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Multiplicative decompositions of the GEKS-AQI and GEKS-GAQI multilateral price indices

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Abstract

Price index decomposition enables national statistical institutes to break down aggregated price movements into contributions of individual commodities or product groups. In particular, decomposing multilateral indices, which combine many different time comparisons within a time window, can be useful in understanding and interpreting them. This paper discusses multiplicative decompositions of relatively new, quality-adjusted GEKS-type indices: GEKS-AQI and GEKS-GAQI. The effects of these decompositions are demonstrated in two real-world scanner datasets from the food product segment.

Key words: Scanner data, GEKS-type indices, multiplicative index decomposition.

1. Introduction

More and more statistical offices are turning to alternative data sources to measure inflation, in particular scanner and web-scraped data (CPI Manual, 2020). Scanner data mean transaction data that specify turnover and numbers of items sold by barcodes, e.g., the Global Trade Item Number (GTIN), formerly known as the European Article Number (EAN) or Stock Keeping Unit (SKU) codes (Chessa, 2015). Scanner data have numerous advantages compared to traditional survey data collection due to the fact that such datasets are much larger than traditional ones and contain complete transaction information, i.e. information about prices and quantities at the bar-code level. One of the main challenges when using scanner data is the proper choice of the price index formula, which should take into account seasonal products together with new and disappearing goods (Eurostat, 2018).

In the case of dynamic scanner data, that is, with a high product turnover, a multilateral index seems to be the most sensible choice (Eurostat, 2022). A multilateral index is compiled over a given time window composed of $T+1$ successive months (typically $T=12$ or $T=24$), and is therefore capable of capturing, for example, the seasonality of the products. Among multilateral indices, GEKS indices are a very popular and frequently used class of indices (Białek, 2022b, 2025a; Eurostat 2022).

The decomposition of the price index enables national statistical institutes to break down aggregated price movements into contributions of individual commodities or product groups. The decomposition of multilateral indices is much more cumbersome and complicated than the decomposition of bilateral indices (Eurostat, 2022). There are many papers in the literature that discuss the decomposition of bilateral indices, e.g. Vartia (1976), Reinsdorf,

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Diewert, and Ehemann (2002) or Diewert (2002). For example, Balk (2004) discusses additive and multiplicative decompositions of the Fisher price index, and Hallerbach (2005) provided an alternative additive decomposition of this index that can be easily generalized to any bilateral price index formula that satisfies the *linear homogeneity* property. However, there is a lack of papers on the decomposition of multilateral indices. The Time Product Dummy (TPD) method advocated by Krsinich (2016), the GEKS method based on the Törnqvist (1936) price index (GEKS-T or CCDI) - see Gini (1931), Eltetö and Köves (1964), Szulc (1964) or Caves, Christensen, and Diewert (1982), and for the Geary-Khamis method proposed by Geary (1958) and Khamis (1972). Multiplicative decompositions of GEKS-type indices are more natural than additive decompositions because GEKS-type indices are multiplicative in form. This paper discusses the multiplicative decompositions of relatively new quality-adjusted GEKS-type indices: GEKS-AQI and GEKS-GAQI. The effects of these decompositions are demonstrated in two real-world scanner datasets from the food product segment.

The structure of the paper is as follows: Section 2 lists more or less well-known GEKS-type indices, including recent proposals of the GEKS-AQI and GEKS-GAQI index methods, Section 3 provides multiplicative decompositions of these indices along with their normalized versions, Section 4 is an empirical study that compares the decompositions of the GEKS-AQI and GEKS-GAQI indices with decompositions obtained for other GEKS-type indices, and Section 5 lists the most important conclusions of the research carried out.

2. The GEKS-type price index family

Let us denote sets of homogeneous products belonging to the same product group in the months s and t by G_s and G_t, respectively, and let $G_{s,t}$ denote a set of matched products in both moments s and t. Let p_i^τ and q_i^τ denote the price and quantity of the i-th product at the time τ and let $N_{s,t}$ be the number of elements of the set $G_{s,t}$. Let $[0,T]$ denote the time window for price and quantity observations and let $G_{[0,T]}$ denote the set of all available products on sale in this interval.

