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Improved estimation of the mean through regressed exponential estimators based on sub-sampling non-respondents

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Abstract

The present study concerns the issue of estimating the population mean and presents novel and improved regressed exponential estimators using different parameters of an auxiliary character based on sub-sampling non-respondents. The bias and mean square error *(MSE)* of the proposed estimators for the most pragmatic simple random sampling without replacement (SRSWOR) scheme have been derived up to the first order of approximation (i.e. the expression containing errors up to the power of two so that the expectation comes only in terms of the mean, variance and covariance). The optimum value of the MSE of the estimators is found, along with the necessary conditions for optimising the *MSE*. The effectiveness of the suggested estimators, outperforming the existing ones in terms of their MSE, has been studied theoretically, while the empirical as well as the simulation studies have confirmed these findings.

Key words: population mean, bias, mean square error, auxiliary character.

1. Introduction

The history of optimal use of auxiliary information to increase the efficiency of the estimators has been established by a variety of research articles in surveys sampling [see Cochran (1940), Tripathi et al. (1994), Khare (2003)]. But practically in a factual scenario, auxiliary information is not only available in the form of a variable but also in the form of an attribute, such as less or more fertility of soil, high or low breed of animals, gender (male or female), tall or short height of person, etc. So, when the auxiliary information is available in the form of attributes, several authors have taken the advantage of point bi-serial correlation coefficient between the study character $'y'$

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and the auxiliary attribute $'\varphi'$ and improvised conventional estimators for estimating the parameters which have been reviewed by Singh et al. (2019).

Recently, using information from auxiliary attributes, Zaman and Kadilar (2019) and Zaman (2020) proposed novel classes of exponential estimators, while Zaman and Kadilar (2021a, b) proposed two phase exponential ratio and product type estimators and class of exponential estimators for estimating population mean. The effectiveness of ratio-type estimators for estimating population has been further improved by Yadav and Zaman (2021) using some conventional and non-conventional parameters.

These days, researchers of various fields are facing problems to reduce non-random bias in the estimation of parameters due to incomplete information on units selected in the sample. One of the main reasons is that nowadays most of the surveys related to different issues of human beings are based on an internet-oriented online program in which respondents are reluctant to reply specially on critical or sensitive matters. In this way, non-response is a massive challenge which creates bias and reduces the exactitude of estimates of parameters. Hansen and Hurwitz (1946) were first to suggest an unbiased estimator by initiating a method of sub-sampling from non-respondents to estimate the population mean.

Following Hansen and Hurwitz's (1946) sub-sampling methodology of non-respondents with known and unknown population means of auxiliary character(s), Rao (1986, 1990), Khare and Srivastava (1993, 1995, 1997, 2000), Khare and Sinha (2009), Singh and Kumar (2009), and Sinha and Kumar (2011, 2014) have made contributions to the estimation of the population by suggesting conventional and alternative ratio, product, regression estimators, generalized and classes of estimators. Furthermore, Khare and Sinha (2002) attempted to estimate the ratio of two population means using an auxiliary character with an unknown population mean. Meanwhile, Sinha and Kumar (2013) and Sinha and Bharti (2021, 2022) suggested some improved estimators using an auxiliary attribute and non-conventional auxiliary parameters to estimate the population mean in the presence of non-response.

Now, we propose a new family of estimators of population mean when the non-response problem occurs – not only in the case of target variable but also in the case of auxiliary attributes expressed usually by relevant categorical variables. It is assumed that an additional feature expressed by a binary variable is investigated and that some part of respondents has not provided some or all data (i.e. that item non-response or unit non-response occur) concerning target or auxiliary variables. The suggested estimators combine a regression estimator with an exponential function of auxiliary information that has two optimizing constants for two distinct non-response scenarios. Their efficiency is verified using empirical data from 1981 Census in India and a simulation study based on some population data in the same country.

2. Preliminary Sample Selection and Literature Review

Consider a finite population of size N from which a simple random sample of size n is taken without replacement. In surveys of human populations, it happens frequently that n_1 of the units respond on the first try to the questions being asked, while the remaining n_2 $(= n - n_1)$ units do not respond at all. Hansen and Hurwitz (1946) considered a double sampling strategy for estimating population mean consisting of the steps outlined when non-response occurs in the initial attempt. A simple random sample of size n is chosen, and the survey is mailed to the sample units. A subsample of size $n_\omega (= n_2\omega^{-1};\ \omega > 1)$ from the n_2 units that did not respond in the initial attempt is then contacted and information is obtained through personal interviews. For the purposes of this procedure, consider a population of size N that is split into two non-overlapping responding (N_1 units) and non-responding (N_2 units) groups with population means of $\bar{Y}_{(1)}$ and $\bar{Y}_{(2)}$ respectively. Although the proportional weights of the response $W_1 = N_1N^{-1}$ and the non-response $W_2 = N_2N^{-1}$ are not known, they can be estimated by $w_1 = n_1n^{-1}$ and $w_2 = n_2n^{-1}$, respectively. On the basis of readily available data for $(n_1 + n_\omega)$ units, Hansen and Hurwitz (1946) proposed an unbiased estimator for estimating the population mean $(\bar{Y} = W_1\bar{Y}_1 + W_2\bar{Y}_2)$ that is given by

$$T_{HH} = \bar{y}^{\#} = w_1\bar{y}_1 + w_2\bar{y}_{(n_\omega)} \tag{1}$$

Its variance up to the first order of approximation $[O(n^{-1})]$ is given by

$$V(T_{HH}) = \bar{Y}^2\left[\pi C_y^2 + \pi^{\#}C_{y(2)}^2\right], \tag{2}$$

where $\pi = (n^{-1} - N^{-1})$, $\pi^{\#} = N_2(\omega - 1)(Nn)^{-1}$, $C_y^2\left(= \frac{S_y^2}{\bar{Y}^2}\right)$ and $C_{y(2)}^2\left(= \frac{S_{y(2)}^2}{\bar{Y}_{(2)}^2}\right)$ are the coefficients of variation while S_y^2 and $S_{y(2)}^2$ are the population mean squares of y for entire and non-responding parts of the population. $\bar{y}_1$ and $\bar{y}_{(n_\omega)}$ are sample means of the study variate depending upon n_1 and n_ω units respectively.

Suppose the population is dichotomous with respect to presence and absence of an attribute $'\varphi'$ which assumes only two values $'1'$ for possessing attribute and $'0'$ otherwise. Let the observations of study character and auxiliary attribute for $i^{th}(i = 1,2,3, \dots, N)$ population unit be denoted by y_i and φ_i.

Let the total number of units possessing the attribute $'\varphi'$ in the population and sample be $T_N = \sum_{i=1}^{N}\varphi_i$ and $T_n = \sum_i^n \varphi_i$ respectively. Let $P\left(= \frac{T_N}{N}\right)$ and $p\left(= \frac{T_n}{n}\right)$ be the proportion of units in the population and sample while $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{y} = \frac{1}{n}\sum_i^n y_i$ be the population mean and sample mean of study variable.

In this manuscript, the unit non-response case is considered, which is the phenomenon described by Bethlehem et al. (2011), in which the questionnaire remains empty for some elements in the sample. Therefore, under the assumption of population division between responding and non-responding groups, let $\bar{P}_{(1)}$ and $\bar{P}_{(2)}$ be the population proportions of the units possessing the attribute for the responding and non-responding groups of the population respectively, even though they are unknown. If the $\bar{p}_1$ and $\bar{p}_{(n_\omega)}$ are the sample proportions of the units possessing φ for the n_1 and n_ω units respectively, then an unbiased estimator for estimating P is given by

$$\bar{p}^{\#} = w_1\bar{p}_1 + w_2\bar{p}_{(n_\omega)} \tag{3}$$

The variance of $\bar{p}^{\#}$ up to $[O(n^{-1})]$ is given by

$$V(\bar{p}^{\#}) = P^2\left[\pi C_p^2 + \pi^{\#} C_{p(2)}^2\right], \tag{4}$$

where $C_p^2 \left(= \frac{S_\varphi^2}{P^2}\right)$ and $C_{p(2)}^2 \left(= \frac{S_{\varphi(2)}^2}{P_{(2)}^2}\right)$ are the coefficients of variation while S_φ^2 and $S_{\varphi(2)}^2$ are the population mean squares of units possessing the attribute φ for entire and non-responding groups of the population.

