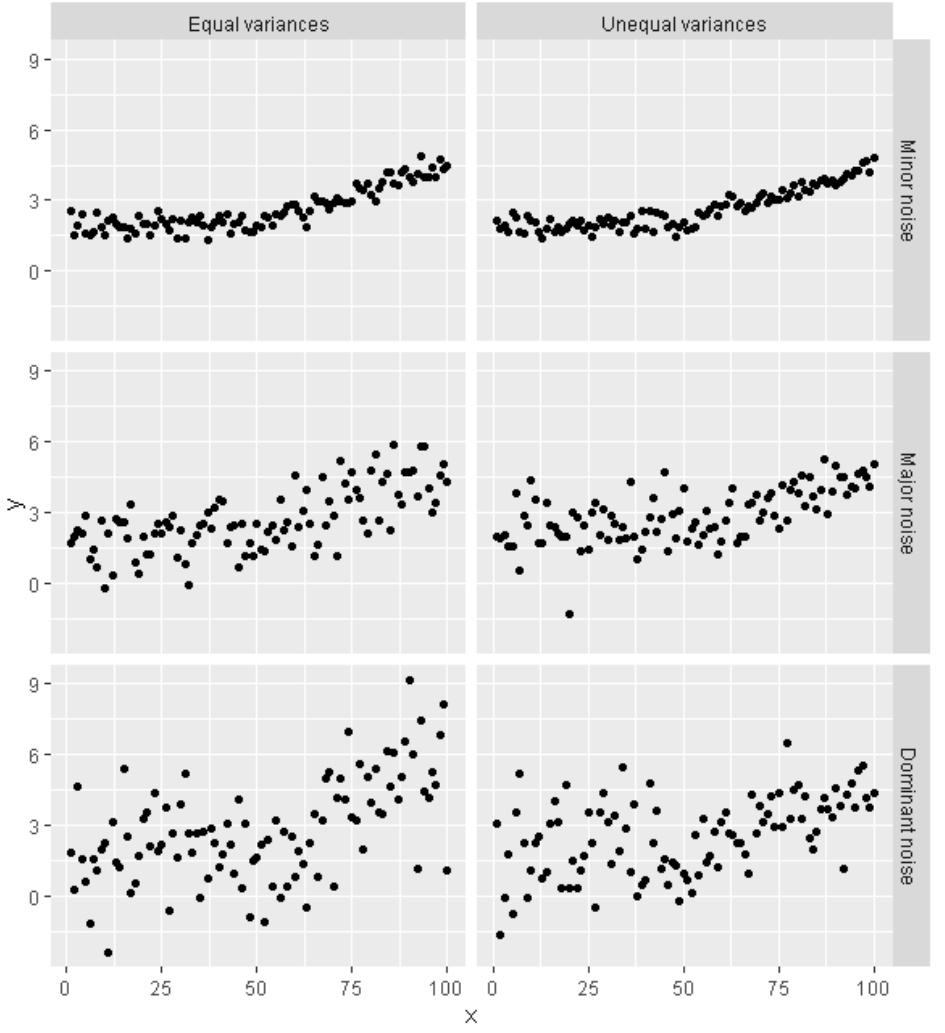

Figure 3. Typical time series with a single changepoint at *chp* = 50 with normal errors (noise: minor – top, major – middle, dominant – bottom, variance: equal – left, unequal – right)

Source: own calculations.

We considered the following steps in the series of computer simulations:

1. We generated $N = 100$ times the datasets according to model (7).
2. We estimated the changepoint with the use of the *permutation* methods for each dataset. For each point $k = 10, 11, \ldots, 90$ variances of parameters of the linear models are estimated based on $N_{perm} = 1000$ permutations of residuals.
3. The changepoint was estimated as a parameter k^* which maximizes Cohen's d size effect as in formula (6).
4. The estimated changepoint was obtained using the *segmented* methods.
5. To compare the two considered methods the standard deviation *SD* and *Bias* were calculated.