2.1. The GEKS, CCDI and GEKS-W indices

The GEKS method for making international index number comparisons was proposed by Gini (1931), although it was derived in a different manner by Eltetö and Köves (1964) and Szulc (1964). The GEKS price index between months 0 and t is an unweighted geometric mean of $T+1$ ratios of bilateral price indices $P^{\tau,t}$ and $P^{\tau,0}$, which are based on the same price index formula. The bilateral price index formula should satisfy the time reversal test, i.e., it should satisfy the condition $P^{a,b} \cdot P^{b,a} = 1$. Typically, the GEKS method uses the superlative Fisher (1922) price index, resulting in the following formula:

$$P_{GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_F^{\tau,t}}{P_F^{\tau,0}} \right)^{\frac{1}{T+1}}, \tag{1}$$

where $P_F^{\tau,t}$ denotes the Fisher price index calculated for products from the set $G_{\tau,t}$.

Feenstra, Ma, and Rao (2009) and de Haan and van der Grient (2011) suggested that

the Törnqvist (1936) price index formula could be used instead of the Fisher price index in the Gini methodology. In Caves, Christensen, and Diewert (1982), the GEKS index is used with the Törnqvist index as a base formula in the context of making quantity comparisons across production units (the CCD method). Inklaar and Diewert (2016) extended the CCD methodology to making price comparisons across production units. Consequently, in Diewert and Fox (2018), the multilateral price comparison method involving the GEKS method based on the Törnqvist price index is called the CCDI method. The GEKS index based on the superlative Walsh (1901) index, which we will refer to as GEKS-W, is also often considered.

2.2. The GEKS-L and GEKS-GL indices

The multilateral indices presented in Section 2.1, although they have good axiomatic properties, do not satisfy the restrictive *identity test* (Eurostat, 2022). This axiom is particularly important in the case of seasonal products, as it requires the price index to have a value of one if the prices of products in the current period return to their base values (CPI Manual, 2004).

New GEKS-type price indices have recently appeared in the literature that challenge traditional assumptions associated with the underlying index found in the body of the GEKS index. These novel GEKS-type indices are based neither on a superlative price index nor on an index that meets the *time reversal test*. In Białek (2022a) and Białek (2022b), a general class of such indices (denoted by GS-GEKS) is proposed, and two special cases, i.e., the GEKS-L and GEKS-GL index. The GEKS-L index is based on the Laspeyres index, while the GEKS-GL index is based on the geometric Laspeyres index. Please note that these indices meet the *identity test* (Białek, 2022b). The identity test is a fundamental requirement for bilateral price indices and is included in all axiomatic systems of minimum requirements for price indices (ILO, 2004). Although this test is more restrictive in the context of multilateral indices, it nonetheless appears to be a natural property to require. In practice, it implies that if the prices of all products return to their initial levels after a certain period, the index should also return to a value of one. This situation may occur, for example, in the case of seasonal products. If prices revert to their original levels while the price index deviates from unity, this would, in practice, imply a biased measurement of inflation.

For example, the GEKS-L index can be expressed using the bilateral Laspeyres index (P_L) as follows:

$$P_{GEKS-L}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_L^{\tau,t}}{P_L^{\tau,0}} \right)^{\frac{1}{T+1}}. \tag{2}$$

The formulation of the GEKS-GL index is analogous, except that it employs the geometric Laspeyres index in place of the conventional Laspeyres price index within its underlying structure.