Under the supposition of unit non-response, Rao (1986) and Khare and Srivastava (1995, 1997, 2000) envisaged ratio, product, and generalized estimators to estimate the mean of the study variable y using the auxiliary variable x. Adopting them, the ratio, product, and generalized estimators are suggested for estimating the population mean $\bar{Y}$ using the known population proportion (P) if non-response only pertains to the study character as follows:

$$T_{r1}^{\#} = \bar{y}^{\#}\frac{P}{p}, \qquad \text{[Ratio estimator]} \tag{5}$$

$$T_{p1}^{\#} = \bar{y}^{\#}\frac{p}{P}, \qquad \text{[Product estimator]} \tag{6}$$

and $$T_{g1}^{\#} = \bar{y}^{\#}\left(\frac{p}{P}\right)^{\gamma_1}, \qquad \text{[Generalized estimator]}, \tag{7}$$

where γ_1 is an optimizing constant for mean square error.

Furthermore, Riaz and Darda (2016) adopted a regression estimator to estimate the population mean using an auxiliary attribute under the non-response on study character with a known population proportion P, which is given as

$$T_{reg1}^{\#} = \bar{y}^{\#} + \beta_1(P - p) \qquad \text{[Regression estimator]}. \tag{8}$$

Under large sample approximation, the $MSEs$ of all the above estimators up to the order of n^{-1} are given by

$$MSE(T_{r1}^{\#}) = \bar{Y}^2\{\pi {C_p}^2 + \left(\pi C_y^2 + \pi^{\#} C_{y(2)}^2\right) - 2\pi\rho_{yp}C_yC_p\}, \tag{9}$$

$$MSE(T_{p1}^{\#}) = \bar{Y}^2\{\pi {C_p}^2 + \left(\pi C_y^2 + \pi^{\#} C_{y(2)}^2\right) + 2\pi\rho_{yp}C_yC_p\}, \tag{10}$$

$$\left[MSE(T_{g1}^{\#})\right]_{min} = V(\bar{y}^{\#}) - \pi\rho_{yp}^2S_y^2 \text{ at } \gamma_{1opt} = -\rho_{yp}\frac{C_y}{C_p} \tag{11}$$

and $$\left[MSE(T_{reg1}^{\#})\right]_{min} = V(\bar{y}^{\#}) - \pi\rho_{yp}^2S_y^2 \tag{12}$$

where ρ_{yp} is the point bi-serial correlation coefficient between y and p.

Advocating the prior discussed notable contributions, the conventional ratio, product, generalized and regression estimators for estimating the population mean with known proportion of auxiliary variable under unit non-response on study as well as auxiliary variates may respectively be adopted and define as

$$T_{r2}^{\#} = \bar{y}^{\#} \frac{P}{p^{\#}}, \tag{13}$$

$$T_{p2}^{\#} = \bar{y}^{\#} \frac{p^{\#}}{P}, \tag{14}$$

$$T_{g2}^{\#} = \bar{y}^{\#} \left(\frac{p^{\#}}{P}\right)^{\gamma_2}, \text{ where } \gamma_2 \text{ is an arbitrary constant,} \tag{15}$$

and $$T_{reg2}^{\#} = \bar{y}^{\#} + \beta_2(P - p^{\#}). \tag{16}$$

The $MSEs$ of the estimators $T_{r2}^{\#}$, $T_{p2}^{\#}$, $T_{reg2}^{\#}$ and $T_{g2}^{\#}$ up to the order of n^{-1} are given as

$$MSE(T_{r2}^{\#}) = \bar{Y}^2\left[\pi\left(C_y^2 + C_p^2 - 2\rho_{yp}C_yC_p\right) + \pi^{\#}\left(C_{y(2)}^2 + C_{p(2)}^2 - 2\rho_{yp(2)}C_{y(2)}C_{p(2)}\right)\right], \tag{17}$$

$$MSE\left(T_{p2}^{\#}\right) = \bar{Y}^2\left[\pi\left(C_y^2 + C_p^2 + 2\rho_{yp}C_yC_p\right) + \pi^{\#}\left(C_{y(2)}^2 + C_{p(2)}^2 + 2\rho_{yp(2)}C_{y(2)}C_{p(2)}\right)\right], \tag{18}$$

$$\left[MSE(T_{g2}^{\#})\right]_{min} = V(\bar{y}^{\#}) - \frac{\left\{\pi\rho_{yp}S_yS_p + \pi^{\#}\rho_{yp(2)}S_{y(2)}S_{p(2)}\right\}^2}{\left\{\pi S_p^2 + \pi^{\#}S_{p(2)}^2\right\}} \tag{19}$$

$$\text{at } (\gamma_2)_{opt} = -\frac{\left\{\pi\rho_{yp}C_yC_p + \pi^{\#}\rho_{yp(2)}C_{y(2)}C_{p(2)}\right\}}{\left\{\pi C_p^2 + \pi^{\#}C_{p(2)}^2\right\}}$$

and $$\left[MSE(T_{reg2}^{\#})\right]_{min} = V(\bar{y}^{\#}) - \frac{\left\{\pi\rho_{yp}S_yS_p + \pi^{\#}\rho_{yp(2)}S_{y(2)}S_{p(2)}\right\}^2}{\left\{\pi S_p^2 + \pi^{\#}S_{p(2)}^2\right\}}. \tag{20}$$

Here, $\rho_{yp(2)}$ is the point bi-serial correlation coefficient between y and p for the non-responding part of the population.

In this sequence, exponential estimators for estimation of population mean using auxiliary attribute have been proposed by Kumar and Kumar (2019) in both the cases of non-response discussed earlier. Exponential ratio, exponential product and generalized estimators for the case when non-response occurs only on study variable are defined as follows:

$$T_{KK1(r)}^{\#} = \bar{y}^{\#}\exp\left(\frac{P-p}{P+p}\right), \tag{21}$$

$$T_{KK1(p)}^{\#} = \bar{y}^{\#}\exp\left(\frac{p-P}{p+P}\right) \tag{22}$$

and $$T_{KK1(g)}^{\#} = \bar{y}^{\#}\exp\left(\alpha_1\frac{P-p}{P+p}\right). \tag{23}$$

The mean square errors of these exponential estimators up to $[O(n^{-1})]$ are derived as

$$MSE\left(T_{KK1(r)}^{\#}\right) = V(\bar{y}^{\#}) + \bar{Y}^2\pi\left\{\frac{C_p^2}{4} - \rho_{yp}C_yC_p\right\} \tag{24}$$

$$MSE\left(T_{KK1(p)}^{\#}\right) = V(\bar{y}^{\#}) + \bar{Y}^2\pi\left\{\frac{C_p^2}{4} + \rho_{yp}C_yC_p\right\} \tag{25}$$

and $$\left[MSE(T_{KK1(g)}^{\#})\right]_{min} = V(\bar{y}^{\#}) - \pi\rho_{yp}^2S_y^2 . \tag{26}$$

Moreover, Kumar and Kumar (2019) suggested the ratio, product, and generalized exponential estimators, which are provided below along with their mean square errors up to $[O(n^{-1})]$ in the case of non-response on both the study variable and the auxiliary attribute:

$$T^{\#}_{KK2(r)} = \bar{y}^{\#}\exp\left(\frac{P-p^{\#}}{P+p^{\#}}\right), \tag{27}$$

$$T^{\#}_{KK2(p)} = \bar{y}^{\#}\exp\left(\frac{p^{\#}-P}{p^{\#}+P}\right) \tag{28}$$

$$T^{\#}_{KK2(g)} = \bar{y}^{\#}\exp\left(\alpha_1\frac{P-p^{\#}}{P+p^{\#}}\right), \tag{29}$$

$$MSE\left(T^{\#}_{KK2(r)}\right) = V(\bar{y}^{\#}) + \bar{Y}^2\left[\pi\left\{\frac{C_p^2}{4} - \rho_{yp}C_yC_p\right\} + \pi^{\#}\left\{\frac{C_{p(2)}^2}{4} - \rho_{yp(2)}C_{y(2)}C_{p(2)}\right\}\right], \tag{30}$$

$$MSE\left(T^{\#}_{KK2(p)}\right) = V(\bar{y}^{\#}) + \bar{Y}^2\left[\pi\left\{\frac{C_p^2}{4} + \rho_{yp}C_yC_p\right\} + \pi^{\#}\left\{\frac{C_{p(2)}^2}{4} + \rho_{yp(2)}C_{y(2)}C_{p(2)}\right\}\right] \tag{31}$$

and $$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = V(\bar{y}^{\#}) - \frac{\left\{\pi\rho_{yp}S_yS_p + \pi^{\#}\rho_{yp(2)}S_{y(2)}S_{p(2)}\right\}^2}{\left\{\pi S_p^2 + \pi^{\#}S_{p(2)}^2\right\}}. \tag{32}$$

