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Power ratio cum median-based ratio estimator of finite population mean with known population median

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Abstract

The search for an efficient estimator of the finite population mean has been a critical problem to the sample survey research community. This study is motivated by the fact that the conducted literature review showed that no research has developed such an average ratio estimator of the population mean that would utilize both the population and the sample medians of study variable, as well as the Srivastava (1967) estimator at a time. In this paper we proposed the power ratio cum median-based ratio estimator of the finite population mean, which is a function of two ratio estimators in the form of an average. The estimator assumes the population to be homogeneous and skewed. The properties (i.e. the Bias and the Mean Squared Error – MSE) of the proposed estimator were derived alongside its asymptotically optimum MSE. We demonstrated the efficiency of the proposed estimator jointly with its efficiency conditions by comparing it to selected estimators described in the literature. Empirically, a real-life dataset from the literature and a simulation study from two skewed distributions (Gamma and Weibull) were used to examine the efficiency gain. The empirical analysis and simulation study demonstrated that the efficiency gain is significant. Hence, the practical application of the proposed estimator is recommended, especially in socio-economic surveys.

Key words: finite population mean, bias, mean squared error, power estimator, median-based, power ratio.

1. Introduction

Sampling is a technique for selecting a sample or subset of the population to make statistical inference on some characteristics of the whole population. The concept of utilizing means of auxiliary variable at estimation stage of a survey is due to Cochran (1940), the author expressed an estimator for population mean of study variable as

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a function of mean per unit estimator of the study variable and ratio of population to sample means of an auxiliary variable, when the relationship between the study and auxiliary variables is positive. Srivastava (1967) defined yet another estimator of population mean of study variable in the form of ratio using a single auxiliary variable. In the developed estimator, the ratio of population to sample mean of the auxiliary variable is expressed in the form of power of a constant, where the constant is obtained in such a way that the mean squared error of the estimator is minimum. Olkin (1958) discussed the concept of ratio estimator with more than one auxiliary variables. The author defined a bivariate ratio estimator which utilizes two auxiliary variables, the estimator is expressed as a function of two ratio estimators in the form of average. The properties of the estimator were expressed and comparison was made with the estimator with one auxiliary variable. Gupta and Shabbir (2008) defined a general class of ratio-type estimator using weight function and some other known parameters of auxiliary variable, they used three real-life dataset to justify the efficiency gain due to the defined estimator, and observed that the developed estimator has the minimum MSE compared to linear regression estimator.

Recently Subramani (2016) defined an efficient median-based estimator of finite population mean using the median of the study variable. The estimator is a function of mean per unit estimator and ratio of the population to sample medians of the study variable. Subramani's estimator does not utilize any auxiliary parameter from auxiliary variable, but rather utilizes an auxiliary parameter from the same variable. Srija and Subramani (2018) defined a median-based estimators using mean, first and third quartiles of the auxiliary variable.

In the same vein, Abdullahi and Ugwuowo (2020) defined an efficient median-based linear regression estimator for population mean under simple random sampling scheme, assuming the population is homogeneous and skewed. Their estimator is expressed as a function of both mean per unit estimator of the study variable, population and sample medians of both study and auxiliary variables respectively. They discussed the properties of the estimator and justified the efficiency gain using both empirical and simulation studies. It is important to note that the difference between the estimator by Abdullahi and Ugwuowo (2020) and the estimator we proposed in this study is that the former is a regression estimator, which assumes that the regression line between the two variable passes through the origin, while the latter assumes that the regression line between the study variable and auxiliary variable does not pass through the origin, and the correlation between the two variables is positive. The strength of the positive correlation between the two variables determines the efficiency of auxiliary variable based estimators.

The search for efficient estimator of finite population mean has been a critical problem to sample survey research community. This study is motivated by the fact that the conducted literature review showed that no research has developed such an average ratio estimator of the population mean that would utilize both the population and the sample medians of study variable, as well as the Srivastava (1967) estimator at a time.