2.3. The GEKS-AQI and GEKS-GAQI indices

Two new multilateral indices, which superficially resemble the GEKS index, were proposed in Białek (2023) (building upon discussions in Białek (2022a) and Białek and Pawelec (2025)). However, their base index structure differs completely from the established

convention of using superlative indices. Moreover, calculating the base index requires *quality adjusting*, which, in turn, is more like the Geary-Khamis index idea (Geary 1958; Khamis 1972). These proposed indices represent a hybrid approach, bridging the gap between the quality-adjusted unit value methods (Chessa, Verburg, and Willenborg, 2017) and the GEKS method. These indices are based on *asynchronous quality-adjusted unit value* (AQUV) proposed in Białek (2023). We will not discuss here the details related to AQUV, which assumes that the consumer response to price changes can be delayed (von Auer, 2019) and accelerated (Białek, 2022a). Since the rest of this paper will focus on the decomposition of these indices, their mathematical formulas are provided below:

$$P_{GEKS-AQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{\sum_{i \in G_{\tau,t}} w_i^Q(\tau,t) \frac{p_i^t}{p_i^\tau}}{\sum_{i \in G_{\tau,0}} w_i^Q(\tau,0) \frac{p_i^0}{p_i^\tau}} \right)^{\frac{1}{T+1}}, \tag{3}$$

$$P_{GEKS-GAQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{\prod_{i \in G_{\tau,t}} \left(\frac{p_i^t}{p_i^\tau} \right)^{w_i^Q(\tau,t)}}{\prod_{i \in G_{\tau,0}} \left(\frac{p_i^0}{p_i^\tau} \right)^{w_i^Q(\tau,0)}} \right)^{\frac{1}{T+1}}, \tag{4}$$

where

$$w_i^Q(\tau,t) = \frac{v_i q_i^\tau}{\sum_{k \in G_{\tau,t}} v_k q_k^\tau}, \tag{5}$$

and a set of factors $v = \{v_i : i \in G_t\}$ is defined as follows (Lamboray 2017; van Loon and Roels 2018):

$$v_i = \frac{\sum_{t=0}^{T} p_i^t q_i^t}{\sum_{t=0}^{T} q_i^t}. \tag{6}$$

Please note that, as in the Geary-Khamis method, prices p_i^τ of different products $i \in G_\tau$ in month τ are transformed into "quality-adjusted prices" $\frac{p_i^\tau}{v_i}$, and quantities q_i^τ are converted into "common units" $v_i q_i^\tau$ using a set of factors $v = \{v_i : i \in G_\tau\}$. Please also note that $w_i^Q(\tau,t)$ can play a role of weights for relative prices $\frac{p_i^t}{p_i^\tau}$ since $\sum_{i \in G_{\tau,t}} w_i^Q(\tau,t) = 1$. It should be added that the GEKS-AQI and GEKS-GAQI indices also satisfy the *identity test* (Białek, 2023).

3. Multiplicative decompositions of the GEKS-type indices

As mentioned above, Webster and Tarnow-Mordi (2019) provided decompositions for three multilateral methods: the Time Product Dummy (TPD), the GEKS method based on the Törnqvist price index (GEKS-T or CCDI), and for the Geary-Khamis method. Bialek (2025b) provided multiplicative decompositions for several GEKS-type indices, including the GEKS, GEKS-W, GEKS-L, and GEKS-GL indices. Sections 3.1 and 3.2 present multiplicative decompositions for the GEKS-AQI and GEKS-GAQI indices.

Note that it is not easy to compare decompositions determined for different index formulas, since the contribution of a given commodity is not normalized by the index value. Assume that for the fixed base period and the current period, we want to compare decompositions of price indices $P_1, P_2, ..., P_N$. Let us denote the multiplicative contributions of n commodities determined for the P_k index formula as $P_k^1, P_k^2, ..., P_k^n$, i.e. we have $P_k = \prod_{i=1}^{n} P_k^i$

for $k \in \{1,2,...,N\}$. Białek (2025b) proposed the *normalized commodity contributions* in the following form:

$$\tilde{P}_k^i = \frac{P_k^i}{\sqrt[n]{P_k}} \quad i = 1,2,...,n, \tag{7}$$

which are normalized to one, that is, we have $\prod_{i=1}^n \tilde{P}_k^i = 1$ for each $k \in \{1,2,...,N\}$. These normalized multiplicative contributions allow for a comparison of commodity impacts on index values across commodities and index formulas. Białek (2025b) also proposed *relative impacts* of individual commodities:

$$\tilde{R}_k^i = (\tilde{P}_k^i - 1)100\%, \quad i = 1,2,...,n, \tag{8}$$

which determines the direction of the influence of commodities on the value of the price index. Positive values of the *relative impact* of the commodity mean that the presence of this commodity in the CPI basket generates inflation, while its negative values lead to the conclusion of generated deflation. Measures (7) and (8) will be used in our empirical study to compare the decompositions of the multilateral indices discussed (Section 4).