4. Results of the Monte Carlo study and discussion

In the Monte Carlo study, the changepoint has been established to $chp = 50$ and the number of replications to $N = 100$. The number of permutations for estimating the standard deviation of the slope coefficients of the regression function was assumed $N_{\mathbf{perm}} = 1000$. Let us denote the estimated changepoint in i-th replication of the model as chp_i. The estimation of standard deviation (*SD*) and *Bias* are also included in the results:

$$SD \approx \sqrt{VAR}, \tag{8}$$

$$Bias \approx \frac{1}{N}\sum_{i=1}^{N} chp_i - chp, \tag{9}$$

where $VAR = \frac{1}{N}\sum_{i=1}^{N}(chp_i - \overline{chp})^2$ and $\overline{chp} = \frac{1}{N}\sum_{i=1}^{N} chp_i$.

Taking into account the lack of change or the occurrence of a variance change and the noise level as minor or dominant, 6 simulation variants were considered (see Fig. 2)

- equal variances and minor noise,
- equal variances and major noise,
- equal variances and dominant noise,
- unequal variances and minor noise,
- unequal variances and major noise,
- unequal variances and dominant noise.

In each case, four types of error distribution were considered in the Monte Carlo study.

4.1. Minor noise

Table 2 presents the values of *SD* and *Bias* of the changepoint estimations by the segmented method and the permutation-based method for the first model considered with four types of error distributions. In all these cases, noticeably smaller (sometimes

by up to 50%) *SDs* were obtained by the permutation-based than with the segmented method. In all cases the values of *Bias* were similar for both methods.

Table 2. Estimated values of SD and Bias of changepoint estimating for minor noise

Errors distributions	Equal variances				Unequal variances			
	Segmented		RESPERM		Segmented		RESPERM	
	SD	Bias	SD	Bias	SD	Bias	SD	Bias
Normal	3.63	0.37	2.26	- 0.14	3.91	- 0.21	2.40	- 0.35
Uniform	2.32	0.16	1.84	- 0.27	2.20	0.00	1.65	- 0.45
Beta (2,2)	2.01	0.06	1.60	- 0.16	1.69	- 2.87	1.65	- 1.76
Beta (2,6)	2.46	0.02	1.76	0.07	1.00	- 1.36	1.41	- 1.13

Source: own calculation in R program.

The top row of Figure 4 shows box-whisker plots for the estimated changepoints for the residuals permutation-based method and the segmented method for the equal variances case. Both methods lead to similar results but the variance of changepoint estimates is much smaller for the permutation-based method than for the segmented method. Maximal errors of changepoint estimation are also greater for the segmented method than for the permutation-based method in each considered case. The advantage of the permutation method is that there are no requirements for the type of error distribution. The bottom row of Figure 4 shows box-whisker plots for the estimated breakpoints for each method for the unequal variance case. The breakpoint estimates from the *permutations-based* method and the *segmented* method led to similar results. For the first three considered error distributions, the variance of the changepoint assessment was smaller for the permutations-based than for the segmented method. Only for the asymmetric beta error distribution, the evaluation variance was smaller for the segmented method. In each case maximal errors of changepoint estimates were greater for the segmented than for the permutations-based method. There was a noticeable estimation bias for both methods in the symmetric beta error distribution. In this case, the bias was greater for the segmented method.