The following conclusions have been drawn when comparing the efficacy of the aforementioned distinct estimators in terms of their MSEs:

(i) from (11), (12) and (26)

$$\left[MSE(T^{\#}_{KK1(g)})\right]_{min} = \left[MSE(T^{\#}_{reg1})\right]_{min} = \left[MSE(T^{\#}_{g1})\right]_{min} \tag{33}$$

and (ii) from (19), (20) and (32)

$$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = \left[MSE(T^{\#}_{reg2})\right]_{min} = \left[MSE(T^{\#}_{g2})\right]_{min} \tag{34}$$

3. Proposed Estimators

Influenced by the methodology of Koyuncu (2012) regression-cum-ratio class estimator and Singh and Solanki (2012) generalized class of estimator, novel ratio and product type improved regressed exponential estimators to estimate the population mean using known proportion of the auxiliary variable for two different cases are proposed as follows:

Case I: Unit non-response observed only on study variable - the proposed estimators for this circumstance are as follows:

$$T_{1\,prop}^{\;r} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left[\frac{(\kappa P-\mathcal{L})-(\kappa p-\mathcal{L})}{(\kappa P-\mathcal{L})+(\kappa p-\mathcal{L})}\right] \quad \text{[Ratio type]} \tag{35}$$

and $$T_{1\,prop}^{\;p} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left[\frac{(\kappa p-\mathcal{L})-(\kappa P-\mathcal{L})}{(\kappa p-\mathcal{L})+(\kappa P-\mathcal{L})}\right] \quad \text{[Product type]} \tag{36}$$

where κ and $\mathcal{L}$ are known constants and $\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2$ are the arbitrary constants.

Furthermore, in accordance with Singh and Taylor (2003), Kadilar and Cingi (2004), Singh et al. (2019), certain members of ${T_1}^{r}_{prop}$ and ${T_1}^{p}_{prop}$ are suggested by giving specific values to κ and $\mathcal{L}$, as shown below.

κ, $\mathcal{L}$	**Ratio Type Estimators**	**Product Type Estimators**
$\kappa = 1$, $\mathcal{L} = \rho$	${T_1}^{r(1)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{(P-p)}{P+p-2\rho}\right\}$	${T_1}^{p(1)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{(p-P)}{p+P-2\rho}\right\}$
$\kappa = C_p$, $\mathcal{L} = \rho$	${T_1}^{r(2)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{C_p(P-p)}{P+p-2\rho}\right\}$	${T_1}^{p(2)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{C_p(p-P)}{p+P-2\rho}\right\}$
$\kappa = \beta_1$, $\mathcal{L} = \rho$	${T_1}^{r(3)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{\beta_1(P-p)}{P+p-2\rho}\right\}$	${T_1}^{p(3)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{\beta_1(p-P)}{p+P-2\rho}\right\}$
$\kappa = \beta_2$, $\mathcal{L} = C_p$	${T_1}^{r(4)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{\beta_2(P-p)}{P+p-2C_p}\right\}$	${T_1}^{p(4)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{\beta_2(p-P)}{p+P-2C_p}\right\}$
$\kappa = \beta_2$, $\mathcal{L} = \rho$	${T_1}^{r(5)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{\beta_2(P-p)}{P+p-2\rho}\right\}$	${T_1}^{p(5)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{\beta_2(p-P)}{p+P-2\rho}\right\}$
$\kappa = \beta_2$, $\mathcal{L} = \beta_1$	${T_1}^{r(6)}_{prop} = \mathcal{A}_1\bar{y}^{\#} + \mathcal{B}_1(P-p)\exp\left\{\frac{\beta_2(P-p)}{P+p-2\beta_1}\right\}$	${T_1}^{p(6)}_{prop} = \mathcal{A}_2\bar{y}^{\#} + \mathcal{B}_2(p-P)\exp\left\{\frac{\beta_2(p-P)}{p+P-2\beta_1}\right\}$

The following approximations under large sample have been assumed to calculate $Bias$ and MSE of the proposed estimators:

$$\frac{\bar{y}^{\#}-\bar{Y}}{\bar{Y}} = \varepsilon_0, \qquad \frac{p-P}{P} = \varepsilon_2, \qquad \text{such that } E(\varepsilon_0) = E(\varepsilon_2) = 0$$

and $\quad E(\varepsilon_0^2) = \pi C_y^2 + \pi^{\#} C_{y(2)}^2, \quad E(\varepsilon_2^2) = \pi C_p^2, \qquad E(\varepsilon_0\varepsilon_2) = \pi\rho_{yp}C_yC_p.$

Using these approximations, the estimators given in (35) and (36) are reduced to

$$ {T_1}^{r}_{prop} = \mathcal{A}_1\bar{Y}(1+\varepsilon_0) - \mathcal{B}_1P(\varepsilon_2 - \theta\varepsilon_2^2) \tag{37}$$

and
$$ {T_1}^{p}_{prop} = \mathcal{A}_2\bar{Y}(1+\varepsilon_0) + \mathcal{B}_2P(\varepsilon_2 + \theta\varepsilon_2^2)\,, \tag{38}$$

where $\theta = \frac{\kappa P}{2(\kappa P - \mathcal{L})}$.

Taking expectation on both sides of (37) and (38) and subtracting $\bar{Y}$ from them, the expressions of $Bias$ of ${T_1}^{r}_{prop}$ and ${T_1}^{p}_{prop}$ up to the first order of approximation are as follows:

$$Bias\left({T_1}^{r}_{prop}\right) = (\mathcal{A}_1 - 1)\bar{Y} + \mathcal{B}_1P\theta v_p \tag{39}$$

and
$$Bias\left({T_1}^{p}_{prop}\right) = (\mathcal{A}_2 - 1)\bar{Y} + \mathcal{B}_2P\theta v_p\,, \tag{40}$$

where $v_p = \pi C_p^2$.

The MSE of ${T_1}^{r}_{prop}$ and ${T_1}^{p}_{prop}$ are calculated up to the $[O(n^{-1})]$ as

$$MSE\left({T_1}^{r}_{prop}\right) = E[\{\mathcal{A}_1\bar{Y}(1+\varepsilon_0) - \mathcal{B}_1P(\varepsilon_2 - \theta\varepsilon_2^2)\} - \bar{Y}]^2$$

and
$$MSE\left({T_1}^{p}_{prop}\right) = E[\{\mathcal{A}_2\bar{Y}(1+\varepsilon_0) + \mathcal{B}_2P(\varepsilon_2 + \theta\varepsilon_2^2)\} - \bar{Y}]^2.$$

After simplifying up to the first order of approximation, the expressions of MSE of $T_{1\,prop}^{\,r}$ and $T_{1\,prop}^{\,p}$ are as follows:

$$MSE\left(T_{1\,prop}^{\,r}\right) = (\mathcal{A}_1 - 1)^2\bar{Y}^2 + \mathcal{A}_1^2\bar{Y}^2V_y + \mathcal{B}_1^2P^2v_p + 2(\mathcal{A}_1 - 1)\mathcal{B}_1P\bar{Y}\theta v_p - 2\mathcal{A}_1\mathcal{B}_1P\bar{Y}c_{yp}, \quad (41)$$

and $$MSE\left(T_{1\,prop}^{\,p}\right) = (\mathcal{A}_2 - 1)^2\bar{Y}^2 + \mathcal{A}_2^2\bar{Y}^2V_y + \mathcal{B}_2^2P^2v_p + 2(\mathcal{A}_2 - 1)\mathcal{B}_2P\bar{Y}\theta v_p + 2\mathcal{A}_2\mathcal{B}_2P\bar{Y}c_{yp}, \quad (42)$$

where $V_y = \pi C_y^2 + \pi^{\#}C_{y(2)}^2$, $\quad c_{yp} = \pi\rho_{yp}C_yC_p$.