2. Preliminaries

We assume that the population is finite of size N and a sample of size n is to be selected using simple random sampling scheme. Each unit of the population is identifiable by means of assigning the number to the population units from 1 to N, the numbers assigned are of nominal scale. We start by discussing some existing estimators to be considered in this study.

The existing estimators and their corresponding properties are presented in Table 1.

Table 1: Estimators

No.	Estimators	Bias	MSE
1.	$\bar{y}=\sum_{i=1}^{n}\frac{y_i}{n}$ Mean per unit	0	$\frac{(1-f)}{n}\bar{Y}^2\left\{C_Y^2\right\}$
2.	$\bar{y}_r=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ Cochran (1940)	$\frac{(1-f)}{n}\bar{Y}\left\{C_X^2-\rho C_Y C_X\right\}$	$\frac{(1-f)}{n}\bar{Y}^2\left\{C_Y^2+C_X^2-2\rho C_Y C_X\right\}$
3.	$\bar{y}_{L-R}=\bar{y}+\beta\left(\bar{X}-\bar{x}\right)$ Hansen, Hurwitz, and Madow (1953)	0	$\frac{(1-f)}{n}\bar{Y}^2 C_Y^2\left\{1-\rho^2\right\}$
4.	$\bar{y}_{Srivastava}=\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^p$ Srivastava (1967)	$\left(\frac{1-f}{n}\right)\bar{Y}\left[\frac{p(p-1)}{2}C_X^2+p\rho C_Y C_X\right]$	$\left(\frac{1-f}{n}\right)\bar{Y}^2 C_Y^2\left(1-\rho^2\right)$
5.	$\bar{y}_{S-med-based}=\bar{y}\left(\frac{M}{m}\right)$ Subramani (2016)	$\bar{Y}\left\{C_m^{'2}-C_{ym}^{'}-\frac{Bias(m)}{M}\right\}$	$V(\bar{y})+R^{'2}V(m)-2R^{'}cov(\bar{y},m)$

Where

Notation

X and Y: auxiliary and study variables,

ρ: correlation coefficient between X and Y,

C_Y and C_X: population coefficient of variation of Y and X respectively,

$\bar{Y}$: population mean of X,

$f=n/N$: sampling fraction,

p: any chosen constant, which is defined in Srivastava (1967)

$R^{'}=\frac{\bar{Y}}{M}$, $\quad R=\frac{\bar{Y}}{\bar{X}}$

2.1. Description of the proposed estimator

We defined power ratio cum median-based ratio estimator for population mean under simple random sampling scheme as

$$\bar{y}_{\Pr opose} = \frac{\bar{y}}{2}\left\{\left(\frac{M}{m}\right)+\left(\frac{\bar{X}}{\bar{x}}\right)^{\theta}\right\} \tag{2.1}$$

Where $\bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$ are mean per unit estimators of Y and X, $\bar{X}$ is the population mean X, M and m are the population and sample medians of Y, 'θ' is a real constant to be determined such that the mean squared error of $\bar{y}_{\Pr opose}$ is minimum.

2.2. Properties of the proposed estimator (bias and Mean Squared Error)

The bias and mean squared error (MSE) of the proposed estimator $\bar{y}_{\Pr opose}$ in (2.1) are presented as

Let $J_{\bar{y}} = \frac{(\bar{y}-\bar{Y})}{\bar{Y}}$, $J_{\bar{x}} = \frac{(\bar{x}-\bar{X})}{\bar{X}}$ and $J_m = \frac{(m-M)}{M}$

Such that $E(J_{\bar{y}}) = E(J_{\bar{x}}) = 0$ and $E(J_m) = \frac{bias(m)}{M}$

Where

$E(J_{\bar{y}}^2) = \frac{Var(\bar{y})}{\bar{Y}^2} = \left(\frac{1-f}{n}\right) C_Y^2 = C_y^{!2}$, $\quad E(J_{\bar{x}}^2) = \frac{Var(\bar{x})}{\bar{X}^2} = \left(\frac{1-f}{n}\right) C_X^2 = C_x^{!2}$