Before presenting the multiplicative decompositions of the GEKS-AQI and GEKS-GAQI, it should be noted that these essentially consist of three stages: 1) an initial multiplicative decomposition of the base index (AQI and GAQI respectively); 2) the transformation of variables determining product prices and their weights, in order to avoid technical problems during decomposition associated with mismatched or disappearing products; 3) the final decomposition that requires the replacement of iterators with respect to time and available products.

3.1. Multiplicative decomposition of the GEKS-AQI index

The asynchronous quality-adjusted unit value index (P_{AQI}), which is found in the numerator and denominator of the formula (3), is a weighted arithmetic mean of relative prices, that is:

$$P_{AQI}^{\tau,t} = \sum_{i \in G_{\tau,t}} w_i^Q(\tau,t) \frac{p_i^t}{p_i^\tau}, \tag{9}$$

where $w_i^Q(\tau,t)$ is defined in (5). Balk (2008) showed that any weighted arithmetic mean of relative prices could also be written as a weighted geometric mean of relative prices. Following his method, we can express the AQI price index as a weighted geometric mean of relative prices, i.e.:

$$P_{AQI}^{\tau,t} = \prod_{i \in G_{\tau,t}} \left(\frac{p_i^t}{p_i^\tau}\right)^{w_i^{AQI}(\tau,t)}, \tag{10}$$

where

$$w_i^{AQI}(\tau,t) = \frac{w_i^Q(\tau) L(p_i^t/p_i^\tau, P_{AQI}^{\tau,t})}{\sum_{j \in G_{\tau,t}} w_j^Q(\tau) L(p_j^t/p_j^\tau, P_{AQI}^{\tau,t})}, \tag{11}$$

$L(x,y)$ denotes the logarithmic mean of x and y and $P_{AQI}^{\tau,t}$ can be computed using (9). Let

us now define:

$$\hat{p}_i^{\tau} = \begin{cases} p_i^{\tau}, & \text{if } i \in G_{\tau} \\ 1, & \text{otherwise} \end{cases} \tag{12}$$

and

$$\hat{w}_i^{AQI}(\tau,t) = \begin{cases} w_i^{AQI}(\tau,t), & \text{if } i \in G_{\tau,t} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

It should be noted that variables $\hat{p}_i^{\tau}$ and $\hat{w}_i^{AQI}(\tau,t)$ are technical adjustments to variables p_i^{τ} and $w_i^{AQI}(\tau,t)$, designed to avoid problems arising from decomposition involving missing prices or mismatched products. Note also that using the symbols (12) and (13), we obtain

$$\begin{aligned} &\prod_{i\in G_{[0,T]}} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)} = \\ &= \prod_{i\in G_{\tau}\setminus G_t} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)} \prod_{i\in G_{\tau,t}} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)} \prod_{i\in G_t\setminus G_{\tau}} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)} \prod_{i\in (G_{\tau}\cup G_t)'} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)} = \\ &= \prod_{i\in G_{\tau}\setminus G_t} \left(\frac{1}{p_i^{\tau}}\right)^0 \prod_{i\in G_{\tau,t}} \left(\frac{p_i^t}{p_i^{\tau}}\right)^{w_i^{AQI}(\tau,t)} \prod_{i\in G_t\setminus G_{\tau}} \left(\frac{p_i^t}{1}\right)^0 \prod_{i\in (G_{\tau}\cup G_t)'} \left(\frac{1}{1}\right)^0 = \prod_{i\in G_{\tau,t}} \left(\frac{p_i^t}{p_i^{\tau}}\right)^{w_i^{AQI}(\tau,t)}. \end{aligned} \tag{14}$$