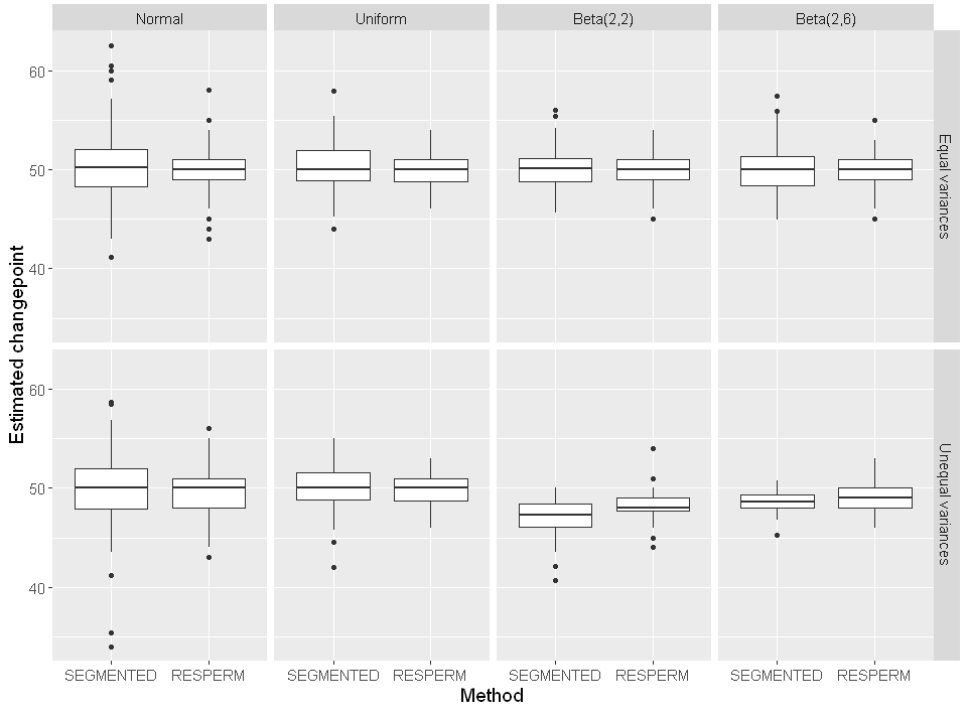

Figure 4. Changepoint estimates for the segmented and permutations -based methods for minor noise and four kinds of distributions of errors. Top: equal variances in both regimes. Bottom: unequal variances.

Source: own calculation in R program.

4.2. Major noise

Table 3 presents *SD* and *Bias* of the changepoint estimation for the segmented and the permutation methods in the cases of equal and unequal variances but at major levels. Four types of error distributions were taken into account as before. In all the error types considered, noticeably smaller errors in the estimation of the changepoint location were obtained using the permutation-based method.

The top row of Figure 5 shows the box-whisker plots for the estimated changepoints when there is major noise for the equal variances case. The mean changepoint estimates with the residuals permutation-based method and the segmented method lead to similar results but the variance is much smaller for the permutation-based method than for the segmented method. The bottom row of Figure 5 shows box-whisker plots for the estimated changepoints for the two methods for the unequal variances case. The variance of the changepoint estimates is consistently larger

for the segmented than for the permutation-based method. Maximal errors of changepoint estimates from the segmented method exceed those from the permutation-based method in each case.

Table 3. *SD* and *Bias* of changepoint estimates for major noise

Errors distributions	Equal variances				Unequal variances			
	Segmented		RESPERM		Segmented		RESPERM	
	SD	Bias	SD	Bias	SD	Bias	SD	Bias
Normal	12.96	0.06	7.86	- 0.63	9.13	- 0.80	6.80	- 1.04
Uniform	11.08	- 0.33	7.71	- 0.10	8.90	- 1.58	5.57	- 1.92
Beta (2,2)	8.07	0.83	4.63	0.16	6.09	- 0.66	3.11	- 1.10
Beta (2,6)	4.09	0.34	2.74	0.21	3.43	- 0.79	1.93	- 0.72

Source: own calculation in R program.