$E(J_m^2) = \frac{Var(m)}{M^2} = C_m^{!2}$, $\quad E(J_{\bar{x}}J_{\bar{y}}) = \frac{Cov(\bar{x},\bar{y})}{\bar{X}\bar{Y}} = \left(\frac{1-f}{n}\right)\rho_{XY} C_X C_Y = C_{xy}^{!}$,

$E(J_{\bar{y}}J_m) = \frac{Cov(\bar{y},m)}{\bar{Y}M} = C_{ym}^{!}$, $\quad E(J_m J_{\bar{x}}) = \frac{Cov(\bar{x},m)}{\bar{X}M} = C_{xm}^{!}$

And expanding (2.1) in terms of $J_{\bar{x}}$'s, we have

$$\bar{y}_{\Pr opose} = \frac{\bar{Y}}{2}(1+J_{\bar{y}})\left\{(1+J_m)^{-1} + (1+J_{\bar{x}})^{-\theta}\right\} \tag{2.2}$$

Note:

$$(1+J_m)^{-1} = 1 - J_m + J_m^2,$$

$$(1+J_{\bar{x}})^{-\theta} = 1 - \theta J_{\bar{x}} + \theta(\theta+1)\frac{J_{\bar{x}}^2}{2}$$

We assume that $|J_m|<1$, $|J_{\bar{x}}|<1$ so that the expression, $(1+J_m)^{-1}$, $(1+J_{\bar{x}})^{-\alpha}$ can be expanded to a convergent infinite series using binomial theorem.

$$\bar{y}_{\Pr opose} = \bar{Y}(1+J_{\bar{y}})\left\{\left(1-J_m+J_m^2\right)+\left(1-\theta J_{\bar{x}}+\theta(\theta+1)\frac{J_{\bar{x}}^2}{2}\right)\right\} \tag{2.3}$$

We also assume that the contribution of terms involving powers in J_m, $J_{\bar{y}}$, $J_{\bar{x}}$ higher than the second is negligible, being of order $1/n^{v}$, where $v>1$. Thus, from the above expression we write to the first order of approximation.

$$=\bar{Y}\left\{\begin{array}{l}\left(1-J_m+J_m{}^2+J_{\bar{y}}-J_mJ_{\bar{y}}+J_m{}^2J_{\bar{y}}\right)\\+\left(1-\theta J_{\bar{x}}+\theta(\theta+1)\dfrac{J_{\bar{x}}{}^2}{2}+J_{\bar{y}}-\theta J_{\bar{x}}J_{\bar{y}}+\theta(\theta+1)\dfrac{J_{\bar{x}}{}^2J_{\bar{y}}}{2}\right)\end{array}\right\} \tag{2.4}$$

$$\bar{y}_{\Pr opose}-\bar{Y}=\frac{\bar{Y}}{2}\left\{\begin{array}{l}\left(-J_m+J_m{}^2+J_{\bar{y}}-J_mJ_{\bar{y}}+J_m{}^2J_{\bar{y}}\right)\\+\left(-\theta J_{\bar{x}}+\theta(\theta+1)\dfrac{J_{\bar{x}}{}^2}{2}+J_{\bar{y}}-\theta J_{\bar{x}}J_{\bar{y}}+\theta(\theta+1)\dfrac{J_{\bar{x}}{}^2J_{\bar{y}}}{2}\right)\end{array}\right\} \tag{2.5}$$

Taking the expectation of both sides of (2.5), we obtained the bias of $\left(\bar{y}_{\Pr opose}\right)$ to the first degree of approximation as

$$bias(\bar{y}_{\Pr opose})=\frac{\bar{Y}}{2}\left\{\left(-C'_m+C'^2_m-C'_{my}\right)+\left(\theta(\theta+1)\frac{C'^2_x}{2}-\theta C'_{xy}\right)\right\} \tag{2.6}$$