To obtain the optimum $MSEs$ of $T_{1\,prop}^{\,r}$ and $T_{1\,prop}^{\,p}$, partially differentiating (41) with respect to $(\mathcal{A}_1, \mathcal{B}_1)$ and (42) with respect to $(\mathcal{A}_2, \mathcal{B}_2)$ and equating them to zero, the optimum values of $\mathcal{A}_i, \mathcal{B}_i$; $i = 1, 2$ are

$$\mathcal{A}_{1(o)} = \frac{v_p - \theta^2v_p^2 + \theta v_pc_{yp}}{v_p + v_pV_y - \theta^2v_p^2 + 2\theta v_pc_{yp} - c_{yp}^2},$$

$$\mathcal{B}_{1(o)} = \frac{\bar{Y}(c_{yp} + \theta v_pV_y)}{P(v_p + v_pV_y - \theta^2v_p^2 + 2\theta v_pc_{yp} - c_{yp}^2)},$$

$$\mathcal{A}_{2(o)} = \frac{v_p - \theta^2v_p^2 - \theta v_pc_{yp}}{v_p + v_pV_y - \theta^2v_p^2 - 2\theta v_pc_{yp} - c_{yp}^2}$$

and $$\mathcal{B}_{2(o)} = \frac{\bar{Y}(-c_{yp} + \theta v_pV_y)}{P(v_p + v_pV_y - \theta^2v_p^2 - 2\theta v_pc_{yp} - c_{yp}^2)}.$$

Substituting the values of $\mathcal{A}_{1(o)}$ and $\mathcal{B}_{1(0)}$ in (41) and $\mathcal{A}_{2(o)}$ and $\mathcal{B}_{2(o)}$ in (42), we get the optimum value of MSE of the proposed estimators as

$$\left[MSE\left(T_{1\,prop}^{\,r}\right)\right]_{min} = \frac{\bar{Y}^2\{v_pV_y - c_{yp}^2 - \theta^2v_p^2V_y\}}{v_p + v_pV_y - \theta^2v_p^2 + 2\theta v_pc_{yp} - c_{yp}^2}, \quad (43)$$

and $$\left[MSE\left(T_{1\,prop}^{\,p}\right)\right]_{min} = \frac{\bar{Y}^2\{v_pV_y - c_{yp}^2 - \theta^2v_p^2V_y\}}{v_p + v_pV_y - \theta^2v_p^2 - 2\theta v_pc_{yp} - c_{yp}^2}. \quad (44)$$

Case II: Unit non-response observed on both study and auxiliary variables - the proposed estimators for this occurrence are as follows:

$$T_{2\,prop}^{\,r} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left[\frac{(\kappa P - \mathcal{L}) - (\kappa p^{\#} - \mathcal{L})}{(\kappa P - \mathcal{L}) + (\kappa p^{\#} - \mathcal{L})}\right] \quad \text{[Ratio type]} \quad (45)$$

and $$T_{2\,prop}^{\,p} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left[\frac{(\kappa p^{\#} - \mathcal{L}) - (\kappa P - \mathcal{L})}{(\kappa p^{\#} - \mathcal{L}) + (\kappa P - \mathcal{L})}\right] \quad \text{[Product type]} \quad (46)$$

where κ and $\mathcal{L}$ are known constants and $\mathcal{A}_3, \mathcal{A}_4, \mathcal{B}_3, \mathcal{B}_4$ are the arbitrary constants.

Proceeding in the same manner as for case I, different members of ${T_2}^{r}_{prop}$ and ${T_2}^{p}_{prop}$ have been suggested by assigning different values to κ and $\mathcal{L}$ as

κ, $\mathcal{L}$	**Ratio type Estimators**	**Product type Estimators**
$\kappa = 1$ $\mathcal{L} = \rho$	${T_2}^{r(1)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{(P-p^{\#})}{P+p^{\#}-2\rho}\right\}$	${T_2}^{p(1)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{(p^{\#}-P)}{p^{\#}+P-2\rho}\right\}$
$\kappa = C_p$ $\mathcal{L} = \rho$	${T_2}^{r(2)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{C_p(P-p^{\#})}{C_p(P+p^{\#})-2\rho}\right\}$	${T_2}^{p(2)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{C_p(p^{\#}-P)}{C_p(p^{\#}+P)-2\rho}\right\}$
$\kappa = \beta_1$ $\mathcal{L} = \rho$	${T_2}^{r(3)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{\beta_1(P-p^{\#})}{\beta_1(P+p^{\#})-2\rho}\right\}$	${T_2}^{p(3)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{\beta_1(p^{\#}-P)}{\beta_1(p^{\#}+P)-2\rho}\right\}$
$\kappa = \beta_2$ $\mathcal{L} = C_p$	${T_2}^{r(4)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{\beta_2(P-p^{\#})}{\beta_2(P+p^{\#})-2C_p}\right\}$	${T_2}^{p(4)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{\beta_2(p^{\#}-P)}{\beta_2(p^{\#}+P)-2C_p}\right\}$
$\kappa = \beta_2$ $\mathcal{L} = \rho$	${T_2}^{r(5)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{\beta_2(P-p^{\#})}{\beta_2(P+p^{\#})-2\rho}\right\}$	${T_2}^{p(5)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{\beta_2(p^{\#}-P)}{\beta_2(p^{\#}+P)-2\rho}\right\}$
$\kappa = \beta_2$ $\mathcal{L} = \beta_1$	${T_2}^{r(6)}_{prop} = \mathcal{A}_3\bar{y}^{\#} + \mathcal{B}_3(P - p^{\#})\exp\left\{\frac{\beta_2(P-p^{\#})}{\beta_2(P+p^{\#})-2\beta_1}\right\}$	${T_2}^{p(6)}_{prop} = \mathcal{A}_4\bar{y}^{\#} + \mathcal{B}_4(p^{\#} - P)\exp\left\{\frac{\beta_2(p^{\#}-P)}{\beta_2(p^{\#}+P)-2\beta_1}\right\}$

In continuation to the approximations assumed in Case I, another large sample approximation for proportion of auxiliary variable is considered as

$$\frac{p^{\#}-P}{P} = \varepsilon_1 \text{ such that } E(\varepsilon_1) = 0,$$
$$E(\varepsilon_1^2) = \pi C_p^2 + \pi^{\#} C_{p(2)}^2 \quad \text{and} \quad E(\varepsilon_0\varepsilon_1) = \pi\rho_{yp}C_yC_p + \pi^{\#}\rho_{yp(2)}C_{y(2)}C_{p(2)}.$$

Now, the estimators given in (45) and (46) can be reduced in terms of ε_0 and ε_1 as

$$ {T_2}^{r}_{prop} = \mathcal{A}_3\bar{Y}(1+\varepsilon_0) - \mathcal{B}_3P(\varepsilon_1 - \theta\varepsilon_1^2) \tag{47}$$

and $$ {T_2}^{p}_{prop} = \mathcal{A}_4\bar{Y}(1+\varepsilon_0) + \mathcal{B}_4P(\varepsilon_1 + \theta\varepsilon_1^2) \tag{48}$$

The *bias* and *MSE* of the estimators ${T_2}^{r}_{prop}$ and ${T_2}^{p}_{prop}$ up to $O(n^{-1})$ can be given as follows:

$$Bias\left({T_2}^{r}_{prop}\right) = (\mathcal{A}_3 - 1)\bar{Y} + \mathcal{B}_3P\theta V_p, \tag{49}$$
$$Bias\left({T_2}^{p}_{prop}\right) = (\mathcal{A}_4 - 1)\bar{Y} + \mathcal{B}_4P\theta V_p\ , \tag{50}$$
$$MSE\left({T_2}^{r}_{prop}\right) = (\mathcal{A}_3 - 1)^2\bar{Y}^2 + \mathcal{A}_3^2\bar{Y}^2V_y + \mathcal{B}_3^2P^2V_p$$
$$+2(\mathcal{A}_3 - 1)\mathcal{B}_3P\bar{Y}\theta V_p - 2\mathcal{A}_3\mathcal{B}_3P\bar{Y}C_{yp} \tag{51}$$

and $$MSE\left({T_2}^{p}_{prop}\right) = (\mathcal{A}_4 - 1)^2\bar{Y}^2 + \mathcal{A}_4^2\bar{Y}^2V_y + \mathcal{B}_4^2P^2V_p$$
$$+2(\mathcal{A}_4 - 1)\mathcal{B}_4P\bar{Y}\theta V_p + 2\mathcal{A}_4\mathcal{B}_4P\bar{Y}C_{yp}. \tag{52}$$

where $V_p = \pi C_p^2 + \pi^{\#}C_{p(2)}^2$, $V_y = \pi C_y^2 + \pi^{\#}C_{y(2)}^2$,
$C_{yp} = \pi\rho_{yp}C_yC_p + \pi^{\#}\rho_{yp(2)}C_{y(2)}C_{p(2)}$.