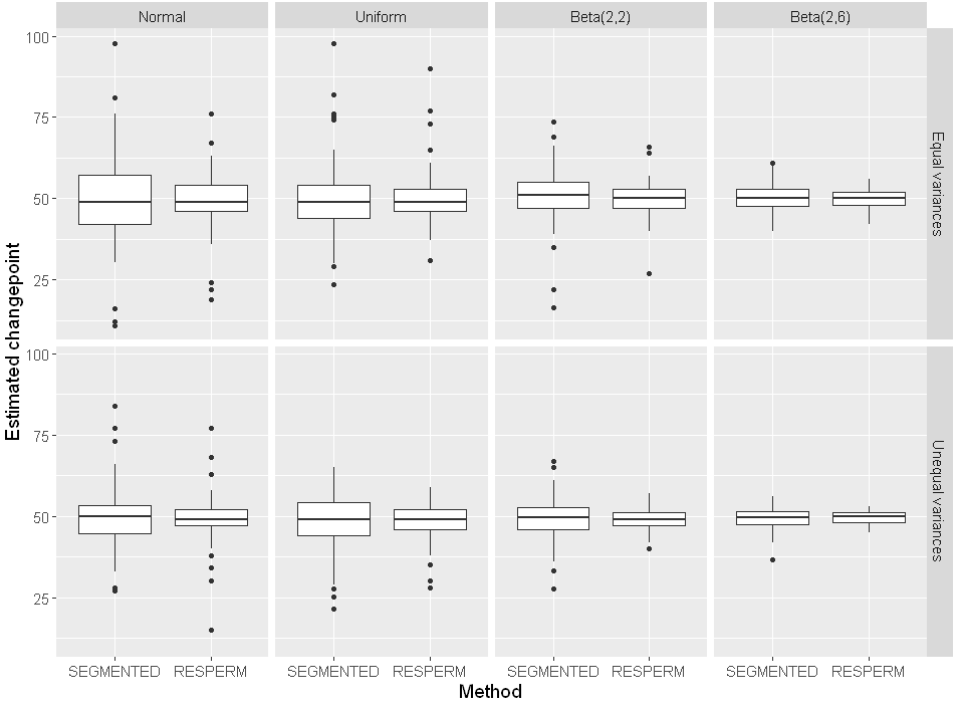

Figure 5. Changepoint estimates for the segmented and permutations-based methods for major noise and four kinds of distributions of errors. Top: equal variances in both regimes. Bottom: unequal variances.

Source: own calculation in R program.

4.3. Dominant noise

Table 4 presents *SD* and *Bias* of the changepoint estimates for the segmented and permutation methods for equal and unequal and dominant noise. In all four error distribution types, changepoint estimation errors were noticeably smaller for the permutation-based method.

Table 4. Estimated values of *SD* and *Bias* of changepoint for dominant noise

Errors distributions	Equal variances				Unequal variances			
	Segmented		RESPERM		Segmented		RESPERM	
	SD	Bias	SD	Bias	SD	Bias	SD	Bias
Normal	20.47	0.51	17.38	0.32	18.41	-1.92	14.66	-2.89
Uniform	19.54	-0.54	15.32	-0.86	16.21	-2.64	13.30	-4.84
Beta (2,2)	15.36	-1.51	10.35	-0.94	12.40	-1.65	8.51	-2.21
Beta (2,6)	8.05	0.90	4.16	-0.27	6.46	-1.28	3.51	-0.90

Source: own calculation in R program.

The top row of Figure 6 shows box-whisker plots for the estimated changepoints for both methods in the case of equal variances. The changepoint estimating with the residuals permutation-based method and the segmented method leads to similar results but the variance of changepoint assessment is much lower for the permutation-based method than for the segmented method. The relative bias of the changepoint assessment in most cases was much less than 2%. For the permutation method with normally distributed errors Average Relative Bias (*ARB*) was 0.64%. The bottom row of Figure 6 shows box-whisker plots for the estimated changepoints for the two methods in the case of unequal variances. The variance of the changepoint estimation was greater for the segmented than for the permutation-based method. Maximal errors of changepoint estimates were larger for the segmented than for the permutation-based method in each case.