Squaring both sides of the equation (2.5) and neglecting the terms of $J's$ having power greater than two we have

$$(\bar{y}_{\Pr opose}-\bar{Y})^2=\bar{Y}^2\left\{J_{\bar{y}}{}^2-J_{\bar{y}}J_m-\theta J_{\bar{x}}J_{\bar{y}}+\frac{\theta J_{\bar{x}}J_m}{2}+\frac{J_m{}^2}{4}+\frac{\theta^2J_{\bar{x}}{}^2}{4}\right\} \tag{2.7}$$

Taking the expectation of both sides of (2.7), we get the MSE of $\bar{y}_{\Pr opose}$ as

$$MSE(\bar{y}_{\Pr opose})=\bar{Y}^2\left\{\frac{Var(\bar{y})}{\bar{Y}}-\frac{Cov(m,\bar{y})}{M\bar{Y}}-\theta\frac{Cov(\bar{x},\bar{y})}{\bar{X}\bar{Y}}+\frac{\theta Cov(\bar{x},M)}{2\bar{X}M}+\frac{Var(m)}{4M}+\frac{\theta^2Var(\bar{x})}{4\bar{X}^2}\right\} \tag{2.8}$$

$$MSE(\bar{y}_{\Pr opose})=\bar{Y}^2\left\{C'^2_y-C'_{my}-\theta C'_{xy}+\frac{\theta C'_{xm}}{2}+\frac{C'^2_m}{4}+\frac{\theta^2C'^2_x}{4}\right\} \tag{2.9}$$

The minimum $MSE(\bar{y}_{\Pr opose})$ is obtained for the optimal value of θ which is

$$\frac{\partial MSE(\bar{y}_{\Pr opose})}{\partial\theta}=\left\{\frac{C'_{xm}}{2}-C'_{xy}+\frac{\theta C'_{xx}}{2}\right\}=0 \tag{2.10}$$

$$\Rightarrow\theta=\frac{2C'_{xy}-C'_{xm}}{C'_{xx}} \tag{2.11}$$

Therefore, the minimum MSE of the proposed estimator is obtained by substituting (2.11) into (2.9)

$$MSE(\bar{y}_{\Pr opose})_{\min} = \bar{Y}^2 \left\{ C^{!}_{yy} - C^{!}_{my} + \frac{C^{!}_{mm}}{4} + \frac{C^{!}_{xm} C^{!}_{xy}}{C^{!}_{xx}} - \frac{C^{!\,2}_{xy}}{C^{!}_{xx}} - \frac{\left(C^{!}_{xm}\right)^2}{4C^{!}_{xx}} \right\} \tag{2.12}$$

3. Efficiency comparison of proposed estimator with some selected estimators

3.1. The mean per unit unbiased estimator

Mean per unit estimator in SRSWOR is less efficient than the proposed estimator if $MSE(\bar{y}_{\Pr opose}) < MSE(\bar{y})$, i.e.

$$\Rightarrow C^{!}_{my} + \theta C^{!}_{xy} \le \frac{\theta C^{!}_{xm}}{2} + \frac{C^{!}_{mm}}{4} + \frac{\theta^2 C^{!}_{xx}}{4}$$

3.2. Cochran (1940) traditional ratio estimator

Traditional ratio estimator is less efficient than the proposed estimator if $MSE(\bar{y}_{\Pr opose}) < MSE(\bar{y}_r)$, i.e.

$$\frac{\theta R R' Cov(m,\bar{x})}{2} + \frac{R'^2 Var(m)}{4} - R' Cov(\bar{y},m) \le -2R\,Cov(\bar{y},\bar{x}) + R^2 Var(\bar{x}) + \theta R\,Cov(\bar{y},\bar{x}) - \frac{\theta^2 R^2 Var(\bar{x})}{4}$$