By analogy, it can be shown that

$$\prod_{i\in G_{[0,T]}} \left(\frac{\hat{p}_i^0}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,0)} = \prod_{i\in G_{\tau,0}} \left(\frac{p_i^0}{p_i^{\tau}}\right)^{w_i^{AQI}(\tau,0)}. \tag{15}$$

As a consequence of (3), (14) and (15), we obtain that

$$\begin{aligned} P_{GEKS-AQI}^{0,t} &= \prod_{\tau=0}^{T} \left(\frac{\prod_{i\in G_{[0,T]}} \left(\frac{\hat{p}_i^t}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,t)}}{\prod_{i\in G_{[0,T]}} \left(\frac{\hat{p}_i^0}{\hat{p}_i^{\tau}}\right)^{\hat{w}_i^{AQI}(\tau,0)}} \right)^{\frac{1}{T+1}} = \\ &= \left\{ \prod_{\tau=0}^{T} \prod_{i\in G_{[0,T]}} \frac{(\hat{p}_i^t)^{\hat{w}_i^{AQI}(\tau,t)}}{(\hat{p}_i^0)^{\hat{w}_i^{AQI}(\tau,0)}} (\hat{p}_i^{\tau})^{\hat{w}_i^{AQI}(\tau,0)-\hat{w}_i^{AQI}(\tau,t)} \right\}^{\frac{1}{T+1}}, \end{aligned} \tag{16}$$

which, due to the iterator relative to i independent of τ, leads to the following decomposition of the GEKS-AQI index:

$$P_{GEKS-AQI}^{0,t} = \prod_{i\in G_{[0,T]}} P_{GEKS-AQI}^{0,t}(i), \tag{17}$$

where

$$P_{GEKS-AQI}^{0,t}(i) = \frac{(\hat{p}_i^t)^{\hat{w}_i^{AQI}(*,t)}}{(\hat{p}_i^0)^{\hat{w}_i^{AQI}(*,0)}} \prod_{\tau=0}^{T} (\hat{p}_i^{\tau})^{\frac{\hat{w}_i^{AQI}(\tau,0)-\hat{w}_i^{AQI}(\tau,t)}{T+1}} \tag{18}$$

and

$$\hat{w}_i^{AQI}(*,0) = \frac{1}{T+1}\sum_{\tau=0}^{T}\hat{w}_i^{AQI}(\tau,0), \quad \hat{w}_i^{AQI}(*,t) = \frac{1}{T+1}\sum_{\tau=0}^{T}\hat{w}_i^{AQI}(\tau,t). \tag{19}$$

3.2. Multiplicative decomposition of the GEKS-GAQI index

The asynchronous quality-adjusted price index (P_{GAQI}), which is found in the numerator and denominator of the formula (4), is a weighted geometric mean of relative prices, that is:

$$P_{GAQI}^{\tau,t} = \prod_{i\in G_{\tau,t}} \left(\frac{p_i^t}{p_i^\tau}\right)^{w_i^Q(\tau,t)}. \tag{20}$$

As a result, the multiplicative decomposition of the GEKS-GAQI index is much simpler than the analogous decomposition of the GEKS-AQI index. To obtain this decomposition, let us first introduce the notation:

$$\hat{w}_i^{GAQI}(\tau,t) = \begin{cases} w_i^Q(\tau,t), & \text{if } i \in G_{\tau,t} \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Similarly to Section 3.1, we can easily obtain the following multiplicative decomposition of the GEKS-GAQI index:

$$P_{GEKS-GAQI}^{0,t} = \prod_{i\in G_{[0,T]}} P_{GEKS-GAQI}^{0,t}(i), \tag{22}$$

where

$$P_{GEKS-GAQI}^{0,t}(i) = \frac{(\hat{p}_i^t)^{\hat{w}_i^{GAQI}(*,t)}}{(\hat{p}_i^0)^{\hat{w}_i^{GAQI}(*,0)}} \prod_{\tau=0}^{T}(\hat{p}_i^\tau)^{\frac{\hat{w}_i^{GAQI}(\tau,0)-\hat{w}_i^{GAQI}(\tau,t)}{T+1}} \tag{23}$$

and

$$\hat{w}_i^{GAQI}(*,0) = \frac{1}{T+1}\sum_{\tau=0}^{T}\hat{w}_i^{GAQI}(\tau,0), \quad \hat{w}_i^{GAQI}(*,t) = \frac{1}{T+1}\sum_{\tau=0}^{T}\hat{w}_i^{GAQI}(\tau,t). \tag{24}$$