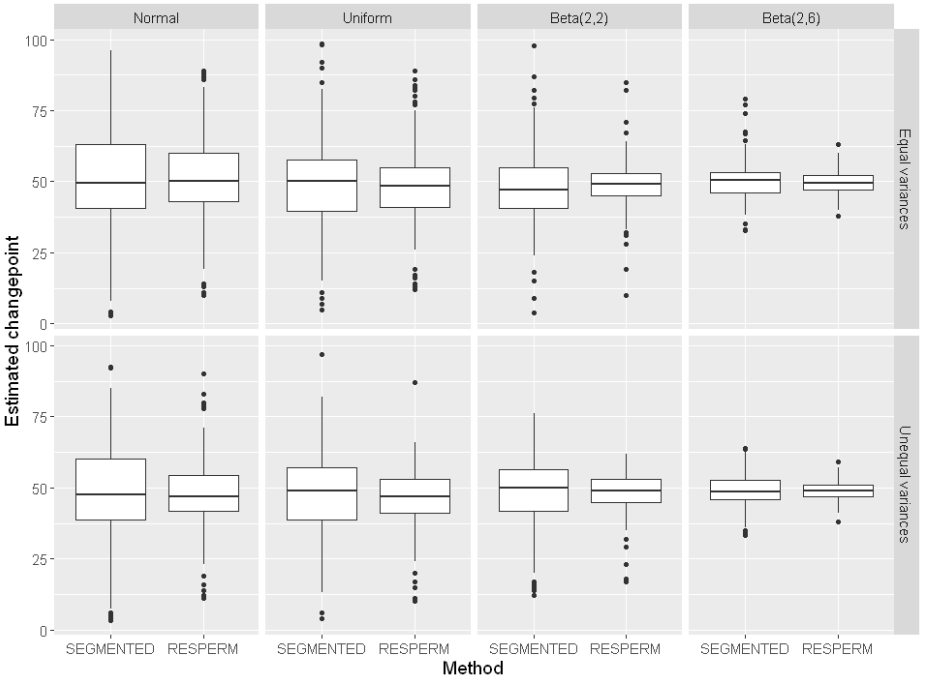

Figure 6. Changepoint estimates for the segmented and permutations-based methods for dominant noise and four kinds of distributions of errors. Top: equal variances in both regimes. Bottom: unequal variances.

Source: own calculation in R program.

5. Sensitivity analysis of the segmented and permutation-based methods

In the previous section, the standard deviation and *Bias* for the assessment of the changepoint location were presented for four types of error distribution: normal, uniform, Beta (2,2), and Beta (2,6). In each of these cases, the variance in the two regimes (before and after the changepoint) was either equal (constant) variances or unequal, that is, it was smaller after the changepoint. All simulations were performed for the changepoint $chp = 50$.