3.3. Hansen, Hurwitz and Madow (1953) linear regression estimator

Linear regression estimator is less efficient than the proposed estimator if $MSE(\bar{y}_{\Pr opose}) \le MSE(\bar{y}_{L-R})$, i.e.

$$\left\{ -R' Cov(\bar{y},m) - \theta R\,Cov(\bar{y},\bar{x}) + \frac{\theta R R' Cov(m,\bar{x})}{2} + \frac{Var(m) R'^2}{4} + \frac{\theta^2 Var(\bar{x}) R^2}{4} \right\} \ge \left(\frac{\left(Cov(\bar{y},\bar{x})\right)^2}{Var(\bar{x})} \right)$$

3.4. Subramani (2016) median ratio estimator

Median-based ratio estimator is less efficient than the proposed estimator if $MSE(\bar{y}_{\Pr opose}) < MSE(\bar{y}_{S-median-based})$, i.e.

$$\frac{3C^{!}_{mm}}{4} - C^{!}_{ym} \ge -\theta C^{!}_{xy} + \frac{\theta C^{!}_{xm}}{2} + \frac{\theta^2 C^{!}_{xx}}{4}$$

4. Numerical Comparison

The merit of the proposed estimator $\bar{y}_{\Pr opose}$ over $\bar{y}$, $\bar{y}_r$ and $\bar{y}_{L-R}$ estimators is presented in this section.

Dataset: The populations considered in this study is a real-life dataset taken from Singh and Chaudhary (1986). The dataset is also used by Srija and Subramani (2018). The area under Wheat cultivation in 1971 is the auxiliary variable while area under

Wheat cultivation in 1974 is the study variable. Table 2 is the summary of the real-life dataset.

Table 2: Summary of the dataset

Parameter		Parameter		Parameter		Parameter	
N	34	R	4.0999	$Cov(\bar{y},m)$	90236.294	C_{ym}	0.1372841
n	3	$R^{!}$	1.1158	$Cov(\bar{y},\bar{x})$	15061.401	C_{yx}	0.0841917
$\bar{Y}$	856.42	$V(\bar{y})$	163356.41	$Cov(\bar{x},m)$	18342.18	C_{xm}	0.1144118
$\bar{X}$	208.88	$V(\bar{x})$	6884.45	C_{yy}	0.222726	$bias(m)$	-19.77774
$\bar{M}$	747.72	$V(m)$	101518.77	C_{xx}	0.1577848	$bias(m)/M$	-0.02576904
M	767.50	ρ	0.4491	C_{mm}	0.1723414		

Percentage Relative Efficiency (PRE)

The Percentage relative efficiency (PRE) of different estimators $T_{.}$ in respect to $\bar{y}_{\text{Propose}}$ is defined as $PRE\left(\bar{y}_{\text{Propose}},T.\right)=\frac{MSE(T.)}{MSE\left(\bar{y}_{\text{Propose}}\right)}\text{X}100$

Table 3 gives the MSE and PRE of the existing and proposed estimators with respect to mean per unit ($\bar{y}$), usual ratio ($\bar{y}_r$) and linear regression ($\bar{y}_{L-R}$) estimators respectively.

Table 3: MSE/Variance of some selected existing estimators and that of proposed estimator

MSE/Variance of some selected existing estimators and that of proposed estimator		PRE with respect to ($\bar{y}$)	PRE with respect to ($\bar{y}_r$)	PRE with to respect ($\bar{y}_{L-R}$)
Estimator				
$\bar{y}$	163356.4	100	<100	<100
$\bar{y}_r$	155583	104.996	100	<100
$\bar{y}_{L-R}$	130408.93	<100	<100	100
$\bar{y}_{\text{Propose}}$	90882.08	179.7455	171.19	143.49

The result from Table 3 reveals that the proposed estimator $\bar{y}_{\text{Propose}}$ has the minimum mean square error compared to some existing estimators and it also shows significant efficiency gain in respect of percentage relative efficiency.