4. Empirical illustration

In the empirical study presented, multiplicative contributions for commodities, their normalized versions (7), and relative impacts of products (8) on the GEKS-type indices considered were determined. We used scanner data from one retail chain in Poland, that is, monthly data on baked goods (6-digit COICOP level: 011131) sold in more than 500 outlets during the period from December 2023 to December 2024. Two data aggregation levels were used for index calculations, that is, the GTIN level and the 7-digit COICOP level. To be more precise: the 7-digit COICOP level means that the homogeneous product is defined one level lower than the 6-digit COICOP level, i.e., we then have broadly defined baked goods (e.g., we consider wheat roll, graham bread, wheat and rye bread, and wholemeal rye bread).

Before calculating the price indices, the datasets were carefully prepared. The product classification was performed using the `data_selecting()` and `data_classification()` functions from the `PriceIndices` R package (Białek, 2021). The first function required manual preparation of dictionaries of keywords and phrases that identified individual product groups. The second function was used for previously unclassified products and required manual preparation of learning samples. The classification itself was based on machine learning using random trees and the XGBoost algorithm (Tianqi and Carlo, 2016). Next, the product matching was carried out based on the available GTIN (Global Trade Item Number) bar codes and product labels (the `data_selecting()` function). The multiplicative GEKS-type index decompositions were obtained by using the `m_decomposition()` function. Section 4.1 presents results for the lowest data aggregation level (GTIN), while Section 4.2 provides index decompositions for the COICOP 6 data aggregation level.

4.1. GTIN data aggregation level

Figure 1 presents two violin plots with normalized multiplicative contributions for all GTINs and relative impacts of the corresponding baked goods on the GEKS-type indices considered. In the empirical illustration, December 2023 is the base period, December 2024 is the current period, and a 13-month time window is considered.

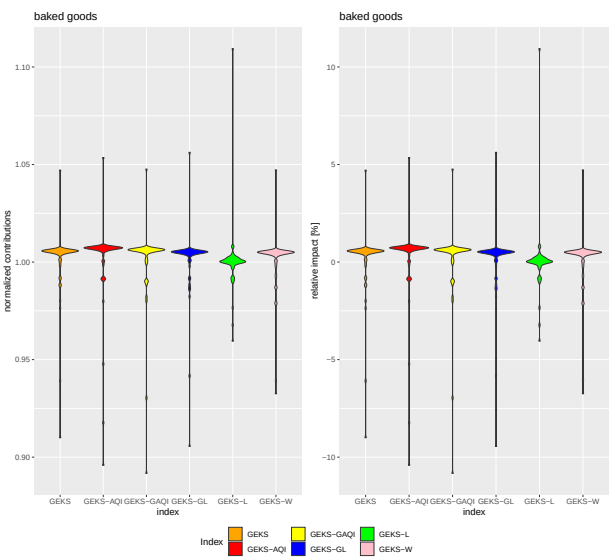


Figure 1. Comparison of relative commodity impacts for selected GEKS-type indices: coffee products

Figure 2 shows the normalized multiplicative contributions for GTINs of four homogeneous subgroups of baked goods. Figure 3 presents the relative impacts of baked goods on the GEKS-type indices for these four subgroups separately. The general observation is

that the contributions assigned to the wheat roll, the graham bread, and the wheat and rye bread products have an asymmetrical distribution, with most of these contributions having small positive values, while individual products generated large negative contribution values. Contributions and relative product impacts computed for wholemeal rye bread have an almost symmetric distribution.