The optimum values of $\mathcal{A}_i$, $\mathcal{B}_i$; $i = 3, 4$ to optimize the MSE of $T_{2prop}^{\ r}$ and $T_{2prop}^{\ p}$, can be obtained by partially differentiating (51) with respect to $(\mathcal{A}_3, \mathcal{B}_3)$ and (52) with respect to $(\mathcal{A}_4, \mathcal{B}_4)$, and equating them to zero we get

$$\mathcal{A}_{3(o)} = \frac{V_p - \theta^2 V_p^2 + \theta V_p C_{yp}}{V_p + V_p V_y - \theta^2 V_p^2 + 2\theta V_p C_{yp} - C_{yp}^2},$$

$$\mathcal{B}_{3(o)} = \frac{\bar{Y}(C_{yp} + \theta V_p V_y)}{P(V_p + V_p V_y - \theta^2 V_p^2 + 2\theta V_p C_{yp} - C_{yp}^2)},$$

$$\mathcal{A}_{4(o)} = \frac{V_p - \theta^2 V_p^2 - \theta V_p C_{yp}}{V_p + V_p V_y - \theta^2 V_p^2 - 2\theta V_p C_{yp} - C_{yp}^2}$$

and $$\mathcal{B}_{4(o)} = \frac{\bar{Y}(-C_{yp} + \theta V_p V_y)}{P(V_p + V_p V_y - \theta^2 V_p^2 - 2\theta V_p C_{yp} - C_{yp}^2)}.$$

Putting the values of $\mathcal{A}_{3(o)}$ and $\mathcal{B}_{3(o)}$ in (51) and $\mathcal{A}_{4(o)}$ and $\mathcal{B}_{4(o)}$ in (52), we get the optimum value of MSEs of the proposed estimators as

$$\left[MSE\left(T_{2prop}^{\ r}\right)\right]_{min} = \frac{\bar{Y}^2\{V_p V_y - C_{yp}^2 - \theta^2 V_p^2 V_y\}}{V_p + V_p V_y - \theta^2 V_p^2 + 2\theta V_p C_{yp} - C_{yp}^2} \tag{53}$$

and $$\left[MSE\left(T_{2prop}^{\ p}\right)\right]_{min} = \frac{\bar{Y}^2\{V_p V_y - C_{yp}^2 - \theta^2 V_p^2 V_y\}}{V_p + V_p V_y - \theta^2 V_p^2 - 2\theta V_p C_{yp} - C_{yp}^2}. \tag{54}$$

It would be remarkable to mention here that the optimum values of the constants $\mathcal{A}_{i(o)}$, $\mathcal{B}_i(o)$; $i = 1,2,3$ & 4 involved in optimizing the MSE of the suggested estimators depend upon unknown population parameters like v_p, c_{yp}, V_y, V_p and C_{yp}, which may be practically obtained from the supposition value based on prior information accessible from past data/pilot survey or replaced with their estimated values [for instance see Reddy (1978) and Srivastava and Jhajj (1983)].

4. Efficiency Comparisons

To show the efficiency of the proposed estimators with respect to the relevant estimators, mathematical conditions are derived by comparing their mean square errors, which are as follows:

(i) From (9), (10), (11), (12) and (2)
$MSE(T_{r1}^{\#}) \le V(T_{HH})$ if $v_p \le 2c_{yp}$, where $v_p = \pi C_p^2$ and $c_{yp} = \pi \rho_{yp} C_y C_p$
$MSE(T_{p1}^{\#}) \le V(T_{HH})$ if $v_p \ge -2c_{yp}$
$\left[MSE(T_{g1}^{\#})\right]_{min} = \left[MSE(T_{reg1}^{\#})\right]_{min} \le V(T_{HH})$ if $c_{yp}^2 \ge 0$, always true.

(ii) From (9), (10) and (12)
$MSE(T_{r1}^{\#}) - \left[MSE(T_{reg1}^{\#})\right]_{min} = \left(v_p - c_{yp}\right)^2 \ge 0$, always true.
$MSE(T_{p1}^{\#}) - \left[MSE(T_{reg1}^{\#})\right]_{min} = \left(v_p + c_{yp}\right)^2 \ge 0$, always true.

Accordingly, using the aforementioned findings with (33), we have

$$\left[MSE(T^{\#}_{KK1(g)})\right]_{min} = \left[MSE(T^{\#}_{g1})\right]_{min} = \left[MSE(T^{\#}_{reg1})\right]_{min} \leq MSE(T^{\#}_{r1})$$

$$\left[MSE(T^{\#}_{KK1(g)})\right]_{min} = \left[MSE(T^{\#}_{g1})\right]_{min} = \left[MSE(T^{\#}_{reg1})\right]_{min} \leq MSE(T^{\#}_{p1}).$$

(iii) Proceeding as in (ii), we have the next two comparisons

From (24) and (33)

$$\left[MSE(T^{\#}_{KK1(g)})\right]_{min} = \left[MSE(T^{\#}_{g1})\right]_{min} = \left[MSE(T^{\#}_{reg1})\right]_{min} \leq MSE\left(T^{\#}_{KK1(r)}\right)$$

as $MSE\left(T^{\#}_{KK1(r)}\right) - \left[MSE\left(T^{\#}_{reg1}\right)\right]_{min} = \left(v_p - 2c_{yp}\right)^2 \geq 0,$ always true.

And from (25) and (33)

$$\left[MSE(T^{\#}_{KK1(g)})\right]_{min} = \left[MSE(T^{\#}_{g1})\right]_{min} = \left[MSE(T^{\#}_{reg1})\right]_{min} \leq MSE\left(T^{\#}_{KK1(p)}\right)$$

as $MSE\left(T^{\#}_{KK1(p)}\right) - \left[MSE\left(T^{\#}_{reg1}\right)\right]_{min} = \left(v_p + 2c_{yp}\right)^2 \geq 0,$ always true.

(iv) From (17), (18), (19), (20) and (2)

$MSE(T^{\#}_{r2}) \leq V(T_{HH})$ if $V_p \leq 2C_{yp}$.

$MSE(T^{\#}_{p2}) \leq V(T_{HH})$ if $V_p \geq -2C_{yp}$.

$\left[MSE\left(T^{\#}_{g2}\right)\right]_{min} = \left[MSE\left(T^{\#}_{reg2}\right)\right]_{min} \leq V(T_{HH})$ if $C^2_{yp} \geq 0,$ always true.

Following a similar path as the comparison in (iii) and (iv), we arrive at the results in (v) and (vi).

(v) From (17) and (34)

$$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = \left[MSE(T^{\#}_{g2})\right]_{min} = \left[MSE(T^{\#}_{reg2})\right]_{min} \leq MSE(T^{\#}_{r2})$$

as $MSE\left(T^{\#}_{r2}\right) - \left[MSE\left(T^{\#}_{reg2}\right)\right]_{min} = \left(V_p - C^2_{yp}\right)^2 \geq 0,$ always true.

And, from (18) and (34)

$$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = \left[MSE(T^{\#}_{g2})\right]_{min} = \left[MSE(T^{\#}_{reg2})\right]_{min} \leq MSE(T^{\#}_{p2})$$

as $MSE\left(T^{\#}_{p2}\right) - \left[MSE\left(T^{\#}_{reg2}\right)\right]_{min} = \left(V_p + C^2_{yp}\right)^2 \geq 0,$ always true.

(vi) From (30) and (34)

$$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = \left[MSE(T^{\#}_{g2})\right]_{min} = \left[MSE(T^{\#}_{reg2})\right]_{min} \leq MSE\left(T^{\#}_{KK2(r)}\right)$$

as $MSE\left(T^{\#}_{KK2(r)}\right) - \left[MSE(T_{reg2}{}^{\#})\right]_{min} = \left(V_p - 2C^2_{yp}\right)^2 \geq 0,$ always true.

From (31) and (34)

$$\left[MSE(T^{\#}_{KK2(g)})\right]_{min} = \left[MSE(T^{\#}_{g2})\right]_{min} = \left[MSE(T^{\#}_{reg2})\right]_{min} \leq MSE\left(T^{\#}_{KK2(p)}\right)$$

as $MSE\left(T^{\#}_{KK2(p)}\right) - \left[MSE(T^{\#}_{reg2})\right]_{min} = \left(V_p + 2C^2_{yp}\right)^2 \geq 0,$ always true.

(vii) From (11), (12) and (43)
$\left[MSE\left(T_{1\,prop}^{\ r}\right)\right]_{min} \leq \left[MSE(T_{g1}^{\#})\right]_{min} = \left[MSE(T_{reg1}^{\#})\right]_{min}$ if $\theta \geq \frac{-v_p V_y + c_{yp}^2}{v_p c_{yp}}$.

(viii) From (11), (12) and (44)
$\left[MSE\left(T_{1\,prop}^{\ p}\right)\right]_{min} \leq \left[MSE(T_{g1}^{\#})\right]_{min} = \left[MSE(T_{reg1}^{\#})\right]_{min}$ if $\theta \geq \frac{v_p V_y - c_{yp}^2}{v_p\, c_{yp}}$.