5. Simulation Study

Additionally, a simulation study is conducted to evaluate the effectiveness of the proposed estimator. The variables are created in accordance with Singh and Horn's (1998) definitions, which were also incorporated into Lamichhane, Singh, and

Diawara's work (2017). Firstly, Gamma distribution and secondly Weibull distribution, the population size is N =3003 with 7 varying sample sizes, while the number of trials is 500. Tables 4 and 5 present the PRE of the proposed estimator of population mean with respectto mean per unit estimator, traditional ratio and linear regression estimator for both Weibull and Gamma distributions respectively.

Table 4: PRE of the proposed and some existing estimators (Weibull Distribution)

PRE With respect to	Sample size (n)	Rho=0.30	Rho=0.50	Rho=0.60	Rho=0.75
$\bar{y}$	12	252.8685	249.8534	249.2091	250.5923
	33	249.5238	241.4079	250.3717	262.6104
	45	212.0155	231.8280	229.4620	237.4103
	79	238.2948	232.6957	238.2306	249.4543
	123	215.7753	195.5499	190.7657	199.4062
	202	238.2451	215.6402	209.2263	215.6172
	243	246.2527	232.7251	235.7246	237.7677
$\bar{y}_{L-R}$	12	240.7894	185.9880	153.2106	99.68960
	33	223.0672	176.8050	157.1801	114.94581
	45	194.9318	171.5238	142.4898	97.75636
	79	233.3049	194.2552	172.0632	124.68826
	123	195.1312	140.9331	114.3663	77.80536
	202	211.8818	155.1586	127.2922	88.57750
	243	215.1578	164.3203	141.0363	96.80333

Table 5: PRE of the proposed and some existing estimators (Gamma distribution)

PRE With respect to	Sample size (n)	Rho=0.30	Rho=0.45	Rho=0.75	Rho=0.9
$\bar{y}$	23	670.9549	548.6637	290.2100	114.34680
	43	542.1858	479.4620	228.1377	93.00426
	53	593.0156	495.9020	250.8398	104.42931
	93	500.1355	399.0709	190.4233	81.13561
	103	508.2268	395.2287	207.1991	85.57811
	203	532.8165	403.5352	215.9648	89.56156
	243	551.3976	443.8553	241.6027	98.21637
$\bar{y}_{L-R}$	(n)	Rho=0.30	Rho=0.45	Rho=0.60	Rho=0.75
	23	281.3494	230.0694	183.2459	121.69287
	43	239.7914	212.0506	153.7183	100.89799
	53	239.3716	200.1716	154.8750	101.25188
	93	225.2151	179.7048	138.7745	85.74912
	103	210.9156	164.0211	126.5875	85.98823
	203	236.4502	179.0785	147.3164	95.83958
	234	210.6481	169.5642	139.2650	92.29847

From the simulation study results in Tables 4 and 5, the proposed estimator $\bar{y}_{\Pr opose}$ shows significant efficiency gain in respect to $\bar{y}$ and $\bar{y}_{L-R}$ compared to some existing estimators.

6. Discussion and Conclusion

In this study, an efficient median-based ratio estimator of population mean with known population median was proposed and named Power Ratio Cum Median-Based Ratio. The bias and MSE of the proposed estimator are derived. Comparing the proposed estimator with various other existing estimators in the literature, we showed that it meets the efficiency criteria. Results from both the real-life dataset and the simulation study show efficiency gain for the proposed estimator that incorporates median of study variable, while for the other estimators the result shows efficiency loss. With the significant performance of proposed estimator, which is function of both medians and mean per unit estimator of the study variable and ratio of population to sample means of auxiliary variable, it is revealed that there is a hidden significant relationship that exists between mean and median of the same variable. Hence, the proposed estimator is recommended for the use in practice when the efficiency conditions are satisfied.

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