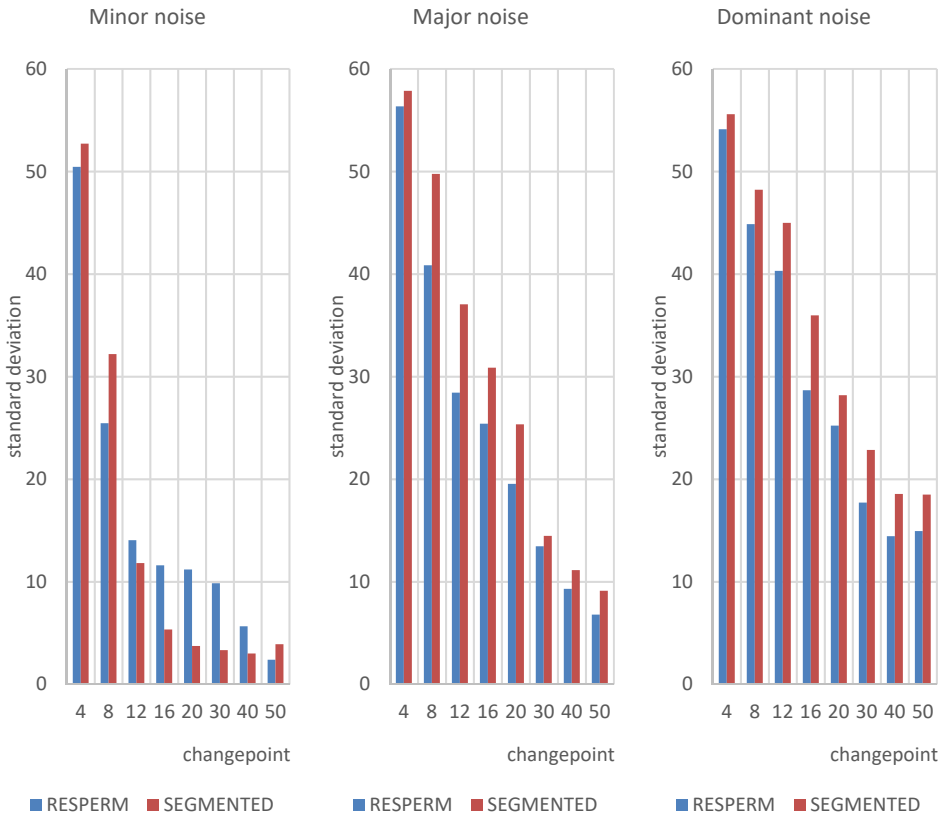

Figure 7. Standard deviation (SD) for RESPERM and SEGMENTED methods for minor, major and dominant noise in the case of normally distributed errors

Source: own calculation in R program.

The results of these simulations showed that for cases with different variance (i.e. reduced in the 2nd part), slightly more precise estimates (i.e. having lower bias) of *chp* were obtained, both for the segmented method and the permutation. Therefore, the following section focuses on time series only with unequal variance. Noticeable biases occurred only in a few cases in both methods. The further study thus considers only a comparative standard deviation (*SD*) analysis. The standard deviation analysis was performed for different changepoints: *chp* = 4, 8, 12, 20, 30, 40, 50. For these changepoints, only errors with normal distribution and diminished variance after the *chp* were considered. Three levels of variance in the distribution of random errors were taken into account: minor, major and dominant errors.

The purpose of this part of the computer simulations was to analyse the sensitivity of the RESPERM method. Sensitivity analysis determines how different values of the independent variable (changepoint) affect a particular dependent variable (estimated changepoint) under a given set of assumptions. In other words, sensitivity analyses examine how different sources of uncertainty in a mathematical model contribute to the overall uncertainty of the model.

It should be noted that in many cases (especially for *chp* = 4, 8, 12) the segmented method did not find the change point, so we had to run this method multiple times until a changepoint was found. The standard deviation of estimation (*SD*) of the changepoint (*chp*) for the above-mentioned cases is shown in Figure 7. In the case of minor error, the segmented method is mostly characterized by a smaller standard deviation than the permutation-based method. However, in the case of greater variance of random errors (minor and dominant), the permutation-based method is characterized by a consistently smaller standard error than the segmented method for each changepoint analysed. Overall, it can be seen that a higher level of random errors leads to a greater standard error in the changepoint assessment, but still with less error for the permutation method.

6. Conclusions

We considered the problem of changepoint detection in linear regression models based on noisy data. This residuals permutation-based method maximizes Cohen's effect size measure *d* with the parameters estimated by the permutation of residuals in the linear model. The residuals permutation-based method was compared in a number of computer simulations. In the simulation study six variants of noise were considered from normal, uniform and two variants of beta distributions together with two cases of equal and unequal variances. Three levels of variance in the distribution of random errors were taken into account: minor, major and dominant errors. The simulations were performed for different locations of changepoint in time series.