Following the same steps as in (vii) and (viii), we have:

(ix) From (19), (20) and (53)
$\left[MSE\left(T_{2\,prop}^{\ r}\right)\right]_{min} \leq \left[MSE\left(T_{g2}^{\#}\right)\right]_{min} = \left[MSE\left(T_{reg2}^{\#}\right)\right]_{min}$ if $\theta \geq \frac{-V_p V_y + C_{yp}^2}{V_p\, C_{yp}}$.

(x) From (19), (20) and (54)
$\left[MSE\left(T_{2\,prop}^{\ p}\right)\right]_{min} \leq \left[MSE\left(T_{g2}^{\#}\right)\right]_{min} = \left[MSE\left(T_{reg2}^{\#}\right)\right]_{min}$ if $\theta \geq \frac{V_p V_y - C_{yp}^2}{V_p\, C_{yp}}$.

From these theoretical comparisons, it has been observed that the proposed estimators would be more efficient than the mean unbiased estimator, ratio, product, regression, generalized and classes of estimators under the specified conditions.

A momentous remark in the overall comparison is that all the members of suggested ratio estimators will be more efficient than corresponding product estimators in respective cases if either $\theta \geq 0$ and $C_{yp} \geq 0$ or $\theta \leq 0$ and $C_{yp} \leq 0$ otherwise results will be reverse.

5. Empirical Study

An empirical study using the real data set has been conducted to show evidence of theoretical comparison and result derivation. The purpose of the study's data set is merely to provide illustrations; analysis is not intended for these data.

Data Description-

We have taken into consideration the Census Data -1981, released by the Government of India of Orissa, Police Station - Baria, Tahsil - Champua. This data set includes the number of agricultural laborers and occupied houses in 109 villages under the jurisdiction of the Baria police station. Data representing upper 25% of all villages (i.e. 27 villages) are taken into account for the population's unit non-respondents.

The study variable (y) is the number of agricultural laborers employed in a village; the auxiliary variable (x) is the number of houses that are occupied in that village. Because the number of occupied homes varies from village to village, villages have been classified as either big or small based on the number of occupied houses. In this instance, a village receives the attribute (φ) of a big village if it has more than 70 occupied houses; if not, it is categorized as a small village.

The parameters for this data are:

$N = 109$	$n = 30$	$\bar{Y} = 41.2385$	$P = 0.5229$
$\lambda_n = 0.02416$ $S_y = 46.64779$ $S_{p_{(2)}} = 0.46532$	$W_2 = 0.2477$ $S_p = 0.50178$ $\rho_{yp(2)} = 0.227$ $C_y = 1.1312$	$\bar{Y}_2 = 51.7037$ $\rho_{yp} = 0.426$ $\beta_1 = 2.4103$ $C_p = 0.9596$	$P_{(2)}$=0.7037 $S_{y(2)} = 38.42857$ $\beta_2 = 6.912$

To show the efficiency of the proposed estimators, minimum mean square errors are calculated along with the relevant existing estimators. The percentage relative efficiency (PRE) of the proposed $(prop)$ and relevant existing (ex) estimators with respect to conventional mean per unit unbiased estimator (T_{HH}) is calculated by the formula:

$$PRE(T_{ex/prop}) = \frac{V(T_{HH})}{[MSE(T_{ex/prop})]_{min}} \times 100.$$

For cases I and II under the considered data set, the minimum MSE and PRE of the proposed and existing estimators are obtained and provided in Tables 1 and 2.

Table 1: $\boldsymbol{MSE}$ **and** $\boldsymbol{PRE}$ of estimators for different values of $1/_\omega$ (for Case I)

Estimator	$\boldsymbol{MSE(PRE)}$ **and constants**			
	$1/_\omega = 1/_5$	$1/_\omega = 1/_4$	$1/_\omega = 1/_3$	$1/_\omega = 1/_2$
T_{HH}	101.345(100%)	89.152(100%)	76.959(100%)	64.7656(100%)
$T_{r1}^{\#}$	101.181(100.2%)	88.988(100.2%)	76.795(100.2%)	64.602(100.2%)
$T_{p1}^{\#}$	177.178(57.2%)	164.985(54%)	152.792(50.4%)	140.599(46.1%)
$T_{g1}^{\#}$	91.804(110.4%)	79.611(112%)	67.418(114.2%)	55.225(117.3%)
$T_{reg1}^{\#}$	91.804(110.4%)	79.611(112%)	67.418(114.2%)	55.225(117.3%)
$T_{KK1(r)}^{\#}$	91.804(110.4%)	79.611(112%)	67.418(114.2%)	55.225(117.3%)
$T_{KK1(p)}^{\#}$	129.803(78.1%)	117.611(75.8%)	105.417(73%)	93.224(69.5%)
$T_{KK1(g)}^{\#}$	91.804(110.4%)	79.611(112%)	67.418(114.2%)	55.225(117.3%)
$\boldsymbol{T_{1_{prop}}^{r(1)}}$	**64.254(157.7%)**	**57.631(154.9%)**	**50.944(151.1%)**	**44.191(146.6%)**
$\boldsymbol{T_{1_{prop}}^{r(2)}}$	**61.432(165.0%)**	**55.006(162.1%)**	**48.512(158.6%)**	**41.950(154.4%)**
$T_{1_{prop}}^{r(3)}$	69.672(145.5%)	62.635(142.3%)	55.537(138.6%)	48.376(133.9%)
$T_{1_{prop}}^{r(4)}$	69.748(145.3%)	62.705(142.2%)	55.600(138.4%)	48.432(133.7%)
$T_{1_{prop}}^{r(5)}$	70.006(144.8%)	62.941(141.6%)	55.813(137.9%)	48.623(133.2%)
$T_{1_{prop}}^{r(6)}$	66.501(152.4%)	59.716(149.3%)	52.867(145.6%)	45.955(140.9%)
$T_{1_{prop}}^{p(1)}$	73.686(137.5%)	66.139(134.8%)	58.507(131.5%)	50.788(127.5%)
$T_{1_{prop}}^{p(2)}$	73.958(137.0%)	66.292(134.5%)	58.530(131.5%)	50.667(127.8%)
$T_{1_{prop}}^{p(3)}$	71.991(140.8%)	64.730(137.7%)	57.402(134.1%)	50.008(129.5%)
$T_{1_{prop}}^{p(4)}$	71.943(140.9%)	64.687(137.8%)	57.366(134.2%)	49.977(129.6%)
$T_{1_{prop}}^{p(5)}$	71.771(141.2%)	64.534(138.1%)	57.233(134.5%)	49.865(129.9%)
$T_{1_{prop}}^{p(6)}$	73.261(138.3%)	65.816(135.4%)	58.295(132.0%)	50.698(127.7%)

Source: Own work.

Table 2: $\boldsymbol{MSE}$ and $\boldsymbol{PRE}$ of estimators for different values of $1/_{\omega}$ (for Case II)