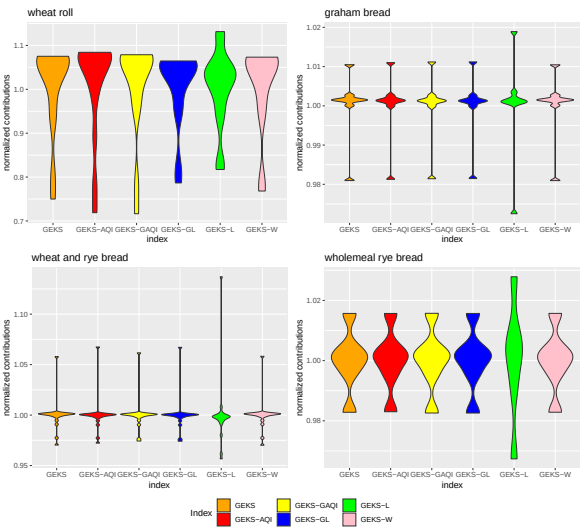


Figure 2. Comparison of relative commodity impacts for selected GEKS-type indices: coffee products

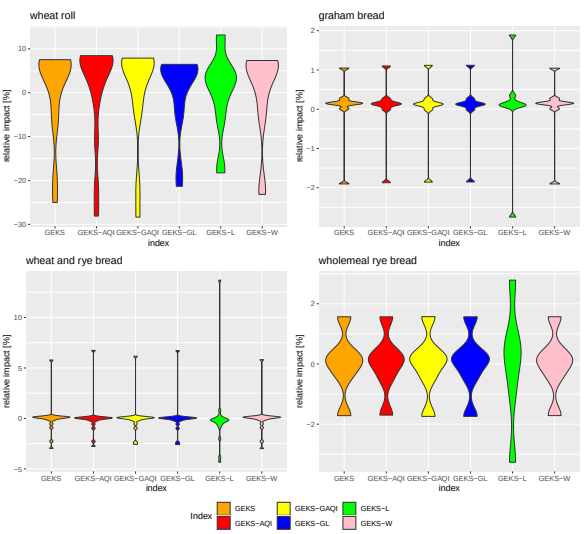


Figure 3. Comparison of relative commodity impacts for selected GEKS-type indices: coffee products

Consequently, the distribution of the normalized contributions of individual commodities computed for all baked goods is also asymmetric (Fig. 1), with a predominance of small positive values. Nevertheless, across the entire set of baked goods, we observe longer left tails of the distribution of contributions (sometimes indicating values even lower than 0.9), which consequently leads to deflation in the period analyzed (see Table 1 in Section 4.2, where the values of all GEKS-type indices are less than one).

It can be seen that both the normalized contributions of individual commodities and the relative impact of the product on the price indices are quite similar between the GEKS-type indices, except for the values determined for the GEKS-L index. The normalized product contributions determined for the GEKS-AQI and GEKS-GAQI indices appear to be slightly higher for the wheat roll with a longer left tail of the distribution (compared to the rest of categories of baked goods).

4.2. 7-digit COICOP data aggregation level

In this section, a homogeneous product is defined much more broadly than at the GTIN level, that is, the identification of products here is done at the level of baked goods grades (local 7-digit COICOP data aggregation level). To be more precise, let us add that we consider four broadly defined products: wheat roll, graham bread, wheat and rye bread, and wholemeal rye bread.

Table 1 presents the multiplicative decompositions of the GEKS-type indices discussed for the four products mentioned above. Table 2 provides normalized versions of these decompositions, while Table 3 additionally presents the relative impacts of individual products on the index values. Normalized contributions and the corresponding relative product impacts allow comparison of the decompositions of the GEKS-AQI and GEKS-GAQI indices with the decompositions of other GEKS-type indices.