The results showed that for cases with different variance (i.e. reduced in the 2nd part), slightly less biased estimates of *chp* are obtained, both for the segmented method and the permutation. In the case of minor errors, that is for relatively clean data, the segmented method in most cases is characterized by a smaller standard deviation than the residuals permutation-based method. For more noisy data, that is in the case of major and dominant greater variance of random errors, the permutation method is superior and characterized by a smaller standard error than segmented for each of the analysed changepoints (i.e. independent of the location of *chp*). In these cases the biases of both methods are comparable. Although not systematically explored,

for early changepoints it was often hard to find such a changepoint with the regimented methods. Note that the RESPERM method was planned to work with 1 regressor only.

The current article presents the results of a simulation experiment designed to study the properties and behaviour of the RESPERM method designed and used earlier on for the purpose of detecting points of interest on the time course of the EEG signal. And it was in the paper Sommer et al. (2022) that the application of the method to real data from an EEG experiment conducted by the authors was presented.

In summary, the present results from the simulation study indicate that the proposed residuals permutation-based method shows a better performance in identifying a changepoint in noisy data and therefore may be recommended in such scenarios. The RESPERM method in such cases is more precise and the loadings of both methods are comparable. For data with minor noise, the results of the two methods are comparable.

References

Aminikhanghahi, S., Cook, D. J. (2017). A Survey of Methods for Time Series Changepoint Detection. *Knowl Inf Syst.,* 2017 May, 51(2), pp. 339–367. doi:10.1007/s10115-016-0987-z.

Applied Regression Analysis, (2023). https://online.stat.psu.edu/stat462/node/185/ (24.07.2023).

Bacon, D. W., Watts, D. G., (1971). Estimating the Transition Between Two Intersecting Straight Lines. *Biometrika*, 58, pp. 525–534.

Chow, G., (1960) Tests of Equality Between Sets of Coefficients in Two Linear Regressions. *Econometrica*, 28, pp. 591–05.

Cohen, J., (1988). Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates, *Publishers*, New York.

Davidson, A. C., Hinkley, D. V., (1997). Bootstrap Methods and Their Application. *Cambridge University Press*, New York.

Julious, S. A., (2001) Inference and Estimation in a Changepoint Regression Problem. *The Statistician*, 50, pp. 51–61.

Lerman, P. M., (1980). Fitting Segmented Regression Models by Grid Search. *Applied Statistics*, 29, pp. 77–84.

Muggeo, V. M. R., (2003). Estimating Regression Models with Unknown Break-Points. *Statistics in Medicine*, 22, pp. 3055–3071.

Muggeo, V. M. R., (2008). Segmented: an R package to fit regression models with broken-line relationships. *R News,* 8/1, pp. 20–25.

Quandt, R. E., (1958). The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes. *Journal of American Statistical Association*, 53, pp. 873–880.

Quandt, R. E., (1960). Tests of the Hypotheses that a Linear Regression System Obeys Two Separate Regimes. *Journal of American Statistical Association* 55, p. 324.

Sheykhfard A., Haghighi F., Nordfiaern T., Soltaninejad M., (2020). Structural equation modelling of potential risk factors for pedestrian accidents in rural and urban roads, *International Journal of Injury Control and Safety Promotion*, 10.1080/17457300.2020.1835991, 28, 1, pp. 46–57.

Sommer, W., Stapor K., Kończak G., Kotowski K., Fabian P., Ochab J., Bereś A., Ślusarczyk G., (2022). Changepoint Detection in Noisy Data Using a Novel Residuals Permutation-Based Method (RESPERM). Benchmarking and Application to Single Trial ERPs, *Brain Sciences*, Vol. 12, iss. 5, nr art. 525.

Truong, Ch., Oudre L., Vayatis N., (2020). Selective Review of Offline Changepoint Detection Methods. *Signal Processing*, Vol. 167, February 2020, 107299.

Ulm, K., (1991). A Statistical Method for Assessing a Threshold in Epidemiological Studies. *Statistics in Medicine*, 10, pp. 341–349.