Estimator	$\boldsymbol{MSE(PRE)}$ **and constants**			
	$1/_{\omega} = 1/_{5}$	$1/_{\omega} = 1/_{4}$	$1/_{\omega} = 1/_{3}$	$1/_{\omega} = 1/_{2}$
T_{HH}	101.345(100%)	89.1518(100%)	76.959(100%)	64.7656(100%)
$T_{r2}^{\#}$	124.513(81.4%)	106.487(83.7%)	88.461(87.0%)	70.4351(92.0%)
$T_{p2}^{\#}$	242.800(41.7%)	214.202(41.6%)	185.603(41.5%)	157.004(43.2%)
$T_{g2}^{\#}$	90.721(111.7%)	78.966(112.9%)	67.141(114.6%)	55.198(117.3%)
$T_{reg2}^{\#}$	90.721(111.7%)	78.966(112.9%)	67.141(114.6%)	55.198(117.3%)
$T_{KK2(r)}^{\#}$	92.351(109.7%)	80.021(111.4%)	67.692(113.7%)	55.362(117.0%)
$T_{KK2(p)}^{\#}$	151.495(66.9%)	133.879(66.6%)	116.26(66.2%)	98.646(65.6%)
$T_{KK2(g)}^{\#}$	90.721(111.7%)	78.966(112.9%)	67.141(114.6%)	55.198(117.3%)
$\boldsymbol{T_{2_{prop}}^{r(1)}}$	**59.611(170.0%)**	**54.512(163.5%)**	**49.108(156.7%)**	**43.398(149.2%)**
$\boldsymbol{T_{2_{prop}}^{r(2)}}$	**53.758(188.5%)**	**49.918(178.6%)**	**45.542(169.0%)**	**43.668(148.3%)**
$T_{2_{prop}}^{r(3)}$	68.557(147.8%)	61.8956(144%)	55.120(139.6%)	48.214(134.3%)
$T_{2_{prop}}^{r(4)}$	68.666(147.6%)	61.9872(143.6%)	55.196(139.4%)	48.276(134.2%)
$T_{2_{prop}}^{r(5)}$	69.028(146.8%)	62.2948(143.1%)	55.453(138.8%)	48.486(133.6%)
$T_{2_{prop}}^{r(6)}$	63.643(159.2%)	57.7873(154.3%)	51.734(148.8%)	45.472(142.4%)
$T_{2_{prop}}^{p(1)}$	73.346(138.2%)	65.8557(135.8%)	58.310(132.0%)	50.694(127.8%)
$T_{2_{prop}}^{p(2)}$	73.254(138.3%)	65.6986(135.7%)	58.101(132.4%)	50.444(128.4%)
$T_{2_{prop}}^{p(3)}$	71.635(141.5%)	64.5046(138.2%)	58.213(132.2%)	49.981(129.6%)
$T_{2_{prop}}^{p(4)}$	71.576(141.6%)	64.455(138.3%)	57.251(134.4%)	49.949(129.7%)
$T_{2_{prop}}^{p(5)}$	71.364(142.0%)	64.276(138.7%)	57.104(134.8%)	49.840(130.0%)
$T_{2_{prop}}^{p(6)}$	73.042(138.7%)	65.653(135.8%)	58.199(132.2%)	50.663(127.8%)

Source: Own work.

The bias of estimators has been calculated and is displayed in Table 3 in order to better support the comparison regarding the efficiency of the suggested estimators.

Table 3: $\boldsymbol{Bias}$ of estimators for different values of $1/_{\omega}$ (for Case I)

Estimator	**Bias**			
	$1/_{\omega} = 1/_{5}$	$1/_{\omega} = 1/_{4}$	$1/_{\omega} = 1/_{3}$	$1/_{\omega} = 1/_{2}$
$\boldsymbol{T_{1_{prop}}^{r(1)}}$	**−1.558**	**−1.398**	**−1.235**	**−1.072**
$\boldsymbol{T_{1_{prop}}^{r(2)}}$	**−1.490**	**−1.334**	**−1.176**	**−1.017**
$T_{1_{prop}}^{r(3)}$	−1.689	−1.519	−1.347	−1.173
$T_{1_{prop}}^{r(4)}$	−1.691	−1.520	−1.348	−1.174
$T_{1_{prop}}^{r(5)}$	−1.698	−1.526	−1.353	−1.179
$T_{1_{prop}}^{r(6)}$	−1.613	−1.448	−1.282	−1.114
$T_{1_{prop}}^{p(1)}$	−1.787	−1.604	−1.419	−1.232
$T_{1_{prop}}^{p(2)}$	−1.793	−1.608	−1.419	−1.229
$T_{1_{prop}}^{p(3)}$	−1.746	−1.570	−1.392	−1.213
$T_{1_{prop}}^{p(4)}$	−1.744	−1.569	−1.391	−1.212
$T_{1_{prop}}^{p(5)}$	−1.740	−1.565	−1.388	−1.209
$T_{1_{prop}}^{p(6)}$	−1.776	−1.596	−1.414	−1.299

Source: Own work.

Table 4: $\boldsymbol{Bias}$ of estimators for different values of $\mathbf{1}/_{\boldsymbol{\omega}}$ (for Case II)

Estimator	Bias			
	$1/_\omega = 1/_5$	$1/_\omega = 1/_4$	$1/_\omega = 1/_3$	$1/_\omega = 1/_2$
$\boldsymbol{T_2}{}_{\boldsymbol{prop}}^{\boldsymbol{r(1)}}$	$\mathbf{-1.446}$	$\mathbf{-1.322}$	$\mathbf{-1.191}$	$\mathbf{-1.052}$
$\boldsymbol{T_2}{}_{\boldsymbol{prop}}^{\boldsymbol{r(2)}}$	$\mathbf{-1.304}$	$\mathbf{-1.210}$	$\mathbf{-1.104}$	$\mathbf{-0.986}$
$T_2{}_{prop}^{r(3)}$	−1.662	−1.501	−1.337	−1.169
$T_2{}_{prop}^{r(4)}$	−1.665	−1.503	−1.338	−1.171
$T_2{}_{prop}^{r(5)}$	−1.674	−1.511	−1.345	−1.176
$T_2{}_{prop}^{r(6)}$	−1.543	−1.401	−1.254	−1.103
$T_2{}_{prop}^{p(1)}$	−1.778	−1.597	−1.414	−1.229
$T_2{}_{prop}^{p(2)}$	−1.776	−1.593	−1.409	−1.223
$T_2{}_{prop}^{p(3)}$	−1.737	−1.564	−1.389	−1.212
$T_2{}_{prop}^{p(4)}$	−1.736	−1.563	−1.388	−1.211
$T_2{}_{prop}^{p(5)}$	−1.730	−1.559	−1.385	−1.208
$T_2{}_{prop}^{p(6)}$	−1.771	−1.592	−1.411	−1.228

Source: Own work.

Tables 1 and 2 show that in the two distinct cases of non-response, the estimators for regression ($T_{reg1}^{\#}$ and $T_{reg2}^{\#}$), generalized ($T_{g1}^{\#}$ and $T_{g2}^{\#}$), and Kumar and Kumar ($T_{KK1(g)}^{\#}$ and $T_{KK2(g)}^{\#}$) exhibited equal efficiency among all the predominating existing estimators. Tables 1 and 2 also show that in both non-response scenarios, every member of the suggested estimators is more efficient than every member of the pre-defined estimators currently in use at every level of sub-sampling fraction (ω^{-1}). Furthermore, the bias of each member of suggested estimators under the two distinct non-response cases is presented in Tables 3 and 4, where it is evident that the estimators ($T_1{}_{prop}^{r(1)}$ and $T_1{}_{prop}^{r(2)}$) and ($T_2{}_{prop}^{r(1)}$ and $T_2{}_{prop}^{r(2)}$) achieve the lowest bias value among all suggested members of the proposed estimators for all values of ω^{-1}.

6. Simulation Study

A simulation study has been carried out to provide the reliability of the comparison of the efficacy of the suggested estimators by real data. According to the District Census Handbook from 1981, 96 villages in the rural area under Police Station Singur in the District of Hooghly, West Bengal, have been taken into consideration for the simulation study [Source: Khare and Sinha (2011)]. The first 25% of the villages, or 24 villages, have been deemed the population's non-respondent group.

Here, the village's population is used as the study character (y), and its area is used as an auxiliary character (x_1). In this case, if a village has an area larger than 80 hectares, it is given the attribute (φ) of a big area village; otherwise, it is classified as a small area village. The parameters of this study are:

$N = 96$	$n = 40$	$\bar{Y} = 1993.3$	$P = 0.7292$
$\lambda_n = 0.5833$ $S_y = 2308.3484$ $S_{p_{(2)}} = 0.3929$	$W_2 = 0.3958$ $S_p = 0.4467$ $\rho_{yp(2)} = 0.251$ $C_y = 1.1581$	$\bar{Y}_2 = 2394.8$ $\rho_{yp} = 0.341$ $\beta_1 = 1.0642$ $C_p = 0.6126$	$P_{(2)} = 0.8158$ $S_{y(2)} = 2971.6196$ $\beta_2 = 2.0640$

Through the use of R software, a random sample of size 40 is drawn from this population. The estimators' values $\left(T_{ex/prop}\right)$ have been calculated using 3000 replications, and their $MSEs$ have been calculated using the following formula:

$$MSE(T_{ex/prop}) = \frac{1}{3000}\sum_{i}^{3000}\left(T_{ex/prop} - \bar{Y}\right)^2.$$

The minimum MSE and PRE in conjunction with the constants involved in proposed and existing estimators for case I and II are given in Tables 5 and 6 respectively.