Table 1. Multiplicative decompositions of the discussed GEKS-type indices for baked goods at the 7-digit COICOP data aggregation level

product	GEKS	GEKS_W	GEKS_L	GEKS_GL	GEKS_AQI	GEKS_GAQI
wheat roll	0.841871	0.851567	0.907957	0.856777	0.841091	0.843632
graham bread	0.999510	0.999495	1.003126	0.999486	0.999502	0.999496
wheat and rye bread	1.031856	1.032322	1.086617	1.026912	1.028287	1.026109
wholemeal rye bread	1.000072	1.000069	1.001724	1.000161	1.000131	1.000157
index value	0.868327	0.878707	0.991392	0.879524	0.864565	0.865358

Table 2. Normalized multiplicative decompositions of the discussed GEKS-type indices for baked goods at the 7-digit COICOP data aggregation level

product	GEKS	GEKS_W	GEKS_L	GEKS_GL	GEKS_AQI	GEKS_GAQI
wheat roll	0.872117	0.879544	0.909922	0.884720	0.872255	0.874690
graham bread	1.035420	1.032332	1.005296	1.032084	1.036536	1.036292
wheat and rye bread	1.068928	1.066238	1.088968	1.060404	1.066387	1.063884
wholemeal rye bread	1.036002	1.032925	1.003891	1.032781	1.037188	1.036978

Table 3. Relative impacts of individual products on the discussed GEKS-type indices at the 7-digit COICOP data aggregation level

product	GEKS	GEKS_W	GEKS_L	GEKS_GL	GEKS_AQI	GEKS_GAQI
wheat roll	-12.788291	-12.045556	-9.007810	-11.528019	-12.774516	-12.530992
graham bread	3.541963	3.233217	0.529647	3.208358	3.653586	3.629169
wheat and rye bread	6.892768	6.623756	8.896760	6.040397	6.638674	6.388437
wholemeal rye bread	3.600174	3.292497	0.389148	3.278051	3.718815	3.697755

As in the case of the GTIN code level analysis, it appears that the decomposition of the GEKS-L index is the most atypical among all decompositions for a more broadly defined product. It seems that the decompositions of the GEKS-AQI and GEKS-GAQI indices are the most similar to the decomposition of the GEKS index (see Tables 2 and 3). The relative impacts for all GEKS-type indices, presented in Table 3, are consistent, that is, all decompositions indicated that: a) the changes in the prices of wheat rolls were deflationary; b) the changes in the prices of other baked goods were inflationary, with the changes in the prices of wheat and rye bread having the strongest impact on the increase in the price index.

5. Conclusions

This paper fills a gap in the literature by proposing multiplicative decompositions of the recently proposed multilateral GEKS-AQI and GEKS-GAQI indices. Multiplicative decompositions of the GEKS-AQI and GEKS-GAQI indices, like other decompositions of multilateral indices (including also GEKS-type indices), allow for the assessment of the impact of price fluctuations of individual products on the final value of the price index. This can be done at any level of aggregation, so it is possible to assess the impact of both a single GTIN code and a homogeneous subgroup of products from a given elementary group. Moreover, the normalized decompositions allow for interindex comparisons, so they can explain the mutual approximations of multilateral indices.

The first general conclusion from the empirical analyzes is that, with the exception of the GEKS-L index, the multiplicative decompositions of the GEKS-AQI and GEKS-GAQI indices give values similar to the decompositions of other GEKS-type indices at the GTIN code level. The second general conclusion is that the differences between the normalized commodity contributions and the corresponding relative commodity impacts determined for the indices under consideration are clearly greater for broadly defined products than for the GTIN level. In particular, the analysis performed at the 7-digit COICOP level provides the observation that the decompositions of the GEKS-AQI and GEKS-GAQI indices are the most similar to the decomposition of the GEKS index. This observation may suggest a mutual approximation of these indices but requires further empirical research or formal mathematical proof.

It should be added that the paper considers the full 13-month time window, with no comparative analysis of contributions when using index extension methods. Although an issue related to the impact of the length of the time window was not addressed in the paper, one would expect it to be relevant. This is because, intuitively, as the length of the time window increases, the number of products included in the contribution estimation procedure

also increases. Since at lower levels of data aggregation (particularly at the GTIN level) there is a high turnover of products and high variability in prices and quantities, a longer time window generally introduces even more of this variability into the analysis. Consequently, one would expect that as the length of the time window increases, the differences between the determined shares become larger. However, a thorough examination of this impact should be supported by a larger empirical study. This is a direction for possible further research by the author on the contributions of individual commodities.

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