Table 5: $\boldsymbol{MSE}$ and $\boldsymbol{PRE}$ of estimators for different values of $\boldsymbol{1/_{\omega}}$ (for Case I)

Estimator	$\boldsymbol{MSE(PRE)}$ **and constants**			
	$1/_{\omega} = 1/_5$	$1/_{\omega} = 1/_4$	$1/_{\omega} = 1/_3$	$1/_{\omega} = 1/_2$
T_{HH}	341376.2(100%)	288854.3(100%)	180370.2(100%)	142235.2(100%)
$T_{r1}^{\#}$	332198.3(102.8%)	276059.6(104.6%)	173184.9(104.2%)	136618.4(100.2%)
$T_{p1}^{\#}$	404432.0(84.4%)	338662.3(85.3%)	229098.1(78.7%)	186186.0(76.4%)
$T_{g1}^{\#}$	330045.3(103.4%)	276379.4(104.5%)	170828.3(105.6%)	134345.2(105.9%)
$T_{reg1}^{\#}$	331010.3(103.1%)	276112.0(104.6%)	171045.0(105.4%)	133980.4(106.2%)
$T_{KK1(r)}^{\#}$	338842.9(100.7%)	276908.7(104.3%)	172245.4(104.7%)	140798.5(101.0%)
$T_{KK1(p)}^{\#}$	437661.8(78.0%)	361235.8(80.0%)	23507.3(76.7%)	182300.0(78.0%)
$T_{KK1(g)}^{\#}$	331235.2(103.1%)	275929.2(104.7%)	170223.0(106.0%)	134031.2(106.1%)
${T_1}_{prop}^{r(1)}$	324032.0(105.4%)	265591.3(108.8%)	169100.8(106.7%)	132441.1(107.4%)
${T_1}_{prop}^{r(2)}$	320142.5(106.6%)	262760.4(109.9%)	169776.6(106.2%)	133163.0(106.8%)
${T_1}_{prop}^{r(3)}$	317643.7(107.5%)	256724.2(108.7%)	169147.1(106.6%)	132469.8(107.4%)
${T_1}_{prop}^{r(4)}$	326438.5(104.6%)	265860.8(108.6%)	169196.6(106.6%)	132500.6(107.4%)
${T_1}_{prop}^{r(5)}$	329327.6(103.6%)	266404.8(108.4%)	169409.5(106.5%)	132636.4(107.2%)
${T_1}_{prop}^{r(6)}$	334343.7(102.1%)	263559.4(109.6%)	168842.9(106.8%)	132364.3(107.5%)
${T_1}_{prop}^{p(1)}$	330475.0(103.3%)	270408.2(106.8%)	171445.0(105.2%)	133951.5(106.2%)
${T_1}_{prop}^{p(2)}$	334892.5(101.9%)	273329.1(105.7%)	173222.7(104.1%)	135141.1(105.2%)
${T_1}_{prop}^{p(3)}$	330489.3(103.3%)	270290.4(106.9%)	171378.9(105.2%)	133908.1(106.2%)
${T_1}_{prop}^{p(4)}$	328424.3(103.9%)	270169.0(106.9%)	171310.9(105.3%)	133862.6(106.2%)
${T_1}_{prop}^{p(5)}$	329475.3(103.6%)	269680.2(107.1%)	171039.5(105.5%)	133682.9(106.4%)
${T_1}_{prop}^{p(6)}$	334325.6(102.1%)	272319.7(106.1%)	172579.0(104.5%)	134709.5(105.6%)

Source: Own work.

Table 6: $\boldsymbol{MSE}$ and $\boldsymbol{PRE}$ of estimators for different values of $1/\omega$ (for Case II)

Estimator	$MSE(PRE)$ and constants			
	$1/\omega = 1/5$	$1/\omega = 1/4$	$1/\omega = 1/3$	$1/\omega = 1/2$
T_{HH}	341376.2(100%)	288854.3(100%)	180370.2(100%)	142235.2(100%)
$T_{r2}^{\#}$	322053.0(106%)	257895.6(112.0%)	167848.1(107.5%)	135439.3(105.0%)
$T_{p2}^{\#}$	474133.6(72.0%)	409292.1(70.6%)	264000.0(68.4%)	205304.1(69.3%)
$T_{g2}^{\#}$	318754.2(107.1%)	258343.2(111.8%)	164000.0(110.0%)	131069.2(108.5%)
$T_{reg2}^{\#}$	318754.2(107.1%)	258343.2(111.8%)	164000.0(110.0%)	131069.2(108.5%)
$T_{KK2(r)}^{\#}$	297043.2(114.9%)	259437.3(111.3%)	165035.0(109.3%)	131980.4(107.8%)
$T_{KK2(p)}^{\#}$	502623.8(67.9%)	431075.2(67.0%)	265045.3(68.0%)	206043.5(69.0%)
$T_{KK2(g)}^{\#}$	313147.8(109.0%)	258354.7(111.8%)	164864.3(109.4%)	131145.0(108.4%)
${T_2}_{prop}^{r(1)}$	305618.8(111.7%)	249933.3(115.6%)	163543.6(110.3%)	128772.2(110.4%)
${T_2}_{prop}^{r(2)}$	304876.2(112.0s%)	277232.5(104.2%)	175000.0(103.0%)	132906.2(107.0%)
${T_2}_{prop}^{r(3)}$	289765.2(117.8%)	250023.1(115.5%)	163588.8(110.3%)	128799.6(110.4%)
${T_2}_{prop}^{r(4)}$	288475.3(118.3%)	250163.1(115.5%)	164000.0(110.0%)	128833.3(110.4%)
${T_2}_{prop}^{r(5)}$	289043.6(118.1%)	251007.9(115.1%)	163871.8(110.1%)	129008.9(110.2%)
${T_2}_{prop}^{r(6)}$	292345.2(116.8%)	259563.5(111.3%)	167000.0(108.0%)	129823.9(109.6%)
${T_2}_{prop}^{p(1)}$	326445.0(104.5%)	260355.6(111.0%)	167000.0(108.0%)	131215.4(108.4%)
${T_2}_{prop}^{p(2)}$	301732.3(112.1%)	269485.9(107.2%)	171469.2(105.2%)	133572.9(106.5%)
${T_2}_{prop}^{p(3)}$	307049.2(111.2%)	260028.0(111.1%)	167000.0(108.0%)	131134.5(108.5%)
${T_2}_{prop}^{p(4)}$	298321.3(114.4%)	259694.5(111.2%)	167188.7(107.9%)	131052.5(108.5%)
${T_2}_{prop}^{p(5)}$	295464.8(115.5%)	258392.9(111.8%)	167000.0(108.0%)	130734.1(108.8%)
${T_2}_{prop}^{p(6)}$	303234.7(112.6%)	266000.0(109.0%)	169968.7(106.1%)	132701.8(107.2%)

Source: Own work.

The simulation study results shown in Tables 5 and 6 validate the theoretical findings about the MSE and estimator efficiency calculated using real data and displayed in Tables 1 and 2. Replications have, however, resulted in nominal changes in the MSE and PRE of the estimators ($T_{g1}^{\#}$, $T_{reg1}^{\#}$ and $T_{KK1(g)}^{\#}$) and ($T_{g2}^{\#}$, $T_{reg2}^{\#}$ and $T_{KK2(g)}^{\#}$).

7. Conclusions

From the analytical study of empirical data, it is clear for both the cases I and II that the proposed estimators are more efficient than all the existing estimators. For case I, when non-response occurs only on study variable, ${T_1}_{prop}^{r(2)}$ and ${T_1}_{prop}^{r(1)}$ are more efficient than all other relevant proposed ratio type estimators while in the category of product type estimators, ${T_1}_{prop}^{p(5)}$ and ${T_1}_{prop}^{p(4)}$ are found to be more efficient. For case II, when non-response occurs on both study variable as well as auxiliary attribute, the proposed estimators $\left(T_{2_{prop}}^{r(2)},\ T_{2_{prop}}^{r(1)} \right)$ and $\left(T_{2_{prop}}^{p(5)},\ T_{2_{prop}}^{p(4)} \right)$ are more efficient among all the members of the proposed ratio and product type estimators respectively. Further, it has also been observed that MSE and PRE both decrease when the values of sub-sampling fraction (ω^{-1}) increase. The reason of the decreasing PRE of the proposed estimators is the faster rate of decrease of the variance of T_{HH} compared to the proposed estimators.

A simulation study confirms and reveals that the efficiency of the proposed estimators is significantly higher than all the relevant estimators at every level of sub-sampling fractions (ω^{-1}), however some estimators have average efficiency as the value of the coefficient of skewness is very small.

Therefore, on the basis of theoretical, empirical and simulation studies, the proposed estimators may be recommended for the improved estimation of mean subject to the condition of availability of the suggested constants of auxiliary variable to increase the precision. It means that one can use any available known parameter of the auxiliary variable among the suggested ones to obtain the efficient estimate, since all members of the proposed estimators are efficient with less $\boldsymbol{MSE}$ compared to all conventional adopted as well as predominating existing estimators